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# A QUEUE-BASED DYNAMIC POWER CONTROL APPROACH FOR WIRELESS COMMUNICATION NETWORKS

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## ABSTRACT

A wireless communication network is considered where the data queues of each link are explicitly taken into account. Based on the queue dynamics the optimal transmission powers for minimizing a predefined cost functional are calculated using the theory of optimal control. Thereby certain constraints concerning the transmission powers are explicitly taken into account. The minimization of the cost functional may be related, e.g., to the system-wide maximization of Quality of Service (QoS) in the communication network. It turns out that the necessary conditions for the optimal control strategy are well interpretable and the minimization of the Hamiltonian can be reformulated as a convex optimization problem. The applicability of the method is shown by means of simulation studies.

***Index Terms***— Optimal control, convex optimization, Pontryagin's maximum principle, power control.

## 1. INTRODUCTION

The topic of power control has been extensively studied in the literature. Power control can be utilized for many tasks in multi-user communication such as interference management, energy management, and connectivity management [1]. One possibility is to determine the transmission powers as the solution of a properly chosen optimization problem which is often motivated by achieving an adequate level of QoS for each user in multi-user communication networks [2, 3]. The cost functions need to be accordingly defined, e.g., to maximize the total network throughput, the lowest data rate among all links or the data rate of one specific user [3].

On the one hand, a frequent approach is to consider static optimization problems [2, 3]. Thereby, the optimal transmission powers are typically determined at a specific instant of time taking into account the current status of the network.

On the other hand, dynamic optimization problems or optimal control problems, either in continuous- or discrete-time, can be considered [4]. This approach is particularly interest-ing for wireless communication networks as it enables to determine the optimal transmission powers over a time horizon of finite length, taking into account the fading characteristics of the channel.

In this paper, the data queues in the network are incorporated for the formulation of an optimal control problem (OCP). This is inspired by other works, see, e.g., [4, 5], which show how the data queues in the network are included in the considerations concerning QoS. By utilizing the queue sizes as a measure for QoS it is possible to indirectly account for delays and transmission errors [6]. In the following, the OCP is analyzed revealing several links to previous results in the literature [3]. The basic feasibility of the presented concept is shown by means of simulation studies.

The paper is organized as follows: Section 2 introduces the model of the system which describes the queue dynamics in terms of the transmission powers. The considered OCP is set up in Section 3. Section 4 deals with the solution of the OCP. Sections 5 and 6 contain simulation studies and some conclusions, respectively.

## 2. SYSTEM MODEL

An abstract network with $K$ links is considered where each link describes a transmitter-receiver pair, see Fig. 1. Such a model can be used to describe cellular and ad-hoc networks, see, e.g., [7]. The $i$th link is modelled by means of a queue with size $x_i$ describing the number of bits waiting for transmission. The vector $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_K \end{bmatrix}^\mathrm{T}$ describes the state of the overall system. Moreover, $r_i$ and $u_i$, $i = 1, \dots, K$, denote the time varying bit arrival rates and the transmission powers of link $i$, respectively. The signal-to-interference-plus-noise ratio (SINR) for the $i$th link is given by

$$\text{SINR}_i\left(t, \mathbf{u}\right) = \frac{|h_{ii}|^2 u_i}{\sum\limits_{\substack{j=1 \\ j \neq i}}^{K} |h_{ij}|^2 u_j + \sigma^2} , \qquad (1)$$
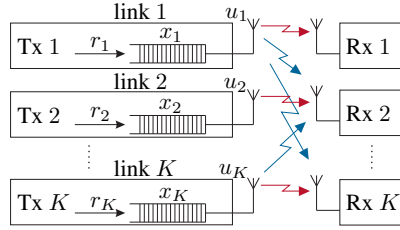
**Fig. 1**. A model for a $K$ link interference channel.

with $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \dots & u_K \end{bmatrix}^{\mathrm{T}}$, where $\sigma^2$ denotes the additive white Gaussian noise (AWGN) power which is assumed to be constant and equal for all receivers. The link gain matrix $\mathbf{H} = [h_{ij}]_{i,j=1,\dots,K}$ results from the channel matrix of a multi-user MIMO broadcast wireless network under consideration of the pre- and decoding matrices. Its elements $h_{ij}$ give the time-varying gains from the $j$th transmitter to the $i$th receiver and contain all losses and modulation factors. The time-varying nature of the link gains $h_{ij}$ is indicated by the argument $t$ of SINR. According to Shannon's capacity formula [8] the achievable data rate of link $i$ is calculated as

$$C_i\left(t,\mathbf{u}\right) = \mathrm{ld}\left(1 + \mathrm{SINR}_i\left(t,\mathbf{u}\right)\right) \qquad (2)$$

where the bandwidth of each link is assumed to be constant and normalized to 1. The queue size for link $i$ is then modelled as an integrator in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}x_i = \dot{x}_i = r_i\left(t\right) - C_i\left(t,\mathbf{u}\right), \quad i = 1,\dots,K . \qquad (3)$$

In the following it is assumed that each link is operating in the high SINR regime, i.e. $\mathrm{SINR}_i\left(t,\mathbf{u}\right) \gg 1 \quad \forall i = 1,\dots,K$ which results in the system differential equations

$$\dot{\mathbf{x}} = \mathbf{f}\left(t,\mathbf{u}\right) = \begin{bmatrix} r_1(t) - \mathrm{ld}\left(\mathrm{SINR}_1\left(t,\mathbf{u}\right)\right) \\ r_2(t) - \mathrm{ld}\left(\mathrm{SINR}_2\left(t,\mathbf{u}\right)\right) \\ \vdots \\ r_K(t) - \mathrm{ld}\left(\mathrm{SINR}_K\left(t,\mathbf{u}\right)\right) \end{bmatrix}. \qquad (4)$$

According to physical limitations the transmission power $\mathbf{u}$ is subject to constraints. In this paper, two types of constraints $\mathbf{u} \in \mathcal{U}_i$ are considered by means of the convex sets

$$\mathcal{U}_1 = \left\{\mathbf{u} \in \mathbb{R}^K \,\middle|\, 0 \le u_i \le u_{i,\max},\ i = 1,\dots,K \right\},$$
$$\mathcal{U}_2 = \left\{\mathbf{u} \in \mathbb{R}^K \,\middle|\, \sum_{i=1}^{K} u_i \le \overline{u}_{\max}, 0 \le u_i, i = 1,\dots,K \right\}. \qquad (5)$$

Here $\mathcal{U}_1$ corresponds to an individual power constraint where $u_{i,\max}$ denotes the maximum transmission power of link $i$, and $\mathcal{U}_2$ describes a sum power constraint with maximum total power $\overline{u}_{\max}$. All subsequent results hold for both, $\mathcal{U}_1$ and $\mathcal{U}_2$. Therefore, a general constraint set $\mathcal{U}$ is introduced which can be equally replaced by $\mathcal{U}_1$ or $\mathcal{U}_2$. The transmit powers will be

allowed to vary over time $t$. Before we formulate the multi-user transmit power control problem, we first need to define the set of admissible time-variant transmit power allocations $\mathcal{U}_{\mathcal{PC}} = \{\mathbf{u}\left(t\right) \in \mathcal{PC}|\ \mathbf{u}\left(t\right) \in \mathcal{U}\ \forall t \in [0,T]\}$ where $T > 0$ specifies a time horizon of finite length and $\mathcal{PC}$ is the vector space of piecewise continuous functions.

## 3. OPTIMAL CONTROL PROBLEM

In the following, an OCP is considered which consists of minimizing a cost functional over a time horizon of finite length subject to the system dynamics (4) and the constraints (5). The manipulable optimization variables are given by the transmission powers $\mathbf{u}$. The OCP reads as

$$\min_{\mathbf{u}(\cdot)} \quad J\left(\mathbf{u}\left(\cdot\right)\right) \qquad (6a)$$

$$\text{s.t.} \quad \dot{\mathbf{x}} = \mathbf{f}\left(t,\mathbf{u}\right), \quad \mathbf{x}\left(0\right) = \mathbf{x}_0 \qquad (6b)$$

$$\mathbf{u}\left(t\right) \in \mathcal{U} \qquad \forall t \in [0,T] \qquad (6c)$$

with $\mathbf{x}_0$ denoting the initial queue sizes at time $t = 0$, the cost functional $J\left(\mathbf{u}\left(\cdot\right)\right) = V\left(\mathbf{x}\left(T\right)\right) + \int_0^T l\left(\mathbf{x}(t),\mathbf{u}(t)\right)\mathrm{d}t$ and $T$ denoting the optimization horizon. The end- and integral cost terms are assumed to be of the form $l\left(\mathbf{x}\left(t\right),\mathbf{u}\left(t\right)\right) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\left(t\right)\mathbf{Q}\mathbf{x}\left(t\right) + \xi\left(\mathbf{u}\left(t\right)\right)$ and $V\left(\mathbf{x}\left(T\right)\right) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\left(T\right)\mathbf{S}\mathbf{x}\left(T\right)$ with the positive definite diagonal weighting matrices $\mathbf{S} = \mathrm{diag}\left(S_1 \ \dots \ S_K\right)$ and $\mathbf{Q} = \mathrm{diag}\left(Q_1 \ \dots \ Q_K\right)$. For reasons which will become clear in Section 4 the function $\xi\left(\cdot\right)$ has to be convex and monotonically increasing.

Solving the OCP (6) requires the knowledge of the link matrix $\mathbf{H}(t)$ and the bit arrival rates $\mathbf{r}(t)$ over the optimization horizon $0 \le t \le T$. We assume perfect channel state information $\mathbf{H}(t)$ and bit arrival rates $\mathbf{r}(t)$ to be available for solving the OCP.

We employ an OCP for transmit power control in order to achieve a desired QoS-level for all links, as this is one of the fundamental objectives in multi-user wireless networks. To this end, QoS may be expressed in terms of the queue sizes $\mathbf{x}$ and/or transmission powers $\mathbf{u}$. The minimization of the queue size of each link corresponds to maximizing QoS because small queue sizes are in general equivalent to a high transmission quality. Incorporating the transmission powers directly into the objective functional representing QoS gives the possibility to achieve a trade-off between high performance and low costs (low transmission powers).

## 4. SOLUTION OF THE OPTIMAL CONTROL PROBLEM

For the solution of the OCP (6) the necessary conditions of the maximum principle of Pontryagin [9] are employed. Henceforth, all optimal system quantities are referred to with the superscript $*$. Therefore, $\mathbf{u}^*$ and $\mathbf{x}^*$ denote the minimizer of the optimization problem (6) with $J\left(\mathbf{u}^*\left(\cdot\right)\right) \le J\left(\mathbf{w}\left(\cdot\right)\right), \forall \mathbf{w}\left(t\right) \in \mathcal{U}_{\mathcal{PC}}$ and the corresponding optimal state

trajectory $\mathbf{x}^*(t)$ with $\mathbf{x}^*(0) = \mathbf{x}_0$, respectively. Necessarily, the optimal solution must satisfy the equations

$$\dot{\mathbf{x}}^* = \left(\frac{\partial H}{\partial \boldsymbol{\lambda}}\right)^{\mathrm{T}} (t, \mathbf{x}^*, \mathbf{u}^*, \boldsymbol{\lambda}^*), \ \mathbf{x}^*(0) = \mathbf{x}_0 \qquad (7a)$$

$$\dot{\boldsymbol{\lambda}}^* = -\left(\frac{\partial H}{\partial \mathbf{x}}\right)^{\mathrm{T}} (t, \mathbf{x}^*, \mathbf{u}^*, \boldsymbol{\lambda}^*), \ \boldsymbol{\lambda}^*(T) = \left(\frac{\partial V}{\partial \mathbf{x}}\right)^{\mathrm{T}} (\mathbf{x}^*(T)) \qquad (7b)$$

$$H(t, \mathbf{x}^*, \mathbf{u}^*, \boldsymbol{\lambda}^*) \leq H(t, \mathbf{x}^*, \mathbf{w}, \boldsymbol{\lambda}^*) \ \ \forall \mathbf{w} \in \mathcal{U}, \ \forall t \in [0, T], \qquad (7c)$$

where $H(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}) = l(\mathbf{x}, \mathbf{u}) + \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{f}(t, \mathbf{u})$ denotes the Hamiltonian with the adjoint states $\boldsymbol{\lambda}(t) \in \mathbb{R}^K$. Evaluation of (7) for the given OCP (6) yields the following two-point boundary value problem

$$\dot{\mathbf{x}}^* = \mathbf{f}(t, \mathbf{u}^*), \quad \mathbf{x}^*(0) = \mathbf{x}_0 \qquad (8a)$$

$$\dot{\boldsymbol{\lambda}}^* = -\mathbf{Q}\mathbf{x}^*, \qquad \boldsymbol{\lambda}^*(T) = \mathbf{S}\mathbf{x}^*(T). \qquad (8b)$$

The optimal input $\mathbf{u}^*$ to the system at time $t \in [0, T]$ follows from solving the static optimization problem (cf. (7c))

$$\mathbf{u}^* = \arg\min_{\mathbf{u} \in \mathcal{U}} \Omega(\mathbf{u}) \qquad (9a)$$

$$\Omega(\mathbf{u}) = \xi(\mathbf{u}) - \sum_{i=1}^{K} \lambda_i^* \, \mathrm{ld}(\mathrm{SINR}_i(t, \mathbf{u})), \qquad (9b)$$

where all terms in $H(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda})$ not explicitly depending on $\mathbf{u}$ are neglected.

Obviously, the minimization of the Hamiltonian (9) is equivalent to the constrained maximization of the weighted sum rate in the network augmented by the cost of the transmission powers $\xi(\mathbf{u})$. Therefore, the optimal transmission powers $\mathbf{u}^*(t)$ maximize the weighted sum rate in the network at every instant $t \in [0, T]$ under additional consideration of minimizing the costs in the sense of $\xi(\mathbf{u})$. This establishes a direct link to existing results on the static optimization of the total weighted sum rate in a communication network, see, e.g., [7]. The weights at time $t$ are given by the value of the adjoint variables $\boldsymbol{\lambda}^*(t)$. According to (8b), the properties of $\mathbf{Q}$ and $\mathbf{S}$ and the fact that $\mathbf{x}^* > \mathbf{0}$ must hold, the weights satisfy $\boldsymbol{\lambda}^*(t) > \mathbf{0} \ \forall t \in [0, T]$.

The crucial part with regard to the numerical solution of the boundary value problem (8) is given by the minimization of the Hamiltonian (9). The cost function (9b) is a non-convex function which means that a solution of (9a) is possibly only a local minimum. However, based on the ideas presented in [10] the optimization problem (9) can be replaced by an equivalent convex optimization problem which results in a unique minimizer $\mathbf{u}^*$. To this end, note that instead of (9) the equivalent optimization problem

$$\min_{\mathbf{u} \in \mathcal{U}, \mathbf{s}} \quad \xi(\mathbf{u}) + \mathbf{s}^{\mathrm{T}} \boldsymbol{\lambda}^* \qquad (10a)$$

$$\text{s.t.} \quad \frac{1}{\mathrm{SINR}_i(t, \mathbf{u})} \leq 2^{s_i}, \ i = 1, \ldots, K \qquad (10b)$$

with additional variables $\mathbf{s} \in \mathbb{R}^K$ can be solved. By introducing the transformation $u_i = g_i(\mathbf{v}) = 2^{v_i}$ with new variables $\mathbf{v} \in \mathbb{R}^K$ and $\mathbf{v} = \mathbf{g}^{-1}(\mathbf{u})$ the optimization problem (10) is reformulated in convex form as

$$\min_{\mathbf{v}, \mathbf{s}} \quad \xi(\mathbf{g}(\mathbf{v})) + \mathbf{s}^{\mathrm{T}} \boldsymbol{\lambda}^* \qquad (11a)$$

$$\text{s.t.} \quad \mathrm{ld}\left(\sum_{\substack{j=1 \\ j \neq i}}^{K} 2^{\mathrm{ld}(c_{ij}) + v_j - v_i - s_i} + 2^{\mathrm{ld}(d_i) - v_i - s_i}\right) \leq 0$$
$$i = 1, \ldots, K \qquad (11b)$$

$$\mathbf{g}(\mathbf{v}) \in \mathcal{U} \qquad (11c)$$

with $c_{ij} = \frac{|h_{ij}|^2}{|h_{ii}|^2}$, $d_i = \frac{\sigma^2}{|h_{ii}|^2}$. Concerning the convexity, note that if $\xi(\cdot)$ is convex and monotonically increasing then the function $\xi(\mathbf{g}(\mathbf{v}))$ is convex in $\mathbf{v}$. Therefore, the cost function as a sum of $\xi(\mathbf{g}(\mathbf{v}))$ and a linear function of $\mathbf{s}$ is convex. The first constraint (11b) is known to be a convex function [11]. The constraint (11c) can be explicitly written as

$$v_i - \mathrm{ld}(u_{i,\max}) \leq 0, \quad i = 1, \ldots, K \qquad (12a)$$

in the case of $\mathcal{U} = \mathcal{U}_1$ and

$$\sum_{i=1}^{K} 2^{v_i} \leq \overline{u}_{\max} \qquad (12b)$$

for $\mathcal{U} = \mathcal{U}_2$ and is therefore convex in both cases.

Based on the results presented in [12] it can be shown under weak assumptions that the conditions of the maximum principle are not only necessary but also sufficient for $\mathbf{u}^*$ to be the optimal solution of the considered problem. Calculating the optimal state trajectories $\mathbf{x}^*(t)$ with inputs $\mathbf{u}^*(t)$ over the horizon $[0, T]$ corresponds to solving the two-point boundary value problem (8) which can be achieved using a classical collocation method. The optimal transmission powers $\mathbf{u}^* = \mathbf{g}(\mathbf{v}^*)$ at each instant $t$ for given $\boldsymbol{\lambda}^*(t)$ can be obtained in an efficient way as the solution of the convex optimization problem (11) and are therefore uniquely determined.

## 5. SIMULATION STUDIES

The presented method of obtaining the transmission powers by solving an OCP is applied to a practical example of a wireless network consisting of $K = 4$ links. The flat Rayleigh fading channel is generated based on the assumption that each user is moving with an average speed of $v_U = 0.83\,\mathrm{m/s}$ (pedestrian). The carrier frequency is chosen according to the 3GPP standard for Long Term Evolution as $f_c = 2.6\,\mathrm{GHz}$. For example, the absolute values of the time variant link matrix entries $|h_{1j}|$, $j = 1, \ldots, 4$ are shown in Fig. 2(a). The bit arrival rates are chosen according to Fig. 2(b).

As already mentioned in Section 3, it is assumed particularly with regard to a practical implementation that suitable
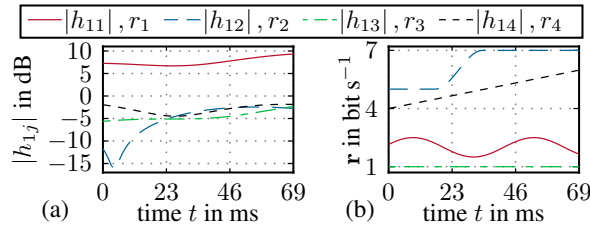
**Fig. 2**. Absolute values of the time variant link matrix entries $|h_{1j}|$, $j = 1, \ldots, 4$, and bit arrival rates.
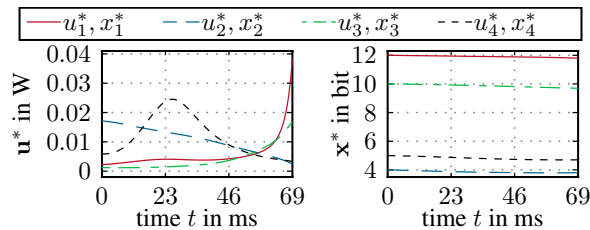


**Fig. 3**. Optimal inputs and queue sizes.

predictions of the bit arrival rates and the link matrix are available. Therefore, the optimization horizon must be restricted to $T_{\max} = \frac{1}{2f_D}$ with the Doppler shift $f_D = \frac{v_U f_c}{c_0}$ and the speed of light $c_0$. In the present case the optimization horizon is set to $T = 69$ ms. The weighting function for the inputs is chosen as

$$\xi(\mathbf{u}) = \frac{1}{2}\mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u} \qquad (13)$$

with $\mathbf{R} = \mathrm{diag}\begin{pmatrix} 1 & 10^3 & 1 & 1 \end{pmatrix} 1/\mathrm{W}^2$ which fulfills the requirement of convexity and is monotonically increasing for $\mathbf{u} > \mathbf{0}$. The remaining parameters are (measurement units are omitted for brevity)

$$\mathbf{x}_0 = \begin{bmatrix} 12 & 4 & 10 & 5 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{Q} = \mathrm{diag}\begin{pmatrix} 1 & 5 & 1 & 3 \end{pmatrix} \quad (14\mathrm{a})$$
$$\sigma^2 = 10^{-4}, \quad \text{and} \quad \mathbf{S} = 10^{-2}\mathrm{diag}\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}. \quad (14\mathrm{b})$$

The admissible set of inputs is chosen as $\mathcal{U} = \mathcal{U}_1$ with $\mathbf{u}_{\max} = \begin{bmatrix} 0.1 & 0.2 & 0.15 & 0.3 \end{bmatrix}^{\mathrm{T}}\mathrm{W}$. The optimal transmission powers as the solution of the boundary value problem (8) and the static optimization problem (11) as well as the optimal state trajectories are depicted in Fig. 3.

## 6. CONCLUSION

This contribution investigated the use of optimal control theory for determining the optimal transmission powers in a $K$ link wireless network. The goal was to calculate the transmission powers in order to minimize a system-wide cost functional. The necessary conditions for the optimal inputs and state trajectories are well interpretable and have nice relations to existing methods for the static optimization of the total weighted sum rate in a communication network. The

minimization of the Hamiltonian was replaced by a convex optimization problem which allows to calculate the unique optimal transmission powers in an efficient way.

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