CIQS Problem Sheet: Chapter 5 (Linear State Space Control)

Lukas Tarra

April 14, 2025

Instructions

- Try to solve the following exercises. If appropriate, use software tools such as Maple or Matlab / Python.
- Focus on understanding and applying the theory established in the lecture notes.
- Your solutions will serve as a basis for discussion in the next Q&A session.

1 Sampling of Nonlinear Systems

Consider the nonlinear system

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = x(t)^2 u(t) \ , \ x(0) = a \tag{1a}$$

$$y(t) = \log\left(x(t)\right) \ . \tag{1b}$$

1. Convert (1) to a sampled-data system of the form (5.12) in the lecture notes.
2. Use the Euler-forward approximation (5.17) from the lecture notes to yield an alternative sampled-data description of (1).
3. Linearize the system (1) around the point $\{x = a, u = 0\}$ (why not $\{x = 0, u = 0\}$?). Discretize this linearized system so that you arrive at the form (5.22) from the lecture notes.
4. Suppose $a = 1$ and $u(t) = \bar{u}\exp(-t)\sigma(t)$, with $\bar{u} = 0.5$ and $\sigma(t)$ being the Heaviside step function. Find a suitable sampling time T_a for discretization. Then simulate the continuous-time system (1), its exact sampled-data representation, the Euler-forward approximation and the sampled-data representation of the linearized system and compare your results.
5. What happens in the case $\bar{u} \geq 1$?
6. Briefly discuss the influence of the chosen sampling time.

2 Sampling of LTI Systems

Complete Exercises 5.4 and 5.5 from the lecture notes.

3 Reachability & Observability

1. Complete Exercises 5.8 and 5.10 from the lecture notes. Make sure you understand why the results for your proofs can already be seen based on the system matrices in Example 5.5 from the lecture notes.

2. Go through Example 5.6 and verify the results.

4 Linear State Space Controllers & Observers

Consider the discrete-time linear, time-invariant system

$$\mathbf{x}_{k+1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3/2 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_k \tag{2a}$$

$$y_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}_k - u_k \ . \tag{2b}$$

1. Is the system (2) globally asymptotically stable as defined by Theorem 5.2 in the lecture notes?

2. Choose desired poles for the closed-loop system that imply global asymptotic stability of the closed loop. Design a state feedback controller directly based on (5.69) and (5.70) from the lecture notes (as the system is already in reachable canonical form by default).

3. Now use Ackermann's formula to derive a controller with the same closed-loop poles. Compare your results.

4. Finally, calculate a full-order observer according to Theorem 5.9 for the desired eigenvalues of the error dynamics matrix at $\lambda_j = 1/20, \ j = 1, \ldots, 4$. Combine the state feedback controller and the full-order state observer according to Figure 5.9 and simulate the closed loop. Compare the result when replacing the full-order observer with the trivial observer of (5.116) from the lecture notes.