Infinite-dimensional decentralized damping control of large-scale manipulators with hydraulic actuation

Infinite-Dimensional Decentralized Damping Control of Large-Scale Manipulators with Hydraulic Actuation ⋆

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Abstract

The control design for decentralized active damping of large-scale manipulators with hydraulic actuation is considered in a distributed-parameter framework. The concepts of modern light-weight construction enable the production of machines like mobile concrete pumps or maritime crane systems with extended operating range and less static load. However, due to the reduced weight the elasticity of the construction elements has a significant influence on the dynamic behavior of the boom. In this paper, a modular decentralized control strategy is presented and the asymptotic stability of the closed-loop system is rigorously proven in the infinite-dimensional setting. The proposed damping control strategy features a robust behavior since it is independent of the number and pose of the boom segments and of the exact knowledge of the system parameters. At the end, the practical implementation of the control strategy is discussed and validated by means of measurements on an industrial mobile concrete pump with four joints and an operating range of about 40 meters.

Key words: flexible link manipulator; hydraulic actuators; concrete pump; feedback control; distributed-parameter systems; passivity; Euler-Bernoulli beams.

1 Introduction

Modern large-scale manipulators such as mobile concrete pumps have a significantly reduced weight compared to classical designs. This is mainly caused by new materials and techniques in construction and production and enables a higher operating range as well as reduced static load. Since at the same time the stiffness of these systems is reduced, they are prone to vibrations, which make the operation more difficult and may lead to the accelerated fatigue of the construction material. For this reason, the development of modern control strategies for active vibration damping or trajectory planning is a topic of current research.

In the literature, many contributions to the modeling of flexible multi-body systems can be found. The existing methods are well developed and are presented in several text books, e.g., [2], [18] and many others. The modeling of flexible structures in general leads to a mathematical description in form of partial differential equations, which are also referred to as distributed-parameter systems. Most of the investigations related to control strategies for flexible multi-body systems deal with electromechanical actuation. In this context, comprehensive literature can be found, which in part is summarized for example in [20] and [21]. For such systems, usually a cascaded control structure with fast current controllers in the innermost control loop justify the assumption that the joint torques serve as control inputs to the system. In contrast, for large-scale manipulators like mobile concrete pumps, hydraulic actuators comprising hydraulic cylinders and valves are commonly used. Their dynamic behavior and the nonlinear characteristics have to be considered in the controller design. The combination of flexible multi-body systems and hydraulic actuators has been studied, e.g., in [6,11]. Therein the modeling of the flexible structure is typically based on a finite-dimensional approximation of the beam deflection. This approach has the advantage that the equations of motion can be derived in a straightforward way by means of computer algebra programs. However, the distributed-parameter nature of the system is lost in the finite-dimensional model. In [23], a control design considering the infinite-dimensional model of a flexible turntable ladder is presented. However, the considered system can be described by only a single rotating beam and the proposed approach is not designed for a multi-body system.

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This paper is motivated by the desire to design a modular advanced damping control strategy for modern mobile concrete pumps as depicted in Figure 1. In particular in view of the practical relevance, we strive for a control strategy that is independent of the number and pose of the boom segments and does not rely on the exact knowledge of the physical parameters. Basically, the design of the damping controller is based on the linearization of the distributed-parameter system around an arbitrary operating point. This is justified by the fact that the movement of the boom, which is manually controlled by the machine operator, is rather slow.

Fig. 1. Mobile concrete pump with four joints.

The control design for distributed-parameter systems is commonly classified into two systematic approaches: In the *early lumping* approach, the distributed-parameter system is first approximated by a finite-dimensional model. Based on this approximation, a controller is developed utilizing well-established design techniques for finite-dimensional systems. However, neglecting the distributed-parameter nature can cause a reduced control performance or even the destabilization of the system due to the well known *spillover*-effects [1]. On the other hand, in the *late lumping* approach the infinite-dimensional system dynamics are explicitly taken into account in the controller design, see, e.g., [4,14,17]. In particular, methods for vibration damping, which *passivate* the closed-loop system, are quite effective, see, e.g., [14]. In this context, the proportional output feedback ensures the dissipativity of the closed-loop system in the case of a so-called *actuator-sensor-collocation*. Here, the control inputs and the output variables build a dual pair of power variables. Numerous results are available for single rotating and clamped beams, see, e.g., [9], [13], [19] and [14]. In particular, in [13] and [14] the feedback of the beam flection in combination with a velocity controlled servo motor is analyzed. Therein, the exponential stability of the closed-loop system is shown. Since it is well known that a velocity control for a hydraulic cylinder piston can be simply realized by means of a servo compensation, this is of special interest in the considered application.

For the damping control of flexible structures with more than one beam, only a few results are available in the *late lumping* setting. The main reason for this is that the derivation of the associated, in general highly nonlinear, partial differential equations is rather complex. An example is given in [22], where the nonlinear infinite-dimensional model of a flexible robot with two beams is considered. In [10], a general approach for the infinite-dimensional modeling and analysis of dynamic elastic multi-link structures is given. Furthermore, the control of networks of serially connected Euler-Bernoulli beams is studied in several papers, e.g., in [3] and [16]. However, these contributions deal with control inputs given by forces and torques, which is not suitable for the systems under consideration.

In this paper, the linearized equations of motion for a planar manipulator composed of a finite number of linked Euler-Bernoulli beams in an arbitrary pose are systematically derived. In order to constrain the complexity of the calculation, the linearization of the system is performed at an early stage. For this purpose, a first order approximation is employed for the rotation matrices used for the description of the kinematic relations. Furthermore, it is assumed that underlying velocity controllers for the joint motion based on a servo compensation for the hydraulic actuators are implemented and thus the joint angle velocities can be considered as control inputs to the system. Application of Hamilton's principle yields the linearized partial differential equations describing the motion of the structure. In order to render the closed-loop system passive, the temporal behavior of the overall energy stored in the system is analyzed. With this, a control law is proposed, which ensures the asymptotic stability of the closed-loop system. The feasibility of the proposed control approach is demonstrated by means of measurement results for an industrial mobile concrete pump.

The paper is organized as follows: In Section 2, the essential steps for the derivation of the mathematical model are presented. The control law, detailed in Section 3, is based on the equations of motion and the energy stored in the overall system. In Section 4, the proof of the asymptotic stability of the closed-loop system is given. The practical implementation as a decentralized modular control law and its validation by means of measurement results on a mobile concrete pump are the content of Section 5 and 6. Finally, a short conclusion is given in Section 7. The Appendices A and B contain some derivations needed for the stability proof. Note that preliminary results motivated by a single rotational beam involving pure simulation results without a stability proof are provided in [7].

2 Energy-based mathematical modeling

In the following, the essential steps for the derivation of the linearized equations of motion of a planar manipulator composed of N linked Euler-Bernoulli beams with lengths L_n, $n = 1, \ldots, N$ are presented. As shown in Figure 2 for a two-link manipulator, the overall motion of the system

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is described with respect to the inertial frame $0_0x_0z_0$. As mentioned before, the system will be linearized around an arbitrary equilibrium. For this reason, the local coordinate frames $0_nx_nz_n$ with $n > 0$ are defined by means of the operating points ψ_n^d of the joint angles. With this, the degrees of freedom are given by the beam deflections $w_n(x_n)$, which, in addition to the elastic deformations of the beams, include the deviations of the joint angles from their operating points ψ_n^d. The joint movement can be controlled by means of hydraulic actuators comprising differential cylinders and servo valves.

Remark 1 *As already discussed in the introduction, the joint angle velocities are supposed to be imposed by underlying controllers such that they may be considered as control inputs to the system.*

In order to derive the equations of motion, the extended Hamilton's principle, see, e.g., [15], will be applied,

$$\int_{t_0}^{t_e} [\delta(E_K - E_P) + \delta E_{NC}]\mathrm{d}t = 0, \tag{1}$$

where δ denotes the variational operator, E_K, E_P and E_{NC} represent the kinetic and the potential energy stored in the boom structure and the virtual work of non-conservative forces due to damping, respectively.

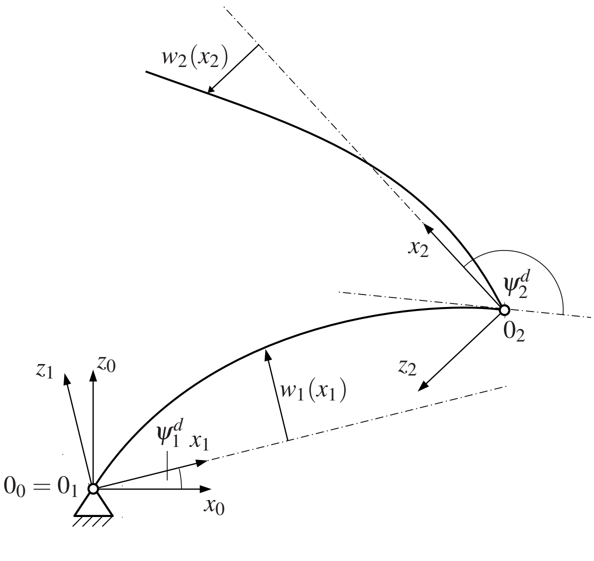

Fig. 2. Planar manipulator with two beams.

The calculation of the kinetic energy requires the knowledge of the velocity of each beam in the inertial frame. The local coordinates $\mathbf{r}_n^n = [x_n, w_n(x_n)]^T$ of a mass particle on the n-th beam can be transformed into the inertial frame using rotation matrices and translation vectors. With the assumption of small deflections, the rotation from the n-th local coordinate frame to the inertial frame is given by

$$\mathbf{R}_0^n = \begin{bmatrix} \cos(\theta_n+\nu_{n-1}) & -\sin(\theta_n+\nu_{n-1}) \\ \sin(\theta_n+\nu_{n-1}) & \cos(\theta_n+\nu_{n-1}) \end{bmatrix}, \tag{2}$$

where $\theta_n = \sum_{k=1}^n \psi_k^d$ and $\nu_n = \sum_{k=1}^n (\partial_{x_k} w_k)(L_k)$, $\nu_0 = 0$. For the damping controller design, the system is linearized around a general equilibrium defined by the constant joint angles ψ_n^d. In this sense, the linear approximation of $\mathbf{R}_0^n$ given by

$$\bar{\mathbf{R}}_0^n = \begin{bmatrix} \cos(\theta_n)-\sin(\theta_n)\nu_{n-1} & -\sin(\theta_n)-\cos(\theta_n)\nu_{n-1} \\ \sin(\theta_n)+\cos(\theta_n)\nu_{n-1} & \cos(\theta_n)-\sin(\theta_n)\nu_{n-1} \end{bmatrix} \tag{3}$$

is used for the further analysis. The translational displacement between the n-th coordinate frame and the inertial frame can be calculated as

$$\bar{\mathbf{d}}_0^n = \sum_{k=1}^{n-1} \bar{\mathbf{R}}_0^k \mathbf{d}_k^{k+1}, \tag{4}$$

with the relative displacement $\mathbf{d}_n^{n+1} = [L_n, w_n(L_n)]^T$ for $n = 1, \ldots, N$ and $\mathbf{d}_0^1 = [0, 0]^T$. Hence, the inertial position of a mass particle on the n-th beam is determined by

$$\mathbf{r}_0^n(x_n) = \bar{\mathbf{R}}_0^n \mathbf{r}_n^n(x_n) + \bar{\mathbf{d}}_0^n. \tag{5}$$

Introducing the variable

$$y_n(x_n) = x_n \sum_{k=1}^{n-1} (\partial_{x_k} w_k)(L_k) + w_n(x_n), \tag{6}$$

the time derivative of (5) can be expressed as

$$\dot{\mathbf{r}}_0^n(x_n) = \begin{bmatrix} -\dot{y}_n(x_n)\sin(\theta_n) - \sum_{k=1}^{n-1} \dot{y}_k(L_k)\sin(\theta_k) \\ \dot{y}_n(x_n)\cos(\theta_n) + \sum_{k=1}^{n-1} \dot{y}_k(L_k)\cos(\theta_k) \end{bmatrix} \tag{7}$$

after some intermediate computations and based on the assumption of small deflections, in particular $\nu_{j-1}\dot{w}_j(x_j) \approx 0$ and $\dot{\nu}_{j-1}w_j(x_j) \approx 0$ in (5) for $j = 1, \ldots, n$. Note that the inverse of (6) is given by

$$w_n(x_n) = y_n(x_n) - x_n(\partial_{x_{n-1}} y_{n-1})(L_{n-1}) \tag{8}$$

for $n = 1, \ldots, N$ and $y_n(x_n) = 0$ for $n = 0$. This can be easily proven by induction starting with $w_1(x_1)$.

Remark 2 *For the sake of a compact presentation of the equations, all variables and parameters referring to a beam n with $n < 1$ or $n > N$, respectively, are equal to zero.*

Henceforth, it will be shown that the transformation (6) yields a compact formulation of the equations of motion.

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With this and the constant mass distribution μ_n the kinetic energy of the overall boom is given by

$$E_K = \sum_{n=1}^{N} \frac{1}{2}\mu_n \int_0^{L_n} \left(\dot{\mathbf{r}}_0^n\right)^T(x_n)\dot{\mathbf{r}}_0^n(x_n)\mathrm{d}x_n. \tag{9}$$

Using the Euler-Bernoulli assumptions, see, e.g., [15], the potential energy due to the beam deflection is determined by

$$E_P = \sum_{n=1}^{N} \frac{1}{2}\Lambda_n \int_0^{L_n} \left(\partial_{x_n}^2 y_n(x_n)\right)^2 \mathrm{d}x_n \tag{10}$$

with the flexural rigidity Λ_n of the n-th beam. Furthermore, viscous damping for the beams is considered in (1) in the form

$$\delta E_{NC} = -\sum_{n=1}^{N} \gamma_n \int_0^{L_n} \dot{y}_n(x_n)\delta y_n(x_n)\mathrm{d}x_n, \tag{11}$$

with the damping coefficients $\gamma_n > 0$.

Remark 3 *As will be shown in Section 4, the assumption of viscous damping can be advantageously utilized for the analysis of the closed-loop stability. Internal Kelvin-Voigt or structural damping can be in principle also be taken into account in the proposed approach but only at the cost of more involved computations.*

The interconnection of the N beam elements implies the boundary conditions

$$y_n(0) = w_n(0) = 0 \tag{12}$$

and according to Remark 1

$$(\partial_{x_n}\dot{w}_n)(0) = u_n, \tag{13}$$

or equivalently (see (8))

$$(\partial_{x_n}\dot{y}_n)(0) - (\partial_{x_{n-1}}\dot{y}_{n-1})(L_{n-1}) = u_n. \tag{14}$$

Note that u_n, $n = 1,\ldots,N$, correspond to the impressed joint angle velocities, which serve as control inputs. Substituting (9), (10) and (11) into (1), and utilizing (12) and (14), after some lengthy but straightforward calculations and re-sort of the finite sums with, e.g., (B.7), results in the system of partial differential equations

$$\begin{aligned}\mu_n\left(\ddot{y}_n(x_n) + \sum_{k=1}^{n-1}\ddot{y}_k(L_k)\cos\left(\theta_n - \theta_k\right)\right)\\ +\gamma_n\dot{y}_n(x_n) + \Lambda_n\partial_{x_n}^4 y_n(x_n) = 0\end{aligned} \tag{15a}$$

and boundary conditions

$$y_n(0) = 0, \tag{15b}$$
$$(\partial_{x_n}\dot{y}_n)(0) = u_n + (\partial_{x_{n-1}}\dot{y}_{n-1})(L_{n-1}), \tag{15c}$$
$$\Lambda_n\left(\partial_{x_n}^2 y_n\right)(L_n) = \Lambda_{n+1}\left(\partial_{x_{n+1}}^2 y_{n+1}\right)(0), \tag{15d}$$
$$\begin{aligned}\Lambda_n\left(\partial_{x_n}^3 y_n\right)(L_n) = \sum_{k=n+1}^{N}\mu_k\int_0^{L_k}\Bigg[\ddot{y}_k(x_k)\cos(\theta_k - \theta_n)\\ +\sum_{j=1}^{k-1}\ddot{y}_j(L_j)\cos(\theta_j - \theta_n)\Bigg]\mathrm{d}x_k\end{aligned} \tag{15e}$$

for $n = 1,\ldots,N$. The geometrical boundary conditions are given by (15b) and (15c), which represent the joint velocities imposed by the underlying controllers. The conditions (15d) and (15e) denote the balance of torques and forces at the boundaries.

3 Passivity-based control law

In the following, a control law is developed which ensures the dissipativity of the closed-loop system. For this purpose, the temporal behavior of the total energy H stored in the system is analyzed in the first step. The time derivative of $H = E_K + E_P$ along a solution of (15) results in, after tedious but straightforward calculations taking into account (B.7),

$$\dot{H} = -\sum_{n=1}^{N}\gamma_n\int_0^{L_n}\dot{y}_n^2(x_n)\mathrm{d}x_n - \sum_{n=1}^{N}\Lambda_n\left(\partial_{x_n}^2 y_n\right)(0)u_n. \tag{16}$$

Equation (16) comprises the dissipation due to the viscous damping and the collocated pairing of the angular velocities and the beam flections [1] $(\partial_{x_n}^2 y_n)(0)$ at the boundaries $x_n = 0$, $n = 1,\ldots,N$. Considering (9) and (10), all $y_n(x_n)$ in form of straight lines $y_n(x_n) = c_n x_n$ with arbitrary c_n yield $H = 0$. Thus, the total energy of the system H is only positive semidefinite and does not directly qualify for a Lyapunov functional. That is, the feedback of only the beam flection $(\partial_{x_n}^2 y_n)(0)$ does not guarantee the asymptotic stability of the closed-loop system. Therefore, an additional position controller is necessary. The following extension (consider (8)) of H,

$$H_e = H + \frac{1}{2}\sum_{n=1}^{N}\alpha_n\left((\partial_{x_n}y_n)(0) - (\partial_{x_{n-1}}y_{n-1})(L_{n-1})\right)^2 \tag{17}$$

with $\alpha_n > 0$ serves as a suitable Lyapunov functional candidate. It can be shown by recursive evaluation with $y_n(x_n) = c_n x_n$ starting at $n = 1$ that $H_e = 0$ implies $c_n = 0$ for $n = 1,\ldots,N$. The time derivative of (17) with (16) and (14) re-

[1] Note that the beam flections at the boundaries $x_n = 0$ are proportional to the respective joint torques.

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sults in

$$\dot{H}_e = -\sum_{n=1}^{N} \gamma_n \int_0^{L_n} \dot{y}_n^2(x_n)\mathrm{d}x_n - \sum_{n=1}^{N} \Big[\Lambda_n\big(\partial_{x_n}^2 y_n\big)(0) \\ -\alpha_n\big((\partial_{x_n} y_n)(0) - (\partial_{x_{n-1}} y_{n-1})(L_{n-1})\big)\Big] u_n. \tag{18}$$

By choosing a feedback law of the form

$$u_n = k_n\Big(\Lambda_n\big(\partial_{x_n}^2 y_n\big)(0) \\ -\alpha_n\big((\partial_{x_n} y_n)(0) - (\partial_{x_{n-1}} y_{n-1})(L_{n-1})\big)\Big) \tag{19}$$

with the control gains $k_n > 0$, it follows that

$$\dot{H}_e = -\sum_{n=1}^{N} \gamma_n \int_0^{L_n} \dot{y}_n^2(x_n)\mathrm{d}x_n - \sum_{n=1}^{N} k_n\Big[\Lambda_n\big(\partial_{x_n}^2 y_n\big)(0) \\ -\alpha_n\big((\partial_{x_n} y_n)(0) - (\partial_{x_{n-1}} y_{n-1})(L_{n-1})\big)\Big]^2 \\ \leq 0. \tag{20}$$

This implies that H_e with (19) is an appropriate Lyapunov functional.

4 Asymptotic stability of the closed-loop system

In the following, a rigorous proof of the asymptotic stability of the closed-loop system is given. For the theoretical background on the analysis of the asymptotic stability of infinite-dimensional systems, the reader is referred to, e.g., [14].

4.1 Abstract formulation

In order to represent the system as an abstract differential equation in the form $\dot{\boldsymbol{y}} = \mathscr{A}\boldsymbol{y}$ with an operator $\mathscr{A}$ and a state vector $\boldsymbol{y}$, the equations of motion (15) have to be solved for $\ddot{y}_1(x_1), \ldots, \ddot{y}_N(x_N)$ and $\ddot{y}_1(L_1), \ldots, \ddot{y}_{N-1}(L_{N-1})$. For this, $\ddot{y}_n(x_n)$ is calculated from (15a) and inserted into the boundary conditions (15e). After some lengthy but straightforward calculations the resulting equations can be represented in the form $\mathbf{M}\ddot{\boldsymbol{y}}_L = \mathbf{b}$ with $\ddot{\boldsymbol{y}}_L = [\ddot{y}_1(L_1), \ldots, \ddot{y}_{N-1}(L_{N-1})]^T$, the matrix $\mathbf{M}$ comprising the elements

$$\mathbf{M}_{n,k\leq n} = \sum_{j=n+1}^{N} \mu_j L_j \big(\cos(\theta_n - \theta_k) \\ -\cos(\theta_j - \theta_n)\cos(\theta_j - \theta_k)\big), \\ \mathbf{M}_{n,k>n} = \sum_{j=k+1}^{N} \mu_j L_j \big(\cos(\theta_k - \theta_n) \\ -\cos(\theta_j - \theta_n)\cos(\theta_j - \theta_k)\big), \tag{21}$$

and the vector $\mathbf{b}$ with the components

$$\mathbf{b}_n = \sum_{k=n+1}^{N} \cos(\theta_k - \theta_n)\Big(\Lambda_k\Big(\big(\partial_{x_k}^3 y_k\big)(L_k) - \big(\partial_{x_k}^3 y_k\big)(0)\Big) \\ + \gamma_k \int_0^{L_k} \dot{y}_k(x_k)\mathrm{d}x_k\Big) + \Lambda_n\big(\partial_{x_n}^3 y_n\big)(L_n). \tag{22}$$

In Appendix A it is proven that the determinant of $\mathbf{M}$ is given by

$$\det(\mathbf{M}) = \prod_{n=2}^{N} \mu_n L_n \sin^2(\theta_n - \theta_{n-1}). \tag{23}$$

Thus, the matrix $\mathbf{M}$ is invertible if no rigid body angle $\psi_n^d = \theta_n - \theta_{n-1}$, $n = 2, \ldots, N$, is a multiple of π, i.e. $\psi_n^d \neq m\pi$, $m \in \mathbb{Z}$.

Remark 4 *From a physical point of view, the singularity of* $\mathbf{M}$ *for* $\psi_n^d = m\pi$ *can be explained by the fact that in this case the forces before and after the joint n only differ by the sign. This brings along two redundant equations in (15e). Thus, no explicit representation of the system in the form* $\dot{\boldsymbol{y}} = \mathscr{A}\boldsymbol{y}$ *can be found with* $\ddot{\boldsymbol{y}}_L$ *being part of the state vector if* $\mathbf{M}$ *is singular. This problem is caused by the lineariziation of the system and the specific choice of the coordinate frames and can be avoided by formulating the deflection of the n-th beam in the coordinate frame* $0_{n-1}x_{n-1}z_{n-1}$*. With this,* $\ddot{w}_{n-1}(L_{n-1})$ *does not appear in the equations of motion so that a regular matrix* $\mathbf{M}$ *can be formulated, see, e.g., the partial differential equations of a system with* $N = 2$ *beams and* $\psi_2^d = 0$*. It seems that this is mainly a problem which originates from the mathematical description and does not restrict the practical imlementation of the control law (19). However, a closed formulation of the equations of motion as a differential equation in the form* $\dot{\boldsymbol{y}} = \mathscr{A}\boldsymbol{y}$ *that includes all possible values for* ψ_n^d *is quite difficult to find. Therefore in the following analysis, we will exclude these points.*

By introducing the state vector

$$\boldsymbol{y} = [y_{1,1}, \ldots, y_{1,N}, y_{2,1}, \ldots, y_{2,N}, y_{3,1}, \ldots, y_{3,N-1}]^T, \tag{24}$$

with $y_{1,n} = y_n(x_n)$, $y_{2,n} = \dot{y}_n(x_n)$ for $n = 1, \ldots, N$, and $y_{3,m} = \dot{y}_m(L_m)$ for $m = 1, \ldots, N-1$ and the assumption of the regularity of $\mathbf{M}$, (15) can be represented by an operator $\mathscr{A}$ in the form

$$\mathscr{A}\boldsymbol{y} = \big[\mathscr{A}_{1,1}\boldsymbol{y}, \ldots, \mathscr{A}_{1,N}\boldsymbol{y}, \mathscr{A}_{2,1}\boldsymbol{y}, \\ \ldots, \mathscr{A}_{2,N}\boldsymbol{y}, \mathscr{A}_{3,1}\boldsymbol{y}, \ldots, \mathscr{A}_{3,N-1}\boldsymbol{y}\big]^T \tag{25a}$$

with

$$\mathscr{A}_{1,n}\boldsymbol{y} = y_{2,n} \\ \mathscr{A}_{2,n}\boldsymbol{y} = -\frac{\gamma_n}{\mu_n} y_{2,n} - \frac{\Lambda_n}{\mu_n}\partial_{x_n}^4 y_{1,n} - \sum_{k=1}^{n-1} \check{\mathbf{M}}_k \mathbf{b}\cos(\theta_n - \theta_k) \\ \mathscr{A}_{3,n}\boldsymbol{y} = \check{\mathbf{M}}_n \mathbf{b}, \tag{25b}$$

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and $\check{\mathbf{M}}_n$ corresponds to the n-th row of the matrix $\check{\mathbf{M}} = \mathbf{M}^{-1}$. The domain of the operator reads as

$$\begin{aligned}\mathscr{D}(\mathscr{A}) = \Big\{ & (y_{1,n}, y_{2,n}, y_{3,m}) | \, y_{1,n} \in \mathscr{H}^4(0,L_n), \\ & y_{2,n} \in \mathscr{H}^2(0,L_n), y_{3,m} \in \mathbb{R}, y_{1,n}(0) = 0, \\ & \Lambda_n \big(\partial^2_{x_n} y_{1,n}\big)(L_n) = \Lambda_{n+1} \big(\partial^2_{x_{n+1}} y_{1,n+1}\big)(0), \\ & (\partial_{x_n} y_{2,n})(0) = (\partial_{x_{n-1}} y_{2,n-1})(L_{n-1}) \\ & + k_n \Big(\Lambda_n \big(\partial^2_{x_n} y_{1,n}\big)(0) - \alpha_n \big((\partial_{x_n} y_{1,n})(0) \\ & - (\partial_{x_{n-1}} y_{1,n-1})(L_{n-1}) \big) \Big), y_{3,m} = y_{2,m}(L_m), \\ & \Lambda_N \big(\partial^3_{x_N} y_{1,N}\big)(L_N) = 0, \\ & n = 1, \ldots, N, m = 1, \ldots, N-1 \Big\}. \end{aligned} \tag{26}$$

The state vector $\boldsymbol{y}$ is defined in the real Sobolev space

$$\begin{aligned}\mathscr{Y} = \; & \mathscr{H}^2_c(0,L_1) \times \ldots \times \mathscr{H}^2_c(0,L_N) \times \\ & \mathscr{L}_2(0,L_1) \times \ldots \times \mathscr{L}_2(0,L_N) \times \mathbb{R}^{N-1}, \end{aligned} \tag{27}$$

with $\mathscr{H}^2_c(0,L_n) = \{y_{1,n} \in \mathscr{H}^2(0,L_n) | y_{1,n}(0) = 0\}$, equipped with the (energy) inner product [2]

$$\begin{aligned}\langle \boldsymbol{y}, \boldsymbol{z} \rangle_{\mathscr{Y}} = \sum_{n=1}^{N} \Bigg[& \int_0^{L_n} \Big(\mu_n \big(\dot{\mathbf{r}}_0^n\big)^T (\boldsymbol{y}) \dot{\mathbf{r}}_0^n(\boldsymbol{z}) + \Lambda_n \partial^2_{x_n} y_{1,n} \partial^2_{x_n} z_{1,n} \Big) \mathrm{d}x_n \\ & + \alpha_n \big((\partial_{x_n} y_{1,n})(0) - (\partial_{x_{n-1}} y_{1,n-1})(L_{n-1}) \big) \times \\ & \big((\partial_{x_n} z_{1,n})(0) - (\partial_{x_{n-1}} z_{1,n-1})(L_{n-1}) \big) \Bigg] \end{aligned} \tag{28}$$

with $\boldsymbol{y}, \boldsymbol{z} \in \mathscr{Y}$ and, see (7),

$$\dot{\mathbf{r}}_0^n(\boldsymbol{y}) = \begin{bmatrix} -y_{2,n} \sin(\theta_n) - \sum_{k=1}^{n-1} y_{3,k} \sin(\theta_k) \\ y_{2,n} \cos(\theta_n) + \sum_{k=1}^{n-1} y_{3,k} \cos(\theta_k) \end{bmatrix}. \tag{29}$$

Due to the special choice of the inner product (28), (17) corresponds to

$$H_e = \frac{1}{2} \langle \boldsymbol{y}, \boldsymbol{y} \rangle_{\mathscr{Y}} = \frac{1}{2} \|\boldsymbol{y}\|^2_{\mathscr{Y}} .$$

The proof that $\langle \boldsymbol{y}, \boldsymbol{y} \rangle_{\mathscr{Y}} = 0$ holds only for $\boldsymbol{y} = \mathbf{0}$ can be done by induction starting with the first beam.

Theorem 5 *The operator $\mathscr{A}$ defined in* (25) *has the following properties:*

(i) $\mathscr{A}$ is the infinitesimal generator of a C_0-semigroup $\mathscr{T}(t)$ of contractions on $\mathscr{Y}$.

(ii) The C_0-semigroup $\mathscr{T}(t)$ is asymptotically stable.

The proof of the asymptotic stability of the closed-loop system is performed in several steps. At first it is shown that the operator $\mathscr{A}$ of (25) is dissipative. Secondly it is verified with the Lumer-Philips theorem, see [12], that $\mathscr{A}$ is the infinitesimal generator of a C_0-semigroup of contractions. In order to apply LaSalle's invariance principle, see [14], the precompactness of the solution orbit has to be proven. Finally, it is shown that the largest positive invariant subset of $\dot{H}_e(\boldsymbol{y}) = 0$ is the equilibrium $\boldsymbol{y} = \mathbf{0}$ itself.

Lemma 6 *The operator $\mathscr{A}$ defined in* (25) *is dissipative.*

Proof In order to prove that the operator $\mathscr{A}$ is dissipative, the validity of the inequality (see [12, Definition 1.1.1])

$$\langle \boldsymbol{y}, \mathscr{A}\boldsymbol{y} \rangle_{\mathscr{Y}} \leq 0 \tag{30}$$

has to be shown. The left-hand side of (30) is equal to the time derivative of the Lyapunov functional $H_e(\boldsymbol{y})$ according to (17), which is rendered negative semidefinite by means of the control law (19), see (20). Thus, inequality (30) is satisfied. □

Lemma 7 *The inverse operator $\check{\mathscr{A}} = \mathscr{A}^{-1}$ is given by*

$$\begin{aligned}\check{\mathscr{A}}\boldsymbol{z} = \big[& \check{\mathscr{A}}_{1,1}\boldsymbol{z}, \ldots, \check{\mathscr{A}}_{1,N}\boldsymbol{z}, \check{\mathscr{A}}_{2,1}\boldsymbol{z}, \\ & \ldots, \check{\mathscr{A}}_{2,N}\boldsymbol{z}, \check{\mathscr{A}}_{3,1}\boldsymbol{z}, \ldots, \check{\mathscr{A}}_{3,N-1}\boldsymbol{z} \big]^T \end{aligned} \tag{31a}$$

with

$$\begin{aligned}\check{\mathscr{A}}_{1,n}\boldsymbol{z} = & -\frac{1}{\Lambda_n} \int_0^{x_n} \int_0^{\xi_n} \int_0^{\eta_n} \int_0^{\varepsilon_n} (\gamma_n z_{1,n} \\ & + \mu_n z_{2,n}) \mathrm{d}\chi_n \mathrm{d}\varepsilon_n \mathrm{d}\eta_n \mathrm{d}\xi_n - \frac{1}{24} \frac{\mu_n}{\Lambda_n} x_n^4 \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) \\ & + \frac{1}{6} \frac{1}{\Lambda_n} C_{1,n} x_n^3 + \frac{1}{2} \frac{1}{\Lambda_n} C_{2,n} x_n^2 + \frac{1}{\Lambda_n} C_{3,n} x_n, \\ \check{\mathscr{A}}_{2,n}\boldsymbol{z} = & \; z_{1,n}, \\ \check{\mathscr{A}}_{3,n}\boldsymbol{z} = & \; z_{1,n}(L_n), \end{aligned} \tag{31b}$$

and the constants

$$\begin{aligned} C_{1,n} = \; & \mathbf{M}_n z_3 + \int_0^{L_n} (\gamma_n z_{1,n} + \mu_n z_{2,n}) \mathrm{d}\xi_n \\ & + \mu_n L_n \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) \\ & + \sum_{k=n+1}^{N} \cos(\theta_k - \theta_n) \Bigg[\int_0^{L_k} \mu_k z_{2,k} \mathrm{d}\xi_k \\ & + \mu_k L_k \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j) \Bigg], \end{aligned} \tag{31c}$$

[2] Formally, the inner product should include complex conjugates to allow for the spectral analysis of the operator. However, the limitation to real spaces has no consequences for the results in this paper and is therefore introduced for a more compact presentation.

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$$C_{2,n} = \sum_{k=n}^{N} \left\{ \int_0^{L_k} \int_0^{\xi_k} (\gamma_k z_{1,k} + \mu_k z_{2,k}) \mathrm{d}\eta_k \mathrm{d}\xi_k + \frac{1}{2}\mu_k L_k^2 \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j) - C_{1,k} L_k \right\}, \tag{31d}$$

$$\begin{aligned} C_{3,n} = \Lambda_n \Bigg\{ \sum_{k=1}^{n} \Bigg\{ \frac{C_{2,k}}{\alpha_k} - \frac{1}{k_k \alpha_k}\big((\partial_{x_k} z_{1,k})(0) \\ - (\partial_{x_{k-1}} z_{1,k-1})(L_{k-1})\big) \\ - \frac{1}{\Lambda_{k-1}} \Bigg(\int_0^{L_{k-1}} \int_0^{\xi_{k-1}} \int_0^{\eta_{k-1}} (\gamma_{k-1} z_{1,k-1} \\ + \mu_{k-1} z_{2,k-1}) \mathrm{d}\varepsilon_{k-1} \mathrm{d}\eta_{k-1} \mathrm{d}\xi_{k-1} \\ + \frac{1}{6}\mu_{k-1} L_{k-1}^3 \sum_{j=1}^{k-2} z_{3,j} \cos(\theta_{k-1} - \theta_j) \\ - \frac{1}{2} C_{1,k-1} L_{k-1}^2 - C_{2,k-1} L_{k-1} \Bigg) \Bigg\}, \end{aligned} \tag{31e}$$

with $z_3 = [z_{3,1}, \ldots, z_{3,N-1}]^T$ *and* $z \in \mathscr{Y}$.

Proof For the determination of the inverse operator $\check{\mathscr{A}}$, the unique solution $\boldsymbol{y}$ of $\mathscr{A}\boldsymbol{y} = \boldsymbol{z}$ has to be found for a known state vector $\boldsymbol{z} \in \mathscr{Y}$. Here it can be directly seen that $y_{2,n} = z_{1,n}$ holds for $n = 1, \ldots, N$ and consequently $y_{3,n} = z_{1,n}(L_n)$ and $\check{\mathbf{M}}_n \mathbf{b} = z_{3,n}$ for $n = 1, \ldots, N-1$. In order to calculate $y_{1,n}$ for $n = 1, \ldots, N$, the elements $\mathscr{A}_{2,n}(\boldsymbol{y})$ of (25) are integrated four times with respect to x_n

$$\Lambda_n \partial_{x_n}^3 y_{1,n} = -\int_0^{x_n} (\gamma_n z_{1,n} + \mu_n z_{2,n}) \mathrm{d}\xi_n - \mu_n x_n \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) + C_{1,n}, \tag{32a}$$

$$\Lambda_n \partial_{x_n}^2 y_{1,n} = -\int_0^{x_n} \int_0^{\xi_n} (\gamma_n z_{1,n} + \mu_n z_{2,n}) \mathrm{d}\eta_n \mathrm{d}\xi_n - \frac{1}{2}\mu_n x_n^2 \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) + C_{1,n} x_n + C_{2,n}, \tag{32b}$$

$$\begin{aligned} \Lambda_n \partial_{x_n} y_{1,n} = -\int_0^{x_n} \int_0^{\xi_n} \int_0^{\eta_n} (\gamma_n z_{1,n} \\ + \mu_n z_{2,n}) \mathrm{d}\varepsilon_n \mathrm{d}\eta_n \mathrm{d}\xi_n - \frac{1}{6}\mu_n x_n^3 \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) \\ + \frac{1}{2} C_{1,n} x_n^2 + C_{2,n} x_n + C_{3,n}. \end{aligned} \tag{32c}$$

The fourth integration with $y_{1,n}(0) = 0$ yields (31b). Due to the domain of the operator (26) the constant $C_{1,N}$ can be directly identified at the position $x_N = L_N$ in (32a). Since $\mathbf{b}_n = \mathbf{M}_n z_3$ with $z_3 = [z_{3,1}, \ldots, z_{3,N-1}]^T$ represents a linear combination of $\Lambda_n \partial_{x_n}^3 y_{1,n}$ at $x_n = 0$ and $x_n = L_n$, see (22), the integration constants $C_{1,n}$ for $n = 1, \ldots, N-1$ as well as $C_{1,N}$ are given by (31c).

Considering (26) at $x_n = L_n$ in (32b), the relation

$$C_{2,n} = C_{2,n+1} + \int_0^{L_n} \int_0^{\xi_n} (\gamma_n z_{1,n} + \mu_n z_{2,n}) \mathrm{d}\eta_n \mathrm{d}\xi_n + \frac{1}{2}\mu_n L_n^2 \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) - C_{1,n} L_n \tag{33}$$

for the constants $C_{2,n}$ follows from $C_{2,n+1} = 0$ for $n = N$. Starting at $n = N$ the constants $C_{2,n}$ can be explicitly determined according to (31d).

With the evaluation of (32c) at $x_n = 0$ and $x_{n-1} = L_{n-1}$ as well as considering $\mathscr{D}(\mathscr{A})$ and $\partial_{x_n} y_{2,n} = \partial_{x_n} z_{1,n}$ the implicit relation for $C_{3,n}$, $n = 2, \ldots, N$ reads as

$$\begin{aligned} \frac{C_{3,n}}{\Lambda_n} = \frac{C_{3,n-1}}{\Lambda_{n-1}} + \frac{C_{2,n}}{\alpha_n} - \frac{1}{k_n \alpha_n}\big((\partial_{x_n} z_{1,n})(0) \\ - (\partial_{x_{n-1}} z_{1,n-1})(L_{n-1})\big) \\ - \frac{1}{\Lambda_{n-1}} \Bigg(\int_0^{L_{n-1}} \int_0^{\xi_{n-1}} \int_0^{\eta_{n-1}} (\gamma_{n-1} z_{1,n-1} \\ + \mu_{n-1} z_{2,n-1}) \mathrm{d}\varepsilon_{n-1} \mathrm{d}\eta_{n-1} \mathrm{d}\xi_{n-1} \\ + \frac{1}{6}\mu_{n-1} L_{n-1}^3 \sum_{k=1}^{n-2} z_{3,k} \cos(\theta_{n-1} - \theta_k) \\ - \frac{1}{2} C_{1,n-1} L_{n-1}^2 - C_{2,n-1} L_{n-1} \Bigg). \end{aligned} \tag{34}$$

The recursive evaluation starting with

$$C_{3,1} = \frac{\Lambda_1}{\alpha_1} C_{2,1} - \frac{\Lambda_1}{k_1 \alpha_1} (\partial_{x_1} z_{1,1})(0) \tag{35}$$

results in the explicit representation (31e). With this, the inverse operator $\check{\mathscr{A}}$ is clearly defined. □

It is shown in Appendix B that the inverse operator $\check{\mathscr{A}}$ is bounded. The boundedness of $\check{\mathscr{A}}$ implies that 0 cannot be an eigenvalue of $\mathscr{A}$ and is therefore an element of the resolvent set. With this, considering Theorem 1.2.4 in [12], which follows directly from the Lumer-Philips theorem, $\mathscr{A}$ is the infinitesimal generator of a C_0-semigroup $\mathscr{T}(t)$ of contractions on $\mathscr{Y}$ and thus (i) in Theorem 5 is shown.

The prerequisite for the application of the invariance principle of LaSalle, see Theorem 3.64 in [14], is the precompactness of the orbit, which is in general not ensured for distributed-parameter systems.

Lemma 8 *The orbit* $\Omega(\boldsymbol{y}) = \bigcup_{t \geq 0} \mathscr{T}(t)\boldsymbol{y}$ *is precompact.*

Proof Pursuant to Theorem 3.65 in [14], the precompactness is given if 0 is an element of the resolvent set of $\mathscr{A}$ and if there exists a $\lambda > 0$ for which the resolvent $(\lambda \mathscr{I} - \mathscr{A})^{-1}$

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is compact. The bounded operator $\check{\mathscr{A}}$ maps bounded subsets from $\mathscr{Y}$ onto bounded subsets of $\mathscr{D}(\mathscr{A})$. According to the Sobolev embedding theorem bounded sets of $\mathscr{D}(\mathscr{A})$ are precompact in $\mathscr{Y}$. Hence, $\check{\mathscr{A}}$ maps bounded subsets of $\mathscr{Y}$ onto precompact subsets of $\mathscr{Y}$ and in consequence the compactness of the inverse operator $\check{\mathscr{A}}$ follows. Therefore, considering Theorem 6.29 in [8, p. 187] the operator $\mathscr{A}$ has a compact resolvent and has only discrete eigenvalues with finite algebraic multiplicity. Hence, there exists a $\lambda > 0$ such that the resolvent $(\lambda\mathscr{I} - \mathscr{A})^{-1}$ is compact. This implies the precompactness of the orbit. □

Lemma 9 *The largest positive invariant subset of* $\{\mathbf{y}|\dot{H}_e(\mathbf{y}) = 0\}$ *is given by* $\mathbf{y} = \mathbf{0}$.

Proof From $\dot{H}_e(\mathbf{y}) = 0$ (see (20)), it follows that

$$-\sum_{n=1}^{N}\gamma_n\int_0^{L_n} y_{2,n}^2 \mathrm{d}x_n - \sum_{n=1}^{N} k_n\Big(\Lambda_n\big(\partial_{x_n}^2 y_{1,n}\big)(0) \\ -\alpha_n\big((\partial_{x_n}y_{1,n})(0)-(\partial_{x_{n-1}}y_{1,n-1})(L_{n-1})\big)\Big)^2 = 0. \tag{36}$$

For (36) to hold, the following conditions

$$\begin{aligned}&\Lambda_n\big(\partial_{x_n}^2 y_{1,n}\big)(0)\\ &-\alpha_n\big((\partial_{x_n}y_{1,n})(0)-(\partial_{x_{n-1}}y_{1,n-1})(L_{n-1})\big) = 0,\\ &y_{2,n} = 0\end{aligned} \tag{37}$$

have to be fulfilled for $n = 1,\ldots,N$. Considering the constraints (37) as well as $y_{3,n} = y_{2,n}(L_n)$, the partial differential equation (15a) with (19) results in

$$\Lambda_n\partial_{x_n}^4 y_{1,n} = 0 \tag{38a}$$

with the boundary conditions

$$y_{1,n}(0) = 0, \tag{38b}$$
$$\Lambda_n\big(\partial_{x_n}^2 y_{1,n}\big)(L_n) = \Lambda_{n+1}\big(\partial_{x_{n+1}}^2 y_{1,n+1}\big)(0), \tag{38c}$$
$$\Lambda_n\big(\partial_{x_n}^3 y_{1,n}\big)(L_n) = 0. \tag{38d}$$

By integration of (38a) four times with respect to x_n, the equations

$$\Lambda_n\partial_{x_n}^3 y_{1,n} = \bar{C}_{1,n}, \tag{39a}$$
$$\Lambda_n\partial_{x_n}^2 y_{1,n} = \bar{C}_{1,n}x_n + \bar{C}_{2,n}, \tag{39b}$$
$$\Lambda_n\partial_{x_n} y_{1,n} = \frac{1}{2}\bar{C}_{1,n}x_n^2 + \bar{C}_{2,n}x_n + \bar{C}_{3,n}, \tag{39c}$$
$$\Lambda_n y_{1,n} = \frac{1}{6}\bar{C}_{1,n}x_n^3 + \frac{1}{2}\bar{C}_{2,n}x_n^2 + \bar{C}_{3,n}x_n + \bar{C}_{4,n} \tag{39d}$$

are given. From (38b) and (38d), it directly follows that $\bar{C}_{4,n} = \bar{C}_{1,n} = 0$, $n = 1,\ldots,N$. A recursive evaluation of the boundary condition (38c) with (39b) beginning at $n = N$ entails $\bar{C}_{2,n} = 0$, $n = 1,\ldots,N$. With these results and (37) the relation

$$(\partial_{x_n}y_{1,n})(0) = (\partial_{x_{n-1}}y_{1,n-1})(L_{n-1}) \tag{40}$$

holds, and thus $\bar{C}_{3,n} = 0$, $n = 1,\ldots,N$ can be deduced by recursive evaluation of (40) with (39c) beginning at $n = 1$. Therefore, $y_{1,n} = 0$, $n = 1,\ldots,N$, is the only possible solution of (38a). With this result, Lemma 9 is shown and thus the asymptotic stability of $\mathscr{T}(t)$ respectively the closed-loop system is proven. □

Remark 10 *It should be noted that for the determination of the largest positive invariant subset the assumption of viscous damping* $\gamma_n > 0$, $n = 1,\ldots,N$, *is essential to avoid unnecessary difficulties in the derivation. Without these non-conservative forces, the proof would be significantly more involved. We are quite sure that this assumption is only of technical nature and that the above results are also valid for systems without viscous damping as well as for systems with internal Kelvin-Voigt or structural damping. However, since some amount of viscous damping could be assumed in all real technical systems, this is no restriction at all.*

5 Decentralized modular realization of the control law

The control law (19) is implemented as a decentralized control strategy for each joint of the planar manipulator. Considering $(\partial_{x_n}y_n)(0) - (\partial_{x_{n-1}}y_{n-1})(L_{n-1}) = (\partial_{x_n}w_n)(0)$ and $\partial_{x_n}^2 y_n(x_n) = \partial_{x_n}^2 w_n(x_n)$ and introducing the absolute joint angle $\psi_n = \psi_n^d + (\partial_{x_n}w_n)(0)$, the corresponding control law (19) for the joints of the boom reads as

$$u_n^c = k_{d,n}\left.\frac{\partial^2 w_n(x_n)}{\partial x_n^2}\right|_{x_n=0} - k_{p,n}\left(\psi_n - \psi_n^d\right), \tag{41}$$

with the controller parameters $k_{d,n} = k_n\Lambda_n$ and $k_{p,n} = k_n\alpha_n$.

In the following, a possible realization of the control strategy for implementation, e.g., to control the boom of a mobile concrete pump, is given. The beam flection $(\partial_{x_n}^2 w_n)(0)$ and the absolute joint angle ψ_n can be measured by means of strain gauges and rotary encoders or inclination sensors, respectively. Due to gravity, the stationary beam flection is in general different from zero and has to be subtracted from the measurement signal prior to feeding it to the damping controller. Due to limited model accuracy, e.g. the exact amount and weight of wet concrete in the pipes is not known, a reliable calculation of the stationary beam flection is not possible. Therefore, the use of a first order high-pass filter is proposed for the elimination of the stationary part of the signal. According to the lowest eigenfrequencies of the system, the cutoff frequency of the filter has to be adjusted such that the dynamic part of the signal remains widely unchanged without noticable phase-lead. Figure 3 illustrates the block diagram of the control strategy for a single boom segment with the index n. The system is represented by the boom and

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the hydraulic actuator HA_n. The beam flection $(\partial_{x_n}^2 w_n)(0)$ is filtered by a high-pass filter HP_n. The resulting control input $u_n = u_n^d + u_n^c$ consists of the desired angular velocity u_n^d, provided by the operator, and the feedback part u_n^c according to (41). The corresponding valve position $s_{v,n}$ is obtained by a subordinate velocity controller VC_n. During a movement of the joint by the operator, the activation of the position controller is not reasonable. For this reason, the desired joint angle is set to the actual measurement $\psi_n^d = \psi_n$ in the case of $u_n^d \neq 0$ and the position controller has no influence on the system. Hence, for $u_n^d = 0$ the desired joint angle ψ_n^d is determined by the position ψ_n where the operator stopped the movement of the particular joint.

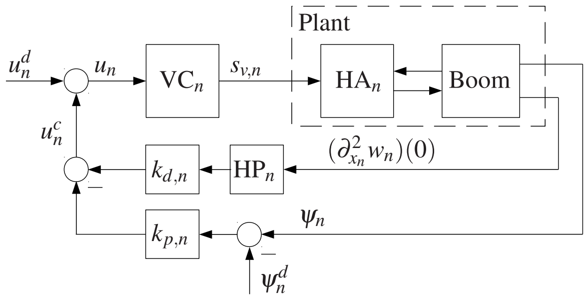

Fig. 3. Control structure for the n-th beam.

Remark 11 *Although the theoretical results allow to choose the controller coefficients $k_{d,n}$ and $k_{p,n}$ arbitrarily high without destabilizing the system, this is not true in reality. In practice, the dynamic behavior of the actuators (here especially the hydraulic actuators of the underlying velocity controllers) as well as of the sensors always constitutes a limiting factor. In addition the control inputs are always bounded in reality. However, even in the case of theoretically ideal actuators and sensors it is probably necessary to limit the controller coefficients in order to avoid a poor closed-loop performance due to the so-called* overdamping phenomenon*, see, e.g., [5]. It has to be noted that the result of the stability proof gives no information about the decay rate of oscillations. Nevertheless, it yields a powerful tool for the engineers to adjust a reasonable damping ratio tailored to the respective application that guarantees a high robustness against model uncertainties and varying boom configurations if the assumptions of the control strategy are essentially fulfilled.*

6 Experimental results

A typical industrial mobile concrete pump according to Figure 1 with four joints and an operating range of about 40 m serves as a test system for the application of the presented control strategy. Since the hydraulic systems typically used for the actuation of mobile concrete pumps have fundamental weaknesses concerning the realization of an active damping control, an alternative hydraulic concept was developed for the actuation of the particular joints. In particular, the control valves are mounted directly on the hydraulic cylinders in order to avoid the dynamic influence of the long hydraulic lines. Furthermore, a controlled constant pressure system is installed instead of the conventional load-sensing system. With this, a feedforward control of the joint angular velocities is realized by means of a servo compensation. A detailed discussion about the disadvantages of the classical system and the proposed hydraulic architecture as well as the implemented algorithms is given in [7].

In the experimental setup, the following sensors were used for the measurement of the system variables required for the feedback controller and the velocity controller:

- Inclination sensors at both ends of each beam in order to determine the joint angles ψ_n.
- Strain gauges for the measurement of the beam flections $(\partial_{x_n}^2 w_n)(0)$.
- Two pressure sensors at each hydraulic cylinder required for the velocity controllers.

In order to validate the proposed control strategy, the system has been excited by means of a sudden release of a 75 kg load at the end of the boom at several poses. This load relates to the mass of wet concrete fitting in the end hose. For all experiments, the same controller parameters were chosen for all joints, $k_d = k_{d,n}$ and $k_p = k_{p,n}$ for $n = 1, \ldots, 4$. The parameters were manually tuned by increasing of k_d and k_p until the boom exhibited the desired damping behavior and an appropriate position control performance.

In the following, the achieved damping of the system is illustrated by means of the strain gauge measurements at three different configurations of the boom presented in Figure 4. In the first configuration (I), the boom was sprawled, which is a pose with low rigidity. Configuration (II) is a typical configuration at a construction site and the third configuration (III) possesses a high rigidity. The initial states of the joint angles are identical to the desired values, i.e. $\psi_n(0) = \psi_n^d$. For comparison, the experiments were performed for the nominal set of parameters with $k_d = \bar{k}_d$, $k_p = \bar{k}_p$, which achieves a good overall behavior of the system, a set of parameters with a lower damping ratio $k_d = \bar{k}_d/4$, $k_p = \bar{k}_p$ and the system without any active control $k_d = 0$, $k_p = 0$. It can be seen in the figures 5, 6 and 7 that the vibrations of the system can be reduced very effectively with the proposed control concept in all boom configurations with identical controller parameters. Even the very rigid configuration with higher natural harmonics is well damped by the controller. Furthermore, the figures illustrate that the damping ratio can be proportionally adjusted by the coefficients $k_{d,n}$. In Figure 8, the control inputs for the experiment with the sprawled boom configuration (I) and the nominal set of parameters are shown. It can be seen that for the damping of the overall structure only low angular velocities are required. The noise of the input variables are caused by the measurement signals of the inclination sensors. It has no negative influence on the control performance.

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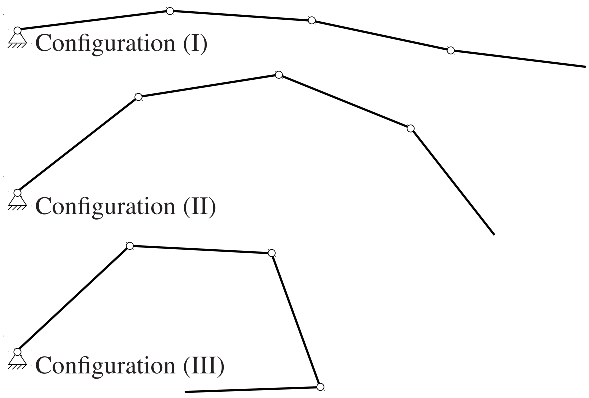

Fig. 4. Boom configurations.

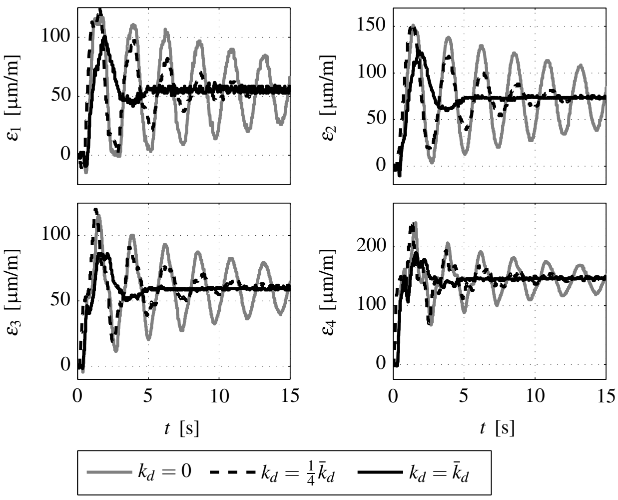

Fig. 5. Strain gauge measurements: boom-configuration (I).

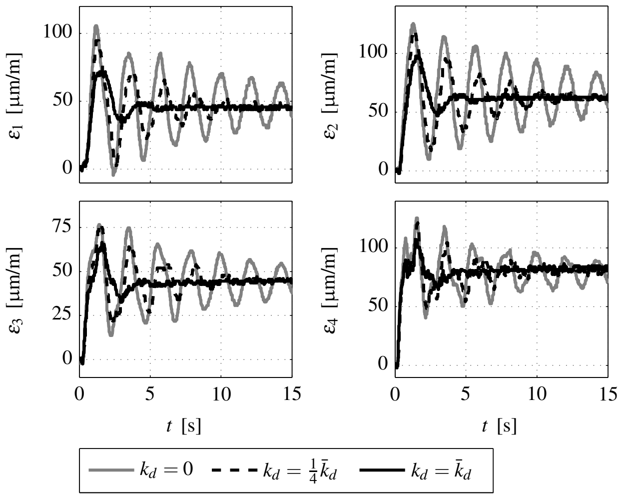

Fig. 6. Strain gauge measurements: boom-configuration (II).

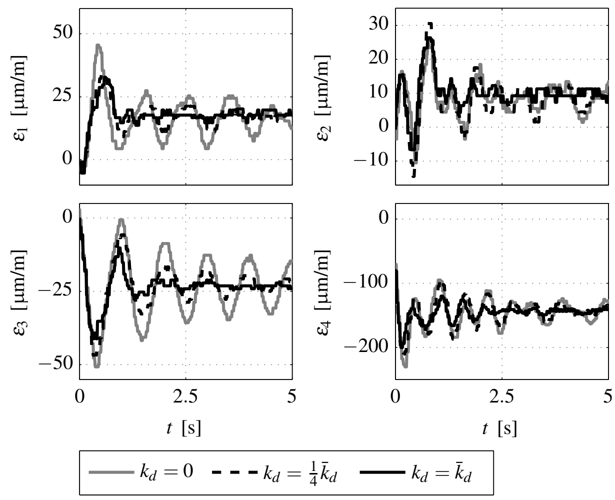

Fig. 7. Strain gauge measurements: boom-configuration (III).

The experimental results demonstrate that the controllers for the particular joints are very easy to parameterize. The performance of the particular position controllers is directly related to the coefficients $k_{p,n}$ whereas the damping ratio can be proportionally adjusted by the coefficients $k_{d,n}$. Due to the fact that the control law is modular and independent of the number and the pose of the boom segments as well as the exact knowledge of the system parameters this control concept proved to be very effective and robust for the industrial use. Furthermore, the practical experiences show that the system could also be damped very effectively if only a few joints are used for the active control. This makes the implementation of this system very flexible and fail safe.

Remark 12 *Considering Remark 4, the joint angles of the boom-configuration (I) are close to the values* $\psi_n = 0$ *for* $n = 2, \ldots, N$ *at which the matrix* $\mathbf{M}$ *gets singular. Since the operating range of the joints 2 and 3 of the considered mobile concrete pump is approximately limited by* $\psi_2, \psi_3 \in [-\pi, 0]$*, a reasonable test at exactly these positions is not possible. However, the experimental results support the assertion that the singularity of* $\mathbf{M}$ *is rather a problem of the mathematical formulation and has no consequence for the controller design or the closed-loop performance.*

7 Conclusions and outlook

In this paper, a control strategy for the damping of the elastic vibrations of large-scale manipulators with hydraulic actuation based on an infinite-dimensional model was presented. The linearized partial differential equations with boundary conditions for a boom with a general number of beams in an arbitrary pose was derived by means of Hamilton's principle. Thereby, an ideal subordinate velocity controller for the hydraulic actuators was presumed and thus the joint angle ve-

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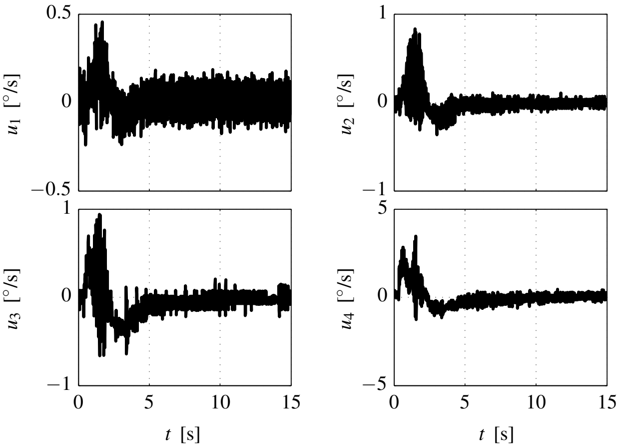

Fig. 8. Control inputs: boom-configuration (I).

locities serve as input variables to the system. It was shown that a modular decentralized feedback of the beam curvature in combination with a position controller renders the closed-loop system dissipative. Furthermore, it was proven that the closed-loop system is asymptotically stable. The proposed control strategy was validated by means of measurements for a typical industrial mobile concrete pump with a modified hydraulic actuation design. For the measurement of the required beam curvature strain gauges were used. The results demonstrate that with the proposed approach the system can be damped in a very effective way independent of the pose of the boom and the number of boom segments.

Although strong and general statements about the stability of planar manipulators composed by serially linked Euler-Bernoulli beams are given in this paper, the theoretical results show the way for further improvements: A stronger theoretical result could be obtained if the largest positive invariant subset can be determined without any damping model. Moreover, the extension of the stability proof to systems with internal Kelvin-Voigt damping or structural damping also constitutes an interesting topic of research. Furthermore, due to the proof of asymptotic stability no information about the decay rate of the oscillations is given. In this context, the proof of exponential stability of the closed-loop system is of interest for the considered systems. However, due to the model complexity, this is much more involved for systems with an arbitray number of serially connected beams. In fact, typically used techniques to prove the exponential stability for single beam structures as described e.g. in [14] are hard to apply or generalize to multi-beam structures. Finally, it was already indicated in the paper that the oscillations of the industrial concrete pump could also be damped if only a few joints are used for the active control. Since this is of strong interest to the practitioners, the extension of the stability proof to systems with a subset of active controllers seems worth striving for.

A Determinant of the matrix M

In the following, the determinant of the matrix (21) will be calculated by means of Gaussian elimination. A closer look at (21) reveals that the elements of the main diagonal can be simplified to

$$\mathbf{M}_{n,n} = \sum_{j=n+1}^{N} \mu_j L_j \sin^2(\theta_j - \theta_n) \tag{A.1}$$

and that $\mathbf{M}$ is symmetric. Since the number of summation elements decreases with a higher row and column index, the Gaussian elimination is getting started at the last column. Assuming $\theta_N - \theta_{N-1} \neq m\pi$, $m \in \mathbb{Z}$, the first step is given by

$$\mathbf{L}_1\mathbf{M} = \begin{bmatrix} 1 & \dots & 0 & -\frac{\mathbf{M}_{N-1,1}}{\mathbf{M}_{N-1,N-1}} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & -\frac{\mathbf{M}_{N-1,N-2}}{\mathbf{M}_{N-1,N-1}} \\ 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{M}_{1,1} & \dots & \mathbf{M}_{N-1,1} \\ \vdots & \ddots & \vdots \\ \mathbf{M}_{N-2,1} & \dots & \mathbf{M}_{N-1,N-2} \\ \mathbf{M}_{N-1,1} & \dots & \mathbf{M}_{N-1,N-1} \end{bmatrix}$$
$$= \begin{bmatrix} \bar{\mathbf{M}}_{1,1} & \dots & \bar{\mathbf{M}}_{N-2,1} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ \bar{\mathbf{M}}_{N-2,1} & \dots & \bar{\mathbf{M}}_{N-2,N-2} & 0 \\ \mathbf{M}_{N-1,1} & \dots & \mathbf{M}_{N-1,N-2} & \mathbf{M}_{N-1,N-1} \end{bmatrix}, \tag{A.2a}$$

with

$$\bar{\mathbf{M}}_{n,n} = \mathbf{M}_{n,n} - \frac{\mathbf{M}^2_{N-1,n}}{\mathbf{M}_{N-1,N-1}}, \tag{A.2b}$$
$$\bar{\mathbf{M}}_{n,k} = \mathbf{M}_{n,k} - \frac{\mathbf{M}_{N-1,n}\mathbf{M}_{N-1,k}}{\mathbf{M}_{N-1,N-1}}. \tag{A.2c}$$

Inserting (21) into (A.2b) yields

$$\bar{\mathbf{M}}_{n,n} = \sum_{j=n+1}^{N} \mu_j L_j \sin^2(\theta_j - \theta_n) - \big(\cos(\theta_{N-1} - \theta_n) - \cos(\theta_N - \theta_n)\cos(\theta_N - \theta_{N-1})\big)^2 \frac{\mu_N L_N}{\sin^2(\theta_N - \theta_{N-1})}.$$

Utilizing the equivalence

$$\big(\cos(\theta_{N-1} - \theta_n) - \cos(\theta_N - \theta_n)\cos(\theta_N - \theta_{N-1})\big)^2 = \sin^2(\theta_N - \theta_{N-1})\sin^2(\theta_N - \theta_n),$$

$\bar{\mathbf{M}}_{n,n}$, $n = 1, \dots, N-2$, reads as

$$\bar{\mathbf{M}}_{n,n} = \sum_{j=n+1}^{N-1} \mu_j L_j \sin^2(\theta_j - \theta_n). \tag{A.3}$$

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Analogously, the matrix elements $\bar{\mathbf{M}}_{n,k}$ for $n = 1,\dots,N-2$ and $k = 1,\dots,N-3$ follow from (A.2c) with (21) in the form

$$\bar{\mathbf{M}}_{n,k} = \sum_{j=n+1}^{N} \mu_j L_j \big(\cos(\theta_n-\theta_k)-\cos(\theta_j-\theta_n)\cos(\theta_j-\theta_k)\big) \\ -\big(\cos(\theta_{N-1}-\theta_n)-\cos(\theta_N-\theta_n)\cos(\theta_N-\theta_{N-1})\big)\times \\ \big(\cos(\theta_{N-1}-\theta_k)-\cos(\theta_N-\theta_k)\cos(\theta_N-\theta_{N-1})\big)\times \\ \frac{\mu_N L_N}{\sin^2(\theta_N-\theta_{N-1})}.$$

The second term on the right-hand side can be simplified to

$$\big(\cos(\theta_{N-1}-\theta_n)-\cos(\theta_N-\theta_n)\cos(\theta_N-\theta_{N-1})\big)\times \\ \big(\cos(\theta_{N-1}-\theta_k)-\cos(\theta_N-\theta_k)\cos(\theta_N-\theta_{N-1})\big) \\ = \sin^2(\theta_N-\theta_{N-1})\big(\cos(\theta_n-\theta_k)-\cos(\theta_N-\theta_n)\cos(\theta_N-\theta_k)\big)$$

by exploiting the product-to-sum identities of trigonometric functions. Finally, this yields

$$\bar{\mathbf{M}}_{n,k} = \sum_{j=n+1}^{N-1} \mu_j L_j \big(\cos(\theta_n-\theta_k)-\cos(\theta_j-\theta_n)\cos(\theta_j-\theta_k)\big). \tag{A.4}$$

Comparing (A.4) with (21) shows that the matrix $\mathbf{L}_1\mathbf{M}$ has the identical structure as the matrix $\mathbf{M}$. Therefore, on the assumption that $\theta_n - \theta_{n-1} \neq m\pi$, $m \in \mathbb{Z}$, the elimination step (A.2a) can be successively repeated until the matrix $\mathbf{M}$ is transformed into the triangular matrix $\mathbf{M}^{\Delta} = \mathbf{L}_{N-2}\dots\mathbf{L}_2\mathbf{L}_1\mathbf{M}$. Since the main diagonal elements of $\mathbf{M}^{\Delta}$ are given by

$$\mathbf{M}^{\Delta}_{n,n} = \mu_{n+1} L_{n+1} \sin^2(\theta_{n+1}-\theta_n) \tag{A.5}$$

for $n = 1,\dots,N-1$, the determinant of $\mathbf{M}^{\Delta}$ and thus of $\mathbf{M}$ reads as

$$\det(\mathbf{M}) = \prod_{n=2}^{N} \mu_n L_n \sin^2(\theta_n-\theta_{n-1}). \tag{A.6}$$

B Proof of boundedness of the inverse operator

In order to prove the boundedness of the inverse operator $\check{\mathscr{A}}$, the existence of a constant C has to be shown such that the inequality $\|\check{\mathscr{A}}z\| \leq C\|z\|$ holds. The square of the norm of $\check{\mathscr{A}}z$, given by

$$\|\check{\mathscr{A}}z\|^2 = \sum_{n=1}^{N} \Bigg[\int_0^{L_n} \Big\{\mu_n \big(\bar{\mathbf{r}}_0^n\big)^T(z)\bar{\mathbf{r}}_0^n(z) \\ +\Lambda_n\Big(-\frac{1}{\Lambda_n}\int_0^{x_n}\int_0^{\xi_n}\big(\gamma_n z_{1,n}+\mu_n z_{2,n}\big)\mathrm{d}\eta_n\mathrm{d}\xi_n \\ -\frac{\mu_n}{2\Lambda_n}x_n^2\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)+\frac{C_{1,n}}{\Lambda_n}x_n+\frac{C_{2,n}}{\Lambda_n}\Big)^2\Big\}\mathrm{d}x_n \\ +\alpha_n\Big(\frac{C_{3,n}}{\Lambda_n}+\frac{1}{\Lambda_{n-1}}\int_0^{L_{n-1}}\int_0^{\xi_{n-1}}\int_0^{\eta_{n-1}}\big(\gamma_{n-1}z_{1,n-1} \\ +\mu_{n-1}z_{2,n-1}\big)\mathrm{d}\varepsilon_{n-1}\mathrm{d}\eta_{n-1}\mathrm{d}\xi_{n-1} \\ +\frac{\mu_{n-1}}{6\Lambda_{n-1}}L_{n-1}^3\sum_{k=1}^{n-2} z_{3,k}\cos(\theta_{n-1}-\theta_k)-\frac{C_{1,n-1}}{2\Lambda_{n-1}}L_{n-1}^2 \\ -\frac{C_{2,n-1}}{\Lambda_{n-1}}L_{n-1}-\frac{C_{3,n-1}}{\Lambda_{n-1}}\Big)^2\Bigg]$$

with, see (7),

$$\bar{\mathbf{r}}_0^n(z) = \begin{bmatrix} -z_{1,n}\sin(\theta_n)-\sum_{k=1}^{n-1} z_{1,k}(L_k)\sin(\theta_k) \\ z_{1,n}\cos(\theta_n)+\sum_{k=1}^{n-1} z_{1,k}(L_k)\cos(\theta_k)\end{bmatrix}, \tag{B.1}$$

can be simplified by substituting the constants $C_{3,n}$ from (31e)

$$\|\check{\mathscr{A}}z\|^2 = \sum_{n=1}^{N} \Bigg[\int_0^{L_n} \Big\{\mu_n \big(\bar{\mathbf{r}}_0^n\big)^T(z)\bar{\mathbf{r}}_0^n(z) \\ +\Lambda_n\Big(-\frac{1}{\Lambda_n}\int_0^{x_n}\int_0^{\xi_n}\big(\gamma_n z_{1,n}+\mu_n z_{2,n}\big)\mathrm{d}\eta_n\mathrm{d}\xi_n \\ -\frac{\mu_n}{2\Lambda_n}x_n^2\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)+\frac{C_{1,n}}{\Lambda_n}x_n+\frac{C_{2,n}}{\Lambda_n}\Big)^2\Big\}\mathrm{d}x_n \\ +\alpha_n\Big(\frac{C_{2,n}}{\alpha_n}-\frac{1}{k_n\alpha_n}\big((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\big)\Big)^2\Bigg]. \tag{B.2}$$

In the following, some useful inequalities and finite sums are introduced.

- **Cauchy-Schwarz inequality:** In the space of the square-integrable functions, the inequality

$$\Big[\int_0^L x(z)y(z)\mathrm{d}z\Big]^2 \leq \int_0^L \big(x(z)\big)^2\mathrm{d}z\int_0^L \big(y(z)\big)^2\mathrm{d}z \tag{B.3}$$

  holds with $x(z), y(z) \in \mathscr{L}_2(0,L)$. Furthermore, in the Euclidean space $\mathbb{R}^N$ we have

$$\Big(\sum_{n=1}^{N} a_n\Big)^2 \leq N\sum_{n=1}^{N} a_n^2. \tag{B.4}$$

- **Poincaré inequality:** Let $x(z) \in \mathscr{C}(0,L)$, then the following inequalities hold:

$$\int_0^L \big(x(z)\big)^2\mathrm{d}z \leq 2L\big(x(0)\big)^2+4L^2\int_0^L \big(\partial_z x(z)\big)^2\mathrm{d}z, \tag{B.5}$$

$$\int_0^L \big(x(z)\big)^2\mathrm{d}z \leq 2L\big(x(L)\big)^2+4L^2\int_0^L \big(\partial_z x(z)\big)^2\mathrm{d}z. \tag{B.6}$$

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- **Finite sums:**

$$\sum_{m=1}^{M} a_m \sum_{n=1}^{m-1} b_n c_{m,n} = \sum_{m=1}^{M} b_m \sum_{n=m+1}^{M} a_n c_{n,m} \tag{B.7}$$

$$\sum_{m=1}^{M} a_m \sum_{n=1}^{m} b_n c_{m,n} = \sum_{m=1}^{M} b_m \sum_{n=m}^{M} a_n c_{n,m} \tag{B.8}$$

$$\sum_{m=1}^{M} a_m \sum_{n=0}^{m-1} 3^n b_{m-n} = \sum_{m=1}^{M} b_m \sum_{n=m}^{M} 3^{n-m} a_n \tag{B.9}$$

$$\sum_{m=k}^{M} a_m \sum_{n=m+1} b_n = \sum_{m=k}^{M} b_m \sum_{n=k}^{m-1} a_n \tag{B.10}$$

$$\sum_{m=k}^{M} a_m \sum_{n=0}^{m-1} 3^n b_{m-n} = \sum_{m=1}^{k} b_m \sum_{n=k}^{M} 3^{n-m} a_n + \sum_{m=k+1}^{M} b_m \sum_{n=m}^{M} 3^{n-m} a_n \tag{B.11}$$

$$\sum_{m=k+1}^{M} a_{m,k} \sum_{n=1}^{m-1} b_n c_{n,m} = \sum_{m=1}^{k} b_m \sum_{n=k+1}^{M} a_{n,k} c_{m,n} + \sum_{m=k+1}^{M} b_m \sum_{n=m+1}^{M} a_{n,k} c_{m,n} \tag{B.12}$$

$$\sum_{m=k+1}^{M} a_m \sum_{n=m+1}^{M} b_n = \sum_{m=k+1}^{M} b_m \sum_{n=k+1}^{m-1} a_n \tag{B.13}$$

Lemma 13 *Let $z_{2,n} \in \mathscr{L}_2(0,L_n)$ and $z_{3,m} \in \mathbb{R}$ with $z_{1,n}(0) = z_{2,n}(0) = z_{3,m}(0) = 0$ for $n = 1,\dots,N$ and $m = 1,\dots,N-1$. Then the inequality*

$$\left(z_{2,n} + \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k)\right)^2 \leq \left(\dot{\mathbf{r}}_0^n\right)^T(z)\dot{\mathbf{r}}_0^n(z) \tag{B.14}$$

holds.

Proof With (7), the substitutions

$$\begin{aligned} \zeta_{s,n} &= \sum_{k=1}^{n-1} z_{3,k} \sin(\theta_k), \\ \zeta_{c,n} &= \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_k) \end{aligned} \tag{B.15}$$

and the addition and subtraction theorems of trigonometry, inequality (B.14) simplifies to

$$\begin{aligned} &\left(z_{2,n} + \cos(\theta_n)\zeta_{c,n} + \sin(\theta_n)\zeta_{s,n}\right)^2 \\ &\leq \left(z_{2,n}\sin(\theta_n) + \zeta_{s,n}\right)^2 + \left(z_{2,n}\cos(\theta_n) + \zeta_{c,n}\right)^2. \end{aligned}$$

Further evaluation yields

$$\left(\sin(\theta_n)\zeta_{c,n} - \cos(\theta_n)\zeta_{s,n}\right)^2 \geq 0,$$

and thus the validity of (B.14) is shown. □

Lemma 14 *Let $z_{2,n} \in \mathscr{L}_2(0,L_n)$ and $z_{3,m} \in \mathbb{R}$ with $z_{1,n}(0) = z_{2,n}(0) = z_{3,m}(0) = 0$ for $n = 1,\dots,N$ and $m = 1,\dots,N-1$. Then the inequality*

$$\begin{aligned} &\Bigg[\left(z_{2,k} + \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j)\right)\cos(\theta_k - \theta_n) \\ &+ \sum_{j=1}^{k-1} z_{3,j} \sin(\theta_k - \theta_j)\sin(\theta_k - \theta_n)\Bigg]^2 \leq 2\left(\dot{\mathbf{r}}_0^k\right)^T(z)\dot{\mathbf{r}}_0^k(z) \end{aligned} \tag{B.16}$$

holds.

Proof Considering the addition and subtraction theorems of trigonometry, the inequality

$$\begin{aligned} &\Big(\left(z_{2,k} + \cos(\theta_k)\zeta_c + \sin(\theta_k)\zeta_s\right)\cos(\theta_k - \theta_n) \\ &+ \left(\sin(\theta_k)\zeta_c - \cos(\theta_k)\zeta_s\right)\sin(\theta_k - \theta_n)\Big)^2 \\ &\leq 2\left(z_{2,k} + \cos(\theta_k)\zeta_c + \sin(\theta_k)\zeta_s\right)^2 + 2\left(\sin(\theta_k)\zeta_c - \cos(\theta_k)\zeta_s\right)^2 \end{aligned} \tag{B.17}$$

is satisfied for the left side of (B.16) with (B.15), (B.4) and $0 \leq \sin^2(\theta_k - \theta_n) \leq 1$ and $0 \leq \cos^2(\theta_k - \theta_n) \leq 1$. Utilizing the identity

$$\begin{aligned} &2\left(z_{2k} + \cos(\theta_k)\zeta_c + \sin(\theta_k)\zeta_s\right)^2 + 2\left(\sin(\theta_k)\zeta_c - \cos(\theta_k)\zeta_s\right)^2 \\ &= 2\left(z_{2k}\sin(\theta_k) + \zeta_s\right)^2 + 2\left(z_{2k}\cos(\theta_k) + \zeta_c\right)^2 \end{aligned}$$

in (B.17) directly shows the validity of Lemma 14. □

Lemma 15 *Let $z_{1,n} \in \mathscr{H}^2(0,L_n)$ and $n = 1,\dots,N$. Then the inequality*

$$\begin{aligned} \int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n \leq 4L_n \sum_{k=0}^{n-1} 3^k \Bigg[&\Big((\partial_{x_{n-k}} z_{1,n-k})(0) \\ &- (\partial_{x_{n-k-1}} z_{1,n-k-1})(L_{n-k-1})\Big)^2 \\ &+ L_{n-k} \int_0^{L_{n-k}} \left(\partial_{x_{n-k}}^2 z_{1,n-k}\right)^2 \mathrm{d}x_{n-k}\Bigg] \end{aligned} \tag{B.18}$$

holds.

Proof By the use of Poincaré's inequality (B.5), the relation

$$\int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n \leq 2L_n\left((\partial_{x_n} z_{1,n})(0)\right)^2 + 4L_n^2 \int_0^{L_n} \left(\partial_{x_n}^2 z_{1,n}\right)^2 \mathrm{d}x_n,$$

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is given. Hence, with the addition and subtraction of $\partial_{x_{n-1}} z_{1,n-1}$ at $x_{n-1} = L_{n-1}$ as well as (B.4) the inequality

$$\int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n \le 4L_n \big((\partial_{x_n} z_{1,n})(0) - (\partial_{x_{n-1}} z_{1,n-1})(L_{n-1})\big)^2 + 4L_n^2 \int_0^{L_n} \big(\partial_{x_n}^2 z_{1,n}\big)^2 \mathrm{d}x_n + 4L_n \big((\partial_{x_{n-1}} z_{1,n-1})(L_{n-1})\big)^2$$

holds. Considering

$$(\partial_{x_{n-1}} z_{1,n-1})(L_{n-1}) = \int_0^{L_{n-1}} \partial_{x_{n-1}}^2 z_{1,n-1} \mathrm{d}x_{n-1} + (\partial_{x_{n-1}} z_{1,n-1})(0),$$

a further addition and subtraction of $\partial_{x_{n-2}} z_{1,n-2}$ at $x_{n-2} = L_{n-2}$ as well as the use of the inequality of Cauchy-Schwarz (B.3) and (B.4) results in

$$\begin{aligned}\int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n \le {} & 4L_n \big((\partial_{x_n} z_{1,n})(0) - (\partial_{x_{n-1}} z_{1,n-1})(L_{n-1})\big)^2 \\ & + 4L_n^2 \int_0^{L_n} \big(\partial_{x_n}^2 z_{1,n}\big)^2 \mathrm{d}x_n \\ & + 12L_n \big((\partial_{x_{n-1}} z_{1,n-1})(0) - (\partial_{x_{n-2}} z_{1,n-2})(L_{n-2})\big)^2 \\ & + 12L_n L_{n-1} \int_0^{L_{n-1}} \big(\partial_{x_{n-1}}^2 z_{1,n-1}\big)^2 \mathrm{d}x_{n-1} \\ & + 12L_n \big((\partial_{x_{n-2}} z_{1,n-2})(L_{n-2})\big)^2.\end{aligned}$$

With the recursive application of the above steps the validity of Lemma 15 can be shown. □

Lemma 16 *Let $z_{1,n} \in \mathscr{H}^2(0, L_n)$ with $z_{1,n}(0) = 0$ and $n = 1, \ldots, N$. Then the inequality*

$$\begin{aligned}\int_0^{L_n} z_{1,n}^2 \mathrm{d}x_n \le 16L_n^3 \sum_{k=0}^{n-1} 3^k \Big[& \big((\partial_{x_{n-k}} z_{1,n-k})(0) \\ & - (\partial_{x_{n-k-1}} z_{1,n-k-1})(L_{n-k-1})\big)^2 \\ & + L_{n-k} \int_0^{L_{n-k}} \big(\partial_{x_{n-k}}^2 z_{1,n-k}\big)^2 \mathrm{d}x_{n-k} \Big]\end{aligned} \tag{B.19}$$

holds.

Proof Considering Poincaré's inequality (B.5) as well as $z_{1,n}(0) = 0$, the inequality

$$\int_0^{L_n} z_{1,n}^2 \mathrm{d}x_n \le 4L_n^2 \int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n$$

is given and thus the validity of (B.19) can be shown by means of Lemma 15. □

Lemma 17 *Let $z_{1,n} \in \mathscr{H}^2(0, L_n)$ with $z_{1,n}(0) = 0$ and $n = 1, \ldots, N$. Then the inequality*

$$\begin{aligned}\sum_{n=1}^{N} \int_0^{L_n} \mu_n \big(\bar{\mathbf{r}}_0^n\big)^T (z) \bar{\mathbf{r}}_0^n (z) \mathrm{d}x_n \le \sum_{n=1}^{N} \Big\{ & K_{1,n} \Lambda_n \int_0^{L_n} \big(\partial_{x_n}^2 z_{1,n}\big)^2 \mathrm{d}x_n \\ & + K_{2,n} \alpha_n \big((\partial_{x_n} z_{1,n})(0) - (\partial_{x_{n-1}} z_{1,n-1})(L_{n-1})\big)^2 \Big\}\end{aligned} \tag{B.20a}$$

holds with

$$\begin{aligned} K_{1,n} &= 8 \frac{L_n}{\Lambda_n} \sum_{k=n}^{N} 3^{k-n} L_k^2 \Big(4\mu_k L_k + \sum_{j=k+1}^{N} (j-1)\mu_j L_j \Big), \\ K_{2,n} &= K_{1,n} \frac{\Lambda_n}{L_n \alpha_n}. \end{aligned} \tag{B.20b}$$

Proof With (B.1), the left side of (B.20a) is given by

$$\begin{aligned}&\sum_{n=1}^{N} \int_0^{L_n} \mu_n \big(\bar{\mathbf{r}}_0^n\big)^T (z) \bar{\mathbf{r}}_0^n (z) \mathrm{d}x_n \\ &= \sum_{n=1}^{N} \int_0^{L_n} \mu_n \Bigg[\Big(-z_{1,n} \sin(\theta_n) - \sum_{k=1}^{n-1} z_{1,k}(L_k) \sin(\theta_k) \Big)^2 \\ &\quad + \Big(z_{1,n} \cos(\theta_n) + \sum_{k=1}^{n-1} z_{1,k}(L_k) \cos(\theta_k) \Big)^2 \Bigg] \mathrm{d}x_n.\end{aligned}$$

Considering (B.4), the inequality

$$\begin{aligned}&\sum_{n=1}^{N} \int_0^{L_n} \mu_n \big(\bar{\mathbf{r}}_0^n\big)^T (z) \bar{\mathbf{r}}_0^n (z) \mathrm{d}x_n \\ &\le \sum_{n=1}^{N} \int_0^{L_n} 2\mu_n \Big[z_{1,n}^2 + (n-1) \sum_{k=1}^{n-1} z_{1,k}^2(L_k) \Big] \mathrm{d}x_n\end{aligned}$$

holds. With the evaluation of the integral for the terms independent of x_n and considering $z_{1,k}(0) = 0$ we may write

$$\begin{aligned}&\sum_{n=1}^{N} \int_0^{L_n} \mu_n \big(\bar{\mathbf{r}}_0^n\big)^T (z) \bar{\mathbf{r}}_0^n (z) \mathrm{d}x_n \\ &\le \sum_{n=1}^{N} 2\mu_n \Bigg[\int_0^{L_n} z_{1,n}^2 \mathrm{d}x_n + L_n (n-1) \sum_{k=1}^{n-1} \Big(\int_0^{L_k} \partial_{x_k} z_{1,k} \mathrm{d}x_k \Big)^2 \Bigg].\end{aligned}$$

Furthermore, application of (B.7) and the inequality of Cauchy-Schwarz (B.3) yields

$$\begin{aligned}\sum_{n=1}^{N} \int_0^{L_n} \mu_n \big(\bar{\mathbf{r}}_0^n\big)^T (z) \bar{\mathbf{r}}_0^n (z) \mathrm{d}x_n \le \sum_{n=1}^{N} 2 \Bigg[& \mu_n \int_0^{L_n} z_{1,n}^2 \mathrm{d}x_n \\ & + L_n \int_0^{L_n} (\partial_{x_n} z_{1,n})^2 \mathrm{d}x_n \sum_{k=n+1}^{N} (k-1) \mu_k L_k \Bigg].\end{aligned}$$

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Finally, utilizing Lemma 15 and 16 in combination with a rearrangement of the sums according to (B.9) shows the validity of Lemma 17. □

Lemma 18 *The constants $C_{1,n}$ comply with the inequality*

$$C_{1n}^2 \leq 48 L_n^4 \gamma_n^2 \sum_{k=0}^{n-1} 3^k \Big\{ \big((\partial_{x_{n-k}} z_{1,n-k})(0) - (\partial_{x_{n-k-1}} z_{1,n-k-1})(L_{n-k-1})\big)^2 + L_{n-k} \int_0^{L_{n-k}} \big(\partial_{x_{n-k}}^2 z_{1,n-k}\big)^2 \mathrm{d}x_{n-k} \Big\} + 3 L_n \mu_n^2 \int_0^{L_n} \big(\dot{\mathbf{r}}_0^n\big)^T (z) \dot{\mathbf{r}}_0^n(z) \mathrm{d}x_n + 6(N-n) \sum_{k=n+1}^{N} \mu_k^2 L_k \int_0^{L_k} \big(\dot{\mathbf{r}}_0^k\big)^T (z) \dot{\mathbf{r}}_0^k(z) \mathrm{d}x_k. \tag{B.21}$$

Proof In the first step, the constants $C_{1,n}$ according to (31c) will be represented in a more convenient form. The constants contain the term $\mathbf{M}_n z_3$ given by

$$\mathbf{M}_n z_3 = \sum_{k=1}^{n-1} \sum_{j=n+1}^{N} \mu_j L_j \Big(\cos(\theta_n - \theta_k) - \cos(\theta_j - \theta_n) \cos(\theta_j - \theta_k) \Big) z_{3,k} + \sum_{j=n+1}^{N} \mu_j L_j \sin^2(\theta_j - \theta_k) z_{3,n} + \sum_{k=n+1}^{N} \sum_{j=k+1}^{N} \mu_j L_j \Big(\cos(\theta_k - \theta_n) - \cos(\theta_j - \theta_n) \cos(\theta_j - \theta_k) \Big) z_{3,k}. \tag{B.22}$$

Furthermore, with (B.12) the identity

$$\sum_{k=n+1}^{N} \cos(\theta_k - \theta_n) \mu_k L_k \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j) = \sum_{k=1}^{n-1} \sum_{j=n+1}^{N} \mu_j L_j \cos(\theta_j - \theta_k) \cos(\theta_j - \theta_n) z_{3,k} + \sum_{j=n+1}^{N} \mu_j L_j \cos^2(\theta_j - \theta_n) z_{3,n} + \sum_{k=n+1}^{N} \sum_{j=k+1}^{N} \mu_j L_j \cos(\theta_j - \theta_k) \cos(\theta_j - \theta_n) z_{3,k}.$$

holds. With this, see (B.22), the constants $C_{1,n}$ due to (31c) can be simplified to

$$C_{1,n} = \sum_{k=1}^{n-1} \cos(\theta_n - \theta_k) z_{3,k} \sum_{j=n+1}^{N} \mu_j L_j + z_{3,n} \sum_{j=n+1}^{N} \mu_j L_j + \sum_{k=n+1}^{N} \cos(\theta_k - \theta_n) z_{3,k} \sum_{j=k+1}^{N} \mu_j L_j + \int_0^{L_n} (\gamma_n z_{1,n} + \mu_n z_{2,n}) \mathrm{d}x_n + \mu_n L_n \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) + \sum_{k=n+1}^{N} \cos(\theta_k - \theta_n) \int_0^{L_k} \mu_k z_{2,k} \mathrm{d}x_k.$$

This representation can be further simplified in several steps with the help of (B.13) and the addition and subtraction theorems of trigonometry,

$$C_{1,n} = \int_0^{L_n} \gamma_n z_{1,n} \mathrm{d}x_n + \int_0^{L_n} \mu_n \Big(z_{2,n} + \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) \Big) \mathrm{d}x_n + \sum_{k=n+1}^{N} \int_0^{L_k} \mu_k \Big[\Big(z_{2,k} + \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j) \Big) \cos(\theta_k - \theta_n) + \sum_{j=1}^{k-1} z_{3,j} \sin(\theta_k - \theta_j) \sin(\theta_k - \theta_n) \Big] \mathrm{d}x_k.$$

By utilizing (B.4) and the inequality of Cauchy-Schwarz (B.3), the following inequality

$$C_{1,n}^2 \leq 3 L_n \gamma_n^2 \int_0^{L_n} z_{1,n}^2 \mathrm{d}x_n + 3 L_n \mu_n^2 \int_0^{L_n} \Big(z_{2,n} + \sum_{k=1}^{n-1} z_{3,k} \cos(\theta_n - \theta_k) \Big)^2 \mathrm{d}x_n + 3(N-n) \sum_{k=n+1}^{N} L_k \mu_k^2 \int_0^{L_k} \Big[\Big(z_{2,k} + \sum_{j=1}^{k-1} z_{3,j} \cos(\theta_k - \theta_j) \Big) \cos(\theta_k - \theta_n) + \sum_{j=1}^{k-1} z_{3,j} \sin(\theta_k - \theta_j) \sin(\theta_k - \theta_n) \Big]^2 \mathrm{d}x_k.$$

can be derived. With this and Lemma 13, 14 and 16, (B.21) is shown. □

Lemma 19 *The constants $C_{2,n}$ comply with the inequality*

$$C_{2,n}^2 \leq 3(N-n+1) \Big\{ \sum_{k=n}^{N} \mu_k^2 L_k \Big(7 L_k^2 + 6 \sum_{j=n}^{k-1} (N-j) L_j^2 \Big) \int_0^{L_k} \big(\dot{\mathbf{r}}_0^k\big)^T (z) \dot{\mathbf{r}}_0^k(z) \mathrm{d}x_k + 112 \sum_{k=1}^{n} \Big[\big((\partial_{x_k} z_{1,k})(0) - (\partial_{x_{k-1}} z_{1,k-1})(L_{k-1})\big)^2 + L_k \int_0^{L_k} \big(\partial_{x_k}^2 z_{1,k}\big)^2 \mathrm{d}x_k \Big] \sum_{j=n}^{N} 3^{j-k} \gamma_j^2 L_j^6 + 112 \sum_{k=n+1}^{N} \Big[\big((\partial_{x_k} z_{1,k})(0) - (\partial_{x_{k-1}} z_{1,k-1})(L_{k-1})\big)^2 + L_k \int_0^{L_k} \big(\partial_{x_k}^2 z_{1,k}\big)^2 \mathrm{d}x_k \Big] \sum_{j=k}^{N} 3^{j-k} \gamma_j^2 L_j^6 \Big\}. \tag{B.23}$$

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Proof Application of (B.4) two times to (31d) and exploiting the inequalities of Cauchy-Schwarz (B.3) and Poincaré (B.5) yields the relation

$$C_{2,n}^2 \le 3(N-n+1)\sum_{k=n}^{N}\left\{4\gamma_k^2 L_k^3\int_0^{L_k} z_{1,k}^2 \mathrm{d}\xi_k + L_k^2 C_{1,k}^2 + 4\mu_k^2 L_k^3 \int_0^{L_k}\left(z_{2,k}+\sum_{j=1}^{k-1} z_{3,j}\cos(\theta_k-\theta_j)\right)^2 \mathrm{d}\xi_k\right\}.$$

With the use of Lemma 13, 16 and 18 together with the summation formulas (B.10) and (B.11), the inequality (B.23) follows. □

Lemma 20 *Let* $z_{1,n}\in\mathscr{H}^2(0,L_n)$, $z_{2,n}\in\mathscr{L}_2(0,L_n)$ *and* $z_{3,m}\in\mathbb{R}$ *with* $z_{1,n}(0)=z_{2,n}(0)=z_{3,m}(0)=0$ *for* $n=1,\dots,N$ *and* $m=1,\dots,N-1$. *Then the inequality*

$$\begin{aligned}&\sum_{n=1}^{N}\frac{1}{\Lambda_n}\int_0^{L_n}\left(-\int_0^{x_n}\int_0^{\xi_n}(\gamma_n z_{1,n}+\mu_n z_{2,n})\mathrm{d}\eta_n\mathrm{d}\xi_n - \frac{1}{2}\mu_n x_n^2\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)+x_n C_{1,n}+C_{2,n}\right)^2\mathrm{d}x_n\\ &\le \sum_{n=1}^{N}\left\{K_{3,n}\int_0^{L_n}\mu_n\left(\dot{\mathbf{r}}_0^n\right)^T(z)\dot{\mathbf{r}}_0^n(z)\mathrm{d}x_n + K_{4,n}\int_0^{L_n}\Lambda_n\left(\partial_{x_n}^2 z_{1,n}\right)^2\mathrm{d}x_n + K_{5,n}\alpha_n\left((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\right)^2\right\}\end{aligned} \tag{B.24a}$$

holds with

$$\begin{aligned}K_{3,n}&=4\mu_n L_n\left[17\frac{L_n^3}{\Lambda_n}+2\sum_{k=1}^{n-1}\frac{L_k^3}{\Lambda_k}(N-k) + 3\sum_{k=1}^{n}\frac{L_k}{\Lambda_k}(N-k+1)\left(7L_n^2+6\sum_{j=k}^{n-1}(N-j)L_j^2\right)\right],\\ K_{4,n}&=4\frac{L_n}{\Lambda_n}\sum_{k=n}^{N}\frac{1}{\Lambda_k}\left(272\gamma_k^2 L_k^7 3^{k-n} + 336 L_k(N-k+1)\sum_{j=k}^{N}3^{j-n}\gamma_j^2 L_j^6 + 336\gamma_k^2 L_k^6 3^{k-n}\sum_{j=1}^{n-1}L_j(N-j+1)\right),\\ K_{5,n}&=\frac{\Lambda_n}{L_n\alpha_n}K_{4,n}\end{aligned} \tag{B.24b}$$

Proof With (B.4), the inequality

$$\begin{aligned}&\sum_{n=1}^{N}\frac{1}{\Lambda_n}\int_0^{L_n}\left(-\int_0^{x_n}\int_0^{\xi_n}(\gamma_n z_{1,n}+\mu_n z_{2,n})\mathrm{d}\eta_n\mathrm{d}\xi_n - \frac{1}{2}\mu_n x_n^2\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)+x_n C_{1,n}+C_{2,n}\right)^2\mathrm{d}x_n\\ &\le \sum_{n=1}^{N}\frac{4}{\Lambda_n}\int_0^{L_n}\left\{\left(\int_0^{x_n}\int_0^{\xi_n}\gamma_n z_{1,n}\mathrm{d}\eta_n\mathrm{d}\xi_n\right)^2 + x_n^2 C_{1,n}^2 + C_{2,n}^2 + \left[\int_0^{x_n}\int_0^{\xi_n}\mu_n\left(z_{2,n}+\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)\right)\mathrm{d}\eta_n\mathrm{d}\xi_n\right]^2\right\}\mathrm{d}x_n\end{aligned}$$

is given. The evaluation of the integral for the terms with the constants $C_{1,n}$ and $C_{2,n}$ and the application of Poincaré's inequality (B.5) for two times yields

$$\begin{aligned}&\sum_{n=1}^{N}\frac{1}{\Lambda_n}\int_0^{L_n}\left(-\int_0^{x_n}\int_0^{\xi_n}(\gamma_n z_{1,n}+\mu_n z_{2,n})\mathrm{d}\eta_n\mathrm{d}\xi_n - \frac{1}{2}\mu_n x_n^2\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)+x_n C_{1,n}+C_{2,n}\right)^2\mathrm{d}x_n\\ &\le \sum_{n=1}^{N}\frac{4}{\Lambda_n}\left\{16\gamma_n^2 L_n^4\int_0^{L_n} z_{1,n}^2\mathrm{d}x_n + \frac{L_n^3}{3}C_{1,n}^2 + L_n C_{2,n}^2 + 16\mu_n^2 L_n^4\int_0^{L_n}\left(z_{2,n}+\sum_{k=1}^{n-1} z_{3,k}\cos(\theta_n-\theta_k)\right)^2\mathrm{d}x_n\right\}.\end{aligned}$$

With this, the validity of Lemma 20 can be shown after some lengthy but straightforward calculations utilizing Lemma 13, 16, 18 and 19 and (B.7), (B.8) and (B.9). □

Lemma 21 *Let* $z_{1,n}\in\mathscr{H}^2(0,L_n)$, $z_{2,n}\in\mathscr{L}_2(0,L_n)$ *and* $z_{3,m}\in\mathbb{R}$ *with* $z_{1,n}(0)=z_{2,n}(0)=z_{3,m}(0)=0$ *for* $n=1,\dots,N$ *and* $m=1,\dots,N-1$. *Then the inequality*

$$\begin{aligned}&\sum_{n=1}^{N}\alpha_n\left[\frac{C_{2,n}}{\alpha_n}-\frac{1}{k_n\alpha_n}\left((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\right)\right]^2\\ &\le \sum_{n=1}^{N}\left\{K_{6,n}\int_0^{L_n}\mu_n\left(\dot{\mathbf{r}}_0^n\right)^T(z)\dot{\mathbf{r}}_0^n(z)\mathrm{d}x_n + K_{7,n}\int_0^{L_n}\Lambda_n\left(\partial_{x_n}^2 z_{1,n}\right)^2\mathrm{d}x_n + K_{8,n}\alpha_n\left((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\right)^2\right\}\end{aligned} \tag{B.25a}$$

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holds with

$$\begin{aligned}
K_{6,n} &= 6\mu_n L_n \sum_{k=1}^{n} \frac{N-k+1}{\alpha_k}\left(7L_n^2+6\sum_{j=k}^{n-1}(N-j)L_j^2\right)\\
K_{7,n} &= 672\frac{L_n}{\Lambda_n}\left(\sum_{k=n}^{N}\frac{N-k+1}{\alpha_k}\sum_{j=k}^{N}3^{j-n}\gamma_j^2 L_j^6\right.\\
&\qquad\left.+\sum_{j=n}^{N}3^{j-n}\gamma_j^2 L_j^6\sum_{k=1}^{n-1}\frac{N-k+1}{\alpha_k}\right)\\
K_{8,n} &= \frac{\Lambda_n}{L_n\alpha_n}K_{7,n}+\frac{2}{\alpha_n^2 k_n^2}
\end{aligned} \tag{B.25b}$$

Proof The validity of Lemma B.25 can be proven in a straightforward way by means of (B.4), Lemma 19, (B.7), (B.8) and Lemma 21. □

According to Lemma 17, 20 und 21 the square of the norm of $\check{\mathscr{A}}z$ fulfills the inequality

$$\begin{aligned}
\left\|\check{\mathscr{A}}z\right\|^2 \leq \sum_{n=1}^{N}\Bigg\{&(K_{3,n}+K_{6,n})\int_0^{L_n}\mu_n\left(\dot{\mathbf{r}}_0^n\right)^T(z)\dot{\mathbf{r}}_0^n(z)\mathrm{d}x_n\\
&+(K_{1,n}+K_{4,n}+K_{7,n})\int_0^{L_n}\Lambda_n\left(\partial_{x_n}^2 z_{1,n}\right)^2\mathrm{d}x_n\\
&+(K_{2,n}+K_{5,n}+K_{8,n})\alpha_n\left((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\right)^2\Bigg\}.
\end{aligned}$$

Since all constants are positive, the inequality

$$\begin{aligned}
\left\|\check{\mathscr{A}}z\right\|^2 \leq \sum_{n=1}^{N}\Bigg\{&\int_0^{L_n}\left(\mu_n\left(\dot{\mathbf{r}}_0^n\right)^T(z)\dot{\mathbf{r}}_0^n(z)+\Lambda_n\left(\partial_{x_n}^2 z_{1,n}\right)^2\right)\mathrm{d}x_n\\
&+\alpha_n\left((\partial_{x_n}z_{1,n})(0)-(\partial_{x_{n-1}}z_{1,n-1})(L_{n-1})\right)^2\Bigg\}\sum_{k=1}^{N}\sum_{j=1}^{8}K_{j,k}
\end{aligned}$$

holds true. Therefore, the boundedness of the inverse operator $\check{\mathscr{A}}$ is shown since

$$\left\|\check{\mathscr{A}}z\right\|^2 \leq \sum_{k=1}^{N}\sum_{j=1}^{8}K_{j,k}\left\|z\right\|^2$$

or

$$\left\|\check{\mathscr{A}}z\right\| \leq \left(\sum_{k=1}^{N}\sum_{j=1}^{8}K_{j,k}\right)^{\frac{1}{2}}\left\|z\right\|,$$

respectively.

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