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Flatness-Based Torque Control of Saturated Surface-Mounted Permanent Magnet Synchronous Machines

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Flatness-based Torque Control of Saturated Surface-Mounted Permanent Magnet Synchronous Machines

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Abstract-Modern permanent magnet synchronous machines (PMSMs) may also be operated in regimes where significant magnetic saturation occurs. Classical fundamental wave models do not incorporate magnetic saturation in a systematic way. Mostly only heuristic extensions can be found in literature. Control schemes based on such dq0-models are thus often unable to achieve the given control demands. This paper proposes a flatness-based torque control strategy for a saturated surfacemounted permanent magnet synchronous machine. Different to existing works, magnetic saturation is considered in a thorough and physically consistent way. Based on a calibrated magnetic equivalent circuit model of the PMSM, a simplified model suitable for the flatness-based controller design is derived. The proposed two-degrees-of-freedom control scheme inherently accounts for the mutual coupling of the phase windings. Furthermore, the time-varying controller gains are obtained by pole placement technique. In contrast to the majority of controllers used for PMSM control, the proposed control scheme is formulated in the stator-fixed reference frame and hence no coordinate transformation is necessary. The flatness-based torque controller is implemented on an experimental test-bench. An accelerated run-up of the PMSM with about four times the rated torque is performed to highlight the feasibility of the proposed approach. Finally, the flatness-based torque controller is compared with a common vector control implementation based on a dq0-model of the motor.

Index Terms—Electric motor, permanent magnet motor, magnetic saturation, magnetic equivalent circuit, flatness-based control, torque control, optimization, experimental validation.

I. INTRODUCTION

E LECTRIC drive systems composed of permanent magnet synchronous machines (PMSMs) are often used in various technical applications, e.g., industrial machine tools, traction drives or automotive applications. This is mainly due to their high performance and high energy efficiency. Model-based controller designs for PMSMs are primarily based on fundamental wave models. The main assumptions in these models are sinusoidally distributed flux linkages of the stator windings, a magnetic linear behavior of the iron core material (i.e. no saturation effects) and negligible iron losses (including both eddy current and hysteresis losses). Application of the Blondel-Park transformation directly yields the well-known dq0-model. In this model, the electromagnetic quantities are independent of the rotor angle and hence lower bandwidths are

The authors are with the Automation and Control Institute, Technische Universität Wien, Vienna, Austria (email: {faustner,kemmetmueller,kugi}@acin.tuwien.ac.at). Manuscript received June 8, 2015; accepted October 17, 2015. sufficient for the current controllers. Several control strategies have been developed based on classical dq0-models, where the most common is known as field-oriented control (FOC), see, e.g., [1] and [2]. Field-oriented control, also referred to as vector control, typically combines a dynamic decoupling scheme with two proportional-integral (PI) controllers to control the motor currents along desired reference trajectories. More advanced control strategies comprise feedback linearization control [3]–[6], backstepping methods [7]–[9], passivity-based control [10]–[12], sliding-mode approaches [13]–[15] and model-predictive control [16]–[18]. Another frequently used method, which is not based on the dq0-transformation, is known as direct-torque control (DTC), see, e.g., [19]–[21].

Although control systems based on fundamental wave models have been successfully implemented in a variety of applications, there seems to be a paradigm shift in recent years. Traditionally, motors were designed in such a way that the use of fundamental wave models is justified by the construction. In recent years, the emphasis has been set on strategies to reduce the overall costs, for example by motor designs which are more suitable to machine production. Amongst others, such measures comprise the replacement of integer-slot windings by fractional-slot concentrated windings and the application of simpler rotor structures, e.g., rotors with internal-mounted magnets (IPMSM). Besides manufacturing aspects, PMSMs are often designed to generate a maximum torque for a thermal-limited short-time operation in the overload range. Therein, almost all PMSMs suffer from significant magnetic saturation. The aforementioned aspects can result in inhomogeneous air gap geometries, non-sinusoidal flux linkages and iron saturation. As a consequence, the assumptions of the fundamental wave model are more or less violated for many actual motor designs. Therefore, suitable mathematical models that accurately describe these motors are required. Furthermore, advanced (model-based) control strategies are strived for to guarantee high control performance in all operating points of the machine.

In literature, to the best of the authors' knowledge, no systematic model-based approach for the design of appropriate control strategies has been presented that inherently incorporates spatial distributions of the flux linkages and magnetic saturation in a thorough and physically consistent way. Almost all presented works concerning the non-ideal behavior of modern PMSMs rely, more or less, on heuristic extensions of the fundamental wave model. Several works have been

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published on how to reduce torque and speed ripples caused by the non-sinusoidal flux distributions, in particular at low rotor speeds, see, e.g., [22]-[27]. Most of these works are based on harmonic extensions of the fundamental wave of the flux linkages, either calibrated with offline measurements, finite-element (FE) results or online estimation strategies. Magnetic saturation, however, is neglected in these works. A common approach to include magnetic saturation is to vary the electromagnetic quantities (i.e. the inductances and the permanent magnet fluxes) of the fundamental wave model with respect to the load (i.e. currents), see, e.g., [28]-[33]. Again, these models are calibrated numerically or experimentally. A consequence of this approach is that the exact physical meaning of the derived model quantities is to a certain extent lost. To be consistent with physical principles, advanced control strategies based on comprehensive mathematical models seem to be promising to exploit the overall motor performance in the whole operating range.

This paper presents a flatness-based torque control strategy for a surface-mounted permanent magnet synchronous machine (SMPMSM). The operating range includes overload operation where significant magnetic saturation is present. Starting with a calibrated magnetic equivalent circuit model (MEC) in Section II, a simplified model is derived in Section III in view of the demands of the controller design. The proposed two-degrees-of-freedom torque control strategy developed in Section IV is composed of a flatness-based feedforward controller and a time-variant feedback error controller. The controller is implemented on a test bench using an industrial electric drive system and is compared with a common vector control implementation. The corresponding results are presented in Section V.

II. MODELING OF PMSM

The PMSM under consideration is equipped with a rotor comprising 10 surface-mounted neodymium-iron-boron (Nd-FeB) magnets (number of pole pairs p = 5), which are alternately magnetized. Fig. 1 shows a cross-sectional view of the PMSM. The stator teeth are equipped with individual stator coils (number of windings $N_c = 70$), where three consecutive coils are connected in series to form single phase windings (labeled a, b and c). The three phase windings are wye-connected and the neutral point is inaccessible (isolated neutral). Such winding diagrams are usually called fractional-slot concentrated windings.

Besides numerically expensive methods like FE, the systematic incorporation of complex geometries and magnetic saturation into electric machine models still represents a challenging task. In this context, MEC modeling serves as a promising and powerful tool to account for these challenges with a manageable modeling complexity. Although MEC modeling is frequently used in the design of electric machines, it has not been exploited for the controller design. The basic idea of MEC modeling is the representation of the electromagnetic behavior in form of a network composed of concentrated elements, namely magnetic conductances (permeances) and magnetomotive force (mmf) sources. The permeances cover



Fig. 1. Cross-sectional view of the PMSM under consideration.

the specific geometry and material behavior, whereas the mmf sources represent the stator coils and rotor magnets.

In [34], a framework for MEC modeling of a PMSM with internal-mounted magnets was proposed that allows for the calculation of the model equations by means of graph theory, with a systematic choice of a minimal set of state variables and the systematic incorporation of the electrical interconnection of the stator coils. Because the suggested framework is not limited to a specific motor design, this procedure was also applied to the surface-mounted PMSM considered in this paper. Subsequently, the main results are briefly summarized to introduce the set of equations used. For details, the reader is referred to [35].

The permeance network proposed in [35] consists of mmf sources of the stator coils and rotor magnets, magnetically nonlinear iron permeances of the stator teeth and yoke, magnetically linear leakage permeances in the slot-opening areas between adjacent teeth as well as rotor angle dependent magnetically linear air gap permeances covering the magnetic coupling between the stator and the rotor. The systematic derivation of the network is based on graph theory, where a suitable tree has to be defined, which covers all nodes of the network without forming meshes. Additionally, all mmf sources have to be included in the spanning tree, cf. [35]. Elements outside the tree form the co-tree. The fluxes of the tree ϕ_t are grouped into coil fluxes $\phi_{tc} \in \mathbb{R}^{9 \times 1}$, fluxes of the permanent magnets $\phi_{tm} \in \mathbb{R}^{18 \times 1}$ and permeance fluxes the permanent magnets $\varphi_{tm} \in \mathbb{R}^{3}$ and permeaner matrix $\phi_{tg} \in \mathbb{R}^{18\times 1}$. The co-tree only contains permeance fluxes $\phi_c \in \mathbb{R}^{27\times 1}$. The same partitioning is performed for the mmfs, i.e. $\mathbf{u}_{tc} \in \mathbb{R}^{9\times 1}$, $\mathbf{u}_{tm} \in \mathbb{R}^{18\times 1}$, $\mathbf{u}_{tg} \in \mathbb{R}^{18\times 1}$ and $\mathbf{u}_c \in \mathbb{R}^{27\times 1}$. The mmfs of the coils are expressed as $\mathbf{u}_{tc} = \mathbf{N}_c \mathbf{V}_c \bar{\mathbf{i}}_{tc}$ with the winding matrix $\mathbf{N}_c = \operatorname{diag}(N_c)$ and the independent electric phase currents $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$, fulfilling $i_a + i_b + i_c = 0$ due to the isolated neutral point. The non-square interconnection matrix \mathbf{V}_c represents the electrical interconnection of the individual stator coils and also determines the vector of independent electric voltages $\bar{\mathbf{v}}_{tc} = [v_{ac}, v_{bc}]^T$ in the form

$$\mathbf{i}_{tc}^{T}\mathbf{v}_{tc} = \left(\bar{\mathbf{i}}_{tc}\right)^{T} \left(\mathbf{V}_{c}\right)^{T} \mathbf{v}_{tc} = \left(\bar{\mathbf{i}}_{tc}\right)^{T} \bar{\mathbf{v}}_{tc}, \qquad (1)$$

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with the independent electric line-to-line voltages $v_{ac} = v_a - v_c$ and $v_{bc} = v_b - v_c$. The (constant) mmfs of the magnets are determined by the coercive field strength and the magnet height. The magnetic interconnection of the elements of the permeance network can be systematically described with the incidence matrix $\mathbf{D} \in \mathbb{R}^{45 \times 27}$, which, for convenience, is factorized in the form $\mathbf{D}^T = [\mathbf{D}_c^T, \mathbf{D}_m^T, \mathbf{D}_g^T]$ and composed of elements $\{-1, 1, 0\}$. Please note that the incidence matrix \mathbf{D} defines the fundamental relationship

$$\begin{bmatrix} \boldsymbol{\phi}_t \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & -\mathbf{D}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}_c \\ \mathbf{u}_t \end{bmatrix}$$
(2)

of the network, with the magnetic fluxes $\phi_t^T = [\phi_{tc}^T, \phi_{tm}^T, \phi_{tg}^T]$ and the mmfs $\mathbf{u}_t^T = [\mathbf{u}_{tc}^T, \mathbf{u}_{tm}^T, \mathbf{u}_{tg}^T]$ of the tree elements and the magnetic fluxes ϕ_c and the mmfs \mathbf{u}_c of the co-tree elements.

The constitutive equations, i.e. the relationships between the mmfs and the fluxes of the permeances of the tree and co-tree, are given by

$$\begin{bmatrix} \boldsymbol{\phi}_{tg} \\ \boldsymbol{\phi}_{c} \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{t} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{c} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{tg} \\ \mathbf{u}_{c} \end{bmatrix}, \qquad (3)$$

where the elements of the diagonal permeance matrices of the tree $\mathbf{G}_t \in \mathbb{R}^{18 \times 18}$ and co-tree $\mathbf{G}_c \in \mathbb{R}^{27 \times 27}$ depend either on \mathbf{u}_c or \mathbf{u}_t due to saturation (iron permeances), on the rotor angle φ (radial airgap permeances) or are constant (leakage permeances).

Substitution of (3) into (2), consideration of the introduced partitioning of the mmfs and fluxes of the tree and co-tree as well as a straightforward rearrangement directly results in the machine model with current input of the PMSM, which describes the influence of the electric currents and the rotor angle φ on the mmfs and magnetic fluxes in form of a set of nonlinear algebraic equations

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{D}_{c}\mathbf{G}_{c}\mathbf{D}_{g}^{T} \\ \mathbf{0} & \mathbf{I} & \mathbf{D}_{m}\mathbf{G}_{c}\mathbf{D}_{g}^{T} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{t} + \mathbf{D}_{g}\mathbf{G}_{c}\mathbf{D}_{g}^{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}_{tc} \\ \boldsymbol{\phi}_{tm} \\ \mathbf{u}_{tg} \end{bmatrix} = (4)$$
$$-\mathbf{D}\mathbf{G}_{c}\left(\mathbf{D}_{c}^{T}\mathbf{u}_{tc} + \mathbf{D}_{m}^{T}\mathbf{u}_{tm}\right).$$

Therein, the unknown variables are the coil fluxes ϕ_{tc} , the fluxes of the permanent magnets ϕ_{tm} and the mmfs \mathbf{u}_{tg} of the permeances of the tree. The inputs on the right-hand side of (4) are the mmfs of the coils \mathbf{u}_{tc} and the (constant) mmfs of the magnets \mathbf{u}_{tm} . If the electric currents $\mathbf{\bar{l}}_{tc}$ are known for a given rotor angle φ , (4) can be numerically solved for \mathbf{u}_{tg} and the fluxes can be simply calculated from linear equations.

Based on co-energy considerations of the elements of the permeance network and due to the fact that the chosen network only exhibits a dependence on the rotor angle in the radial air gap permeances, the developed electromagnetic torque can be expressed as, see [34],

$$\tau = \frac{1}{2} \mathbf{u}_{tg}^T \frac{\partial \mathbf{G}_t}{\partial \varphi} \mathbf{u}_{tg} + \frac{1}{2} \mathbf{u}_t^T \mathbf{D} \frac{\partial \mathbf{G}_c}{\partial \varphi} \mathbf{D}^T \mathbf{u}_t.$$
 (5)

Modern electric drive systems are typically equipped with a voltage source inverter (VSI) triggered by pulse-width modulation (PWM). Thus, the electric voltages at the machine terminals serve as control inputs and the flux dynamics have

to be added to obtain a machine model with voltage input. In [34] and [35], a comprehensive analysis and a systematic elimination of magnetic and electric redundancies was carried out to obtain a set of differential-algebraic equations (DAE) of minimum dimension. Subsequently, a slightly different and more descriptive approach is introduced to obtain a reduced set of differential equations for the corresponding flux linkages.

Consider a wye-connected three-phase winding system. Application of Faraday's induction law directly yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{abc} = -\mathbf{R}\mathbf{i}_{abc} + \left(\mathbf{v}_{abc} - \mathbf{v}_n\right),\tag{6}$$

with the flux linkages $\psi_{abc} = [\psi_a, \psi_b, \psi_c]^T$ of the phase windings, the phase currents $\mathbf{i}_{abc} = [i_a, i_b, i_c]^T$, the phase winding resistances $\mathbf{R} = \text{diag}(R)$, and the terminal and neutral point voltages $\mathbf{v}_{abc} = [v_a, v_b, v_c]^T$ and $\mathbf{v}_n = [v_n, v_n, v_n]^T$ with respect to the reference potential of the VSI. Due to the isolated neutral point, only two phase currents are independent. Hence, there is a redundancy in the voltage equation (6). This redundancy can be systematically eliminated by the invertible transformation matrix

$$\mathbf{V}_{v} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$
 (7)

Left multiplying the voltage equation (6) with V_v yields

$$\frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{bmatrix} \psi_a - \psi_c \\ \psi_b - \psi_c \end{bmatrix}}_{\tilde{\psi}_{tc}^I} = -\underbrace{\begin{bmatrix} 2R & R \\ R & 2R \end{bmatrix}}_{\tilde{\mathbf{R}}} \underbrace{\begin{bmatrix} i_a \\ i_b \end{bmatrix}}_{\tilde{\mathbf{i}}_{tc}} + \underbrace{\begin{bmatrix} v_a - v_c \\ v_b - v_c \end{bmatrix}}_{\tilde{\mathbf{v}}_{tc}} \quad (8a)$$
$$\frac{\mathrm{d}}{\mathrm{d}t} (\psi_a + \psi_b + \psi_c) = -R(\underbrace{i_a + i_b + i_c}_{=0}) + (v_0 - 3v_n), \quad (8b)$$

with $v_0 = v_a + v_b + v_c$. In (8b), the sum of the flux linkages vanishes for an ideal symmetric three-phase machine without leakage fluxes. In this case, the neutral point voltage v_n is determined by forcing the voltage term in (8b) to zero (i.e. floating neutral). In a real machine, however, $\psi_a + \psi_b + \psi_c$ does not necessarily have to be zero. In any case, the set of differential equations (8a) and (8b) are decoupled and neither $\psi_a + \psi_b + \psi_c$ nor v_0 and v_n have an influence on the torque. Thus, (8b) is of no interest for the further derivations. Please note that an equivalent formulation of (8a) can be obtained by a more general analysis of \mathbf{V}_c and \mathbf{D} as given in [34] and applied in [35]. The vector of independent flux linkages $\tilde{\psi}_{tc}^I$ and the resistance matrix $\mathbf{\bar{R}}$ in (8a) can be calculated in the form

$$\begin{split} \tilde{\boldsymbol{\psi}}_{tc}^{I} &= -\mathbf{V}_{c}^{T} \mathbf{N}_{c} \boldsymbol{\phi}_{tc} \quad (9a) \\ \bar{\mathbf{R}} &= 1/3 \mathbf{V}_{c}^{T} \mathbf{R} \mathbf{V}_{c}. \quad (9b) \end{split}$$

In conclusion, the machine model with voltage input of the PMSM is given by the DAE system (8a), (4) and (5). If the DAE system (8a) and (4) is intended to be used for dynamical simulations with the flux linkages as state variables and the electric voltages as inputs, the independent electric phase currents $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$ have to be calculated from the set of nonlinear algebraic equations (4) for given flux linkages. In order to do so, the magnetic and electric redundancies

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still present in (4) have to be eliminated. A comprehensive discussion on how this can be done systematically is given in [34] and applied to the considered PMSM in [35]. However, as it will be shown in the subsequent sections, this reduction is not necessary for the design of a controller, if the independent electric phase currents can be measured.

To demonstrate the quality of the model and thus the feasibility of the modeling approach, some simulation and experimental results are briefly recapitulated, see [35]. Thereby, the nominal model was parameterized by geometric and material data and calibrated by measurements. To validate the machine model with current input (4) and (5), the PMSM was supplied with direct current $i_a = -i_b$, $i_c = 0 A$ and driven by a harmonic drive system at a constant rotational speed. The speed was chosen sufficiently low, such that the influence of the back electromotive force (back-emf) is negligible. The rotor angle φ was measured by a high resolution encoder and the torque τ was measured using a torque transducer. Fig. 2(a) shows the torque as a function of the rotor angle for $i_a = -i_b = 5 \text{ A}$, $i_c = 0 \text{ A}$. A very good agreement of both the shape and the amplitude of the simulated and measured torque can be identified. As can be further seen in Fig. 2, the main electromagnetic quantities including the currents, the voltages, the flux linkages and the torque show a periodicity of 72°, which results from the number of pole pairs (p = 5). Fig. 2(b) depicts the results for $i_a = -i_b = 20$ A, $i_c = 0$ A. This increased current leads to a significant influence of magnetic saturation of the iron core, which can be seen as the corresponding torque does not increase linearly with the current (as would be the case for an unsaturated motor). The proposed model still accurately resembles the measured torque. To evaluate the influence of magnetic saturation on the amplitude of the torque, this experiment was repeated for different operating points. The resulting current-to-torque characteristic is depicted in Fig. 3. The strong influence of magnetic saturation can be recognized and it can be seen that it is accurately reflected by the proposed mathematical model (4) and (5). To validate the dynamical behavior of the developed model, i.e. the effect of the electric voltages on the electric currents, the induction law (8a) is considered. In the first experiment, the PMSM was driven at rated speed n = 3000 rpm by an external speed-controlled machine and the back-emf was measured at open machine terminals (i.e. $i_a = i_b = i_c = 0$ A). The results of this open-circuit experiment in Fig. 2(c) again show a high accuracy of the proposed machine model. In the final experiment, a shortcircuited motor, i.e. $v_a - v_c = 0$, $v_b - v_c = 0$ was considered, where the PMSM was driven at a constant speed of n = 1600rpm. The measurement results of the currents i_a , i_b and i_c are compared with the simulation results in Fig. 2(d). The very good agreement in this experiment also proves the feasibility of the chosen modeling approach. Looking at the results of Fig. 2, it might seem that the quantities are purely sinusoidal. A fast Fourier transform (FFT) analysis, however, exhibits the presence of harmonics of order $k = 5, 7, 11, 13, \ldots$ with rapid descending amplitudes.

The proposed machine model with voltage input of the PMSM in form of the DAE system (8a), (4) and (5) accurately



Fig. 2. Comparison of the developed model with measurements: torque as a function of the rotor angle φ in (a) for $i_a = -i_b = 5$ A and in (b) for $i_a = -i_b = 20$ A, induced voltages due to open-circuit operation in (c) for n = 3000 rpm and the currents due to short-circuit operation in (d) for n = 1600 rpm.

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Fig. 3. Comparison of the measured and simulated current-to-torque characteristic for a current supply in the form $i_a = -i_b$, $i_c = 0$ A.

reflects the motor behavior in the entire operating range. In particular, the model inherently accounts for a non-sinusoidal flux distribution and magnetic saturation in overload operation. Based on this comprehensive machine model, a simplified model of the PMSM suitable for flatness-based control design will be derived in the next section.

III. OPTIMAL CURRENTS AND FLUX LINKAGES

In this paper, a flatness-based torque control strategy of the PMSM in the entire operating range including overload operation with significant magnetic saturation is discussed. A classical torque control loop utilizing a torque transducer is undesirable in most applications due to additional hardware and cost. Thus, indirect torque control by proper current control will be considered as an appropriate solution. In a first step, optimal currents as functions of the rotor angle φ for a desired torque are calculated by minimizing the Joule losses of the motor. Using the set of nonlinear algebraic equations for \mathbf{u}_{tg} from (4), i.e.

$$\left(\mathbf{G}_t + \mathbf{D}_g \mathbf{G}_c \mathbf{D}_g^T\right) \mathbf{u}_{tg} = -\mathbf{D}_g \mathbf{G}_c \left(\mathbf{D}_c^T \mathbf{u}_{tc} + \mathbf{D}_m^T \mathbf{u}_{tm}\right)$$
(10)

with $\mathbf{u}_{tc} = \mathbf{N}_c \mathbf{V}_c \bar{\mathbf{i}}_{tc}$, the nonlinear constrained optimization problem reads as

$$\begin{array}{ll} \min_{\mathbf{\tilde{i}}_{tc},\mathbf{u}_{tg}} & (\mathbf{i}_{tc})^{T}(\mathbf{i}_{tc}) \\ \text{s.t.} & \text{eq. (10)} \\ & \tau^{d}\left(\varphi\right) - \tau\left(\varphi\right) = 0, \end{array}$$
(11)

with the desired torque $\tau^d(\varphi)$, the calculated torque $\tau(\varphi)$ from (5) and $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$. Minimization of the Joule losses is also commonly known as maximum torque per ampere (MTPA). An active-set algorithm is used to calculate a numerical solution of the optimal currents as functions of the rotor angle φ from the nonlinear constrained optimization problem (11). For this, the rotor angle φ is discretized by 144 points which corresponds to a step size $\Delta \varphi = 0.5^{\circ}$ mechanical angle. Corresponding results are shown in Fig. 4, where in Fig. 4(a) the desired torque $\tau^d = 5 \,\mathrm{Nm}$ was chosen close to the rated value of the motor. The associated optimal current shape is, in fact, almost perfectly sinusoidal. If the desired torque is



Fig. 4. Optimal current shape and corresponding fundamental wave providing minimum Joule losses for a constant desired torque of $\tau^d = 5 \,\mathrm{N\,m}$ in (a) and $\tau^d = 25 \,\mathrm{N\,m}$ in (b).

increased towards overload operation (i.e. $\tau^d = 25 \,\mathrm{N\,m}$) as shown in Fig. 4(b), harmonics of order $k = 5, 7, 11, 13, \ldots$ are more developed. The fundamental wave, however, is still dominating. This behavior is quite common for PMSMs with surface-mounted magnets, where magnetic saturation strongly influences the amplitude of the currents, while the shape still is close to a sinusoidal form. Other motor designs as PMSMs with internal-mounted magnets frequently show dominating influence of harmonics already at low torques, which makes the subsequent control strategy not directly applicable to these motor designs. The subsequent analysis and controller design are based on a fundamental wave approximation of the optimal current shapes, which, of course, is at the expense of a torque error. The small and thus tolerable influence of this approximation will be investigated later in the paper.

It should be noted that it is also possible to consider sinusoidal currents right from the start and to optimize the amplitude and phase of the current by minimizing the resulting (cumulative) torque error over the discretized fundamental period of the rotor angle in the cost function. This approach has not been chosen due to the following reasons: (i) In general, sinusoidal currents do not yield an almost constant torque, e.g., IPMSM with distinctive nonuniform air gap geometry. (ii) The advantage of the chosen approach (to calculate optimal current waveforms first, followed by a sinusoidal approximation) over the above-mentioned approach appears

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Fig. 5. Current-to-torque characteristic due to a sinusoidal current supply according to (12).

in a significant reduction of the dimension of the nonlinear constrained optimization problem in terms of optimization variables and equality constraints with almost the same accuracy.

The chosen optimization and approximation leads to the current-to-torque characteristic as given in Fig. 5. Here again, the nonlinear characteristic due to magnetic saturation at higher loads can be recognized. The fundamental wave approximation of the optimal currents can be expressed as

$$\bar{\mathbf{i}}_{tc}^d = [i_a^d, i_b^d]^T = \hat{i}^d \, \mathbf{T}_{c,i}(\varphi),\tag{12}$$

with $\mathbf{T}_{c,i}(\varphi) = [\cos(\alpha), \cos(\alpha + 2\pi/3)]^T$ and $\alpha = p(\varphi - \varphi_1)$, the amplitude of the current $\hat{i}^d(\tau^d)$ according to the currentto-torque characteristic in Fig. 5 and the phase $\varphi_1 = 40^\circ + 18^\circ \operatorname{sign}(\tau^d)$. If the currents in (12) are supplied to the machine, it can be expected that the PMSM generates the desired torque τ^d . Since the electric voltages at the machine terminals serve as control inputs, the voltage equation (8a) has to be considered to account for the corresponding flux dynamics.

Substituting the optimal sinusoidal currents (12) into (4) allows for the numerical calculation of the corresponding coil fluxes ϕ_{tc} and, based on the relationship (9), the flux linkages $\psi_{ac} = \psi_a - \psi_c$ and $\psi_{bc} = \psi_b - \psi_c$. This procedure applied for different amplitudes \hat{i}^d and rotor angles φ directly yields two-dimensional maps for the flux linkages $\psi_{ac}(\hat{i}^d,\varphi)$ and $\psi_{bc}(\hat{i}^d,\varphi)$, see Fig. 6 for a map of ψ_{ac} . For the flux linkage calculation, a grid with $\Delta \varphi = 0.5^{\circ}$ (a total of 144 grid points) and $\Delta i = 2$ A (a total of 20 grid points) is chosen. The flux linkages ψ_{ac} and ψ_{bc} have identical form but their phase is shifted by $\bar{\varphi}$, i.e. $\psi_{bc}(\hat{i}^d,\varphi) = \psi_{ac}(\hat{i}^d,\varphi + \bar{\varphi})$ with $\bar{\varphi} = 12^{\circ}$.

To analyze the form of the flux linkages, profiles for constant current amplitudes are depicted in Fig. 7. Beside the rising amplitude of the flux linkages with increasing current, an additional load-dependent phase shift occurs. Again, harmonics of order $k = 5, 7, 11, 13, \ldots$ are visible in the overload operating range. The second simplification step in the considered analysis consists of the sinusoidal approximation of the flux linkages in the entire operating range. Thus, the fundamental wave of the flux linkages due to the sinusoidal



Fig. 6. Two-dimensional map of the flux linkage $\psi_{ac}(i^d, \varphi)$ due to a sinusoidal current supply according to (12).



Fig. 7. Profiles of the two-dimensional map of the flux linkage $\psi_{ac}(\hat{i}^d, \varphi)$ for $\hat{i}^d = 5 \,\mathrm{A}$ and $\hat{i}^d = 40 \,\mathrm{A}$.

current supply according to (12) can be formulated as

$$\psi^d_{ac}(\hat{i}^d,\varphi) = \hat{\psi}^d \sin(p(\varphi - \Delta\varphi))$$
 (13a)

$$\psi_{bc}^{d}(\hat{i}^{d},\varphi) = \psi_{ac}^{d}(\hat{i}^{d},\varphi + \bar{\varphi}), \qquad (13b)$$

with a current-dependent amplitude $\hat{\psi}^d(\hat{i}^d)$ and phase shift $\Delta \varphi(\hat{i}^d)$. These functions, as depicted in Fig. 8, are approximated by means of polynomials of order three and five for the phase and the amplitude of the flux linkages, respectively.

Utilizing these approximations of the flux linkages in the voltage equation (8a) results in

$$\frac{\partial \psi_{ac}^d}{\partial \hat{i}^d} \frac{\mathrm{d}\hat{i}^d}{\mathrm{d}t} + \frac{\partial \psi_{ac}^d}{\partial \varphi} \omega = -R\left(2i_a^d + i_b^d\right) + v_{ac} \tag{14a}$$

$$\frac{\partial \psi_{bc}^{d}}{\partial \hat{i}^{d}} \frac{\mathrm{d}i^{d}}{\mathrm{d}t} + \frac{\partial \psi_{bc}^{d}}{\partial \varphi} \omega = -R\left(i_{a}^{d} + 2i_{b}^{d}\right) + v_{bc}, \qquad (14b)$$

with the electric line-to-line voltages $v_{ac} = v_a - v_c$, $v_{bc} = v_b - v_c$, the phase currents i_a^d , i_b^d in the form (12), and the angular velocity ω .

Up to now, optimal currents for a desired torque have been calculated by a nonlinear constrained optimization problem. Based on a sinusoidal approximation of the optimal currents in

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Fig. 8. Load-dependent phase shift and amplitude of the sinusoidal flux linkages (13).

(12), the corresponding flux linkages have been calculated and a sinusoidal approximation of these flux linkages yields the simplified model (14). Note that the simplified model (14) still incorporates the magnetic saturation effect in a systematic and physically consistent way. Thus, this model can be regarded as a good basis for the flatness-based controller design in the next section.

IV. CONTROLLER DESIGN

The proposed flatness-based torque control strategy relies on the measurements of the (independent) electric currents $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$ and of the rotor angle φ . It is based on a two-degrees-of-freedom control structure with a flatness-based feedforward controller in combination with a time-variant feedback error controller. As pointed out in the previous section, torque control is replaced by controlling the currents along the corresponding optimal trajectories. The control inputs v_{ac} and v_{bc} are split into

$$v_{ac} = v_{ac}^d + v_{ac}^c \tag{15a}$$

$$v_{bc} = v_{bc}^d + v_{bc}^c, \tag{15b}$$

with the feedforward control inputs v_{ac}^d , v_{bc}^d and the feedback control inputs v_{ac}^c , v_{bc}^c , respectively. Furthermore, the independent electric phase currents $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$ are written in the form

$$i_a = i_a^d + \Delta i_a \tag{16a}$$

$$b = i_b^a + \Delta i_b, \tag{16b}$$

with the desired currents $\bar{\mathbf{i}}_{tc}^d = [i_a^d, i_b^d]^T$ due to (12) and the tracking errors of the currents Δi_a and Δi_b . The flatness-based feedforward controller can be directly deduced from (14) in the form

$$v_{ac}^{d} = R(2i_{a}^{d} + i_{b}^{d}) + \frac{\partial \psi_{ac}^{d}}{\partial \hat{i}^{d}} \frac{\mathrm{d}\hat{i}^{d}}{\mathrm{d}t} + \frac{\partial \psi_{ac}^{d}}{\partial \varphi} \omega$$
(17a)

$$v_{bc}^{d} = R(i_{a}^{d} + 2i_{b}^{d}) + \frac{\partial \psi_{bc}^{d}}{\partial \hat{i}^{d}} \frac{\mathrm{d}\hat{i}^{d}}{\mathrm{d}t} + \frac{\partial \psi_{bc}^{d}}{\partial \varphi}\omega, \qquad (17b)$$

with the sinusoidal flux linkages ψ^d_{ac} and ψ^d_{bc} from (13), the desired sinusoidal currents $\bar{\mathbf{i}}^d_{tc}$ from (12) in combination with

the desired current amplitude \hat{i}^d and the nominal winding resistance R. The desired value of \hat{i}^d and its time derivative are calculated from the current-to-torque characteristic given in Fig. 5, assuming at least a C^1 -continuous desired torque signal $\tau^d(t)$.

The flatness-based feedforward controller (17) is extended by a (time-variant) feedback error controller to stabilize the tracking error in case of parameter variations due to unmodeled dynamics, measurement uncertainties and external disturbances. In this work, unmodeled dynamics mainly results from iron losses (including both eddy current and hysteresis losses) as well as the temperature behavior of the magnets and winding resistances. Measurement inaccuracies typically occur due to the approximate numerical differentiation of the rotor angle φ when calculating the angular velocity ω , and the current measurement signals are often corrupted by measurement noise and disturbances resulting from the switching of the VSI. Of course, the feedback controller also has to compensate for variations resulting from the simplification steps in the simplified model.

The derivation of the current error dynamics is based on a Taylor series expansion of (8a) and (4). In this context it is advantageous to express the flux linkages as functions of the independent electric phase currents $\bar{\mathbf{i}}_{tc} = [i_a, i_b]^T$. Note that the desired current amplitude \hat{i}^d in (12) can be easily calculated from the desired phase currents i_a^d and i_b^d with the help of $i_a^d + i_b^d + i_c^d = 0$. Using (16), the flux linkages $\psi_{ac}(i_a, i_b, \varphi)$ and $\psi_{bc}(i_a, i_b, \varphi)$ can be written in the form $\psi_{ac} = \psi_{ac}^d + \Delta \psi_{ac}$, $\psi_{bc} = \psi_{bc}^d + \Delta \psi_{bc}$, with

$$\begin{bmatrix} \Delta \psi_{ac} \\ \Delta \psi_{bc} \end{bmatrix} \approx \underbrace{\begin{bmatrix} \frac{\partial \psi_{ac}(i_{a}^{d}, i_{b}^{d}, \varphi)}{\partial i_{a}} & \frac{\partial \psi_{ac}(i_{a}^{d}, i_{b}^{d}, \varphi)}{\partial i_{b}} \\ \frac{\frac{\partial \psi_{bc}(i_{a}^{d}, i_{b}^{d}, \varphi)}{\partial i_{a}} & \frac{\partial \psi_{bc}(i_{a}^{d}, i_{b}^{d}, \varphi)}{\partial i_{b}} \end{bmatrix}}_{\mathbf{L}_{ab}} \begin{bmatrix} \Delta i_{a} \\ \Delta i_{b} \end{bmatrix}.$$
(18)

Given a sufficiently accurate current control, deviations from the desired operating point $\bar{\mathbf{i}}_{tc}^d$ are small and thus the error made by this approximation is also small. Substitution of (15)– (18) into the voltage equation (8a) results in

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \Delta \psi_{ac} \\ \Delta \psi_{bc} \end{bmatrix} = \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{L}_{\psi} \begin{bmatrix} \Delta i_{a} \\ \Delta i_{b} \end{bmatrix} + \mathbf{L}_{\psi} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \Delta i_{a} \\ \Delta i_{b} \end{bmatrix} = -\begin{bmatrix} 2R & R \\ R & 2R \end{bmatrix} \begin{bmatrix} \Delta i_{a} \\ \Delta i_{b} \end{bmatrix} + \begin{bmatrix} v_{ac}^{c} \\ v_{bc}^{c} \end{bmatrix}.$$
(19)

As can be seen from (18), the differential inductance matrix \mathbf{L}_{ψ} depends on the operating point. For a more detailed analysis, the partial derivatives of the flux linkages ψ_{ac} and ψ_{bc} with respect to the currents i_a and i_b have to be evaluated at the desired sinusoidal reference trajectories i_a^d and i_b^d according to (12). Since explicit analytical expressions of these terms are not available, appropriate approximations have to be derived.

It can be shown that the derivatives of the permeance matrices \mathbf{G}_t and \mathbf{G}_c with respect to \mathbf{i}_{tc} and \mathbf{u}_{tg} are small and thus can be neglected. Please note that in the magnetically linear case, these derivatives would be exactly zero. For the considered magnetically nonlinear case, the derivatives are small, since the rate of change of the relative permeability $\mu_r(H)$ of the iron core with respect to the magnetic field strength H is also small. Thus, the errors of this approximation

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Fig. 9. Numerical results of the differential inductance matrix \mathbf{L}_{ψ} for different operating points as a function of the rotor angle φ in (a) and average values over a fundamental period of 72° in (b).

are again small and thus this approximation can be considered feasible. Then, (4) yields the approximations

$$\frac{\partial \phi_{tc}}{\partial \bar{\mathbf{i}}_{tc}} \approx -\mathbf{D}_c \mathbf{G}_c \mathbf{D}_c^T \mathbf{N}_c \mathbf{V}_c - \mathbf{D}_c \mathbf{G}_c \mathbf{D}_g^T \frac{\partial \mathbf{u}_{tg}}{\partial \bar{\mathbf{i}}_{tc}}$$
(20a)

$$\frac{\partial \mathbf{u}_{tg}}{\partial \bar{\mathbf{i}}_{tc}} \approx -(\mathbf{G}_t + \mathbf{D}_g \mathbf{G}_c \mathbf{D}_g^T)^{-1} \mathbf{D}_g \mathbf{G}_c \mathbf{D}_c^T \mathbf{N}_c \mathbf{V}_c.$$
(20b)

With (20) and (9), the differential inductance matrix \mathbf{L}_{ψ} reads as

$$\mathbf{L}_{\psi} = -\mathbf{V}_{c}^{T} \mathbf{N}_{c} \frac{\partial \phi_{tc}}{\partial \bar{\mathbf{i}}_{tc}}.$$
 (21)

A numerical solution of \mathbf{L}_{ψ} as a function of the rotor angle φ and for fixed values of \hat{i}^d can be easily determined from (21). Numerical results for different values of \hat{i}^d (i.e. τ^d) are depicted in Fig. 9(a). The calculations show that the differential mutual inductances $\partial \psi_{ac} / \partial i_b$ and $\partial \psi_{bc} / \partial i_a$ (off-diagonal entries of \mathbf{L}_{ψ}) are identical, which, as is expected, implies that \mathbf{L}_{ψ} is a symmetric matrix. Additionally, average values over a fundamental period of 72° of the entries of the diagonal elements $\partial \psi_{ac} / \partial i_a$ and $\partial \psi_{bc} / \partial i_b$ (differential self-inductances), are identical too. It can be seen from Fig. 9(a) that the differential self and mutual inductances show a periodicity of twice the fundamental period, which

is physically interpretable. The variation of the differential inductance matrix \mathbf{L}_{ψ} with respect to the rotor angle φ is neglected for the design of the feedback controller. One can deduce from Fig. 9(a) that this assumption is clearly feasible for desired torques up to 15 N m, but the variations of the differential inductances increase for higher torques. However, as will be shown subsequently, \mathbf{L}_{ψ} only acts as a gain in the feedback control law and thus the decrease of control accuracy made by this assumption is again negligible. Hence, operating point dependent average values over a fundamental period of 72° , as shown in Fig. 9(b), are used instead. Here, the strong influence of magnetic saturation of the iron core and a ratio $\bar{L}_s/\bar{L}_m \approx 2$ can be observed. Thus, the averaged differential inductance matrix $\bar{\mathbf{L}}_{\psi}$ can be formulated as

$$\bar{\mathbf{L}}_{\psi} = \begin{bmatrix} \bar{L}_s & \frac{1}{2}\bar{L}_s \\ \frac{1}{2}\bar{L}_s & \bar{L}_s \end{bmatrix}.$$
 (22)

Substitution of $\bar{\mathbf{L}}_{\psi}$ into (19) allows for further simplifications. As can be deduced from Fig. 9(b), the variation of $\bar{\mathbf{L}}_{\psi}$ with respect to the current amplitude is rather small even if the current amplitude is changed in a wide range. Consequently, the time derivative of $\bar{\mathbf{L}}_{\psi}$ is rather small. Furthermore, assuming a proper feedback controller, the current errors Δi_a and Δi_b are small as well. As a consequence, the product of these terms can be neglected in (19) without significant loss of accuracy. It should be noted that the approximations performed in this section are also applicable to many other PMSMs with surfacemounted magnets. The current error dynamics (19) may thus be written in the simplified form

$$\bar{\mathbf{L}}_{\psi} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix}}_{\Delta \bar{\mathbf{i}}_{tc}} = -\underbrace{\begin{bmatrix} 2R & R \\ R & 2R \end{bmatrix}}_{\bar{\mathbf{R}}} \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix} + \underbrace{\begin{bmatrix} v_{ac}^2 \\ v_{bc}^2 \end{bmatrix}}_{\bar{\mathbf{v}}_{tc}^c}, \quad (23)$$

where the averaged differential inductance matrix $\bar{\mathbf{L}}_{\psi}$ is a function of the desired current amplitude \hat{i}^d and hence the error dynamics (23) is time-variant. Application of the feedback control law

$$\bar{\mathbf{v}}_{tc}^{c} = \bar{\mathbf{R}}\Delta\bar{\mathbf{i}}_{tc} - \bar{\mathbf{L}}_{\psi}\left(\mathbf{k}_{p}\Delta\bar{\mathbf{i}}_{tc} + \mathbf{k}_{i}\int\Delta\bar{\mathbf{i}}_{tc}\mathrm{d}t\right)$$
(24)

directly yields the second-order closed-loop error dynamics

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta\bar{\mathbf{i}}_{tc} + \mathbf{k}_p \frac{\mathrm{d}}{\mathrm{d}t}\Delta\bar{\mathbf{i}}_{tc} + \mathbf{k}_i\Delta\bar{\mathbf{i}}_{tc} = \mathbf{0}.$$
(25)

The positive controller gains $\mathbf{k}_p = \text{diag}(k_p)$ and $\mathbf{k}_i = \text{diag}(k_i)$ can be calculated by choosing desired eigenvalues of the error dynamics (25). The overall control law of the PMSM is composed of the flatness-based feedforward controller (17) and the time-variant feedback error controller (24). A block diagram of the proposed control structure is depicted in Fig. 10.

V. EXPERIMENTAL RESULTS

This section demonstrates the quality and feasibility of the proposed flatness-based torque controller. The controller is compared with a common vector control implementation based on the well-known dq0-model of the motor. Both

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Fig. 10. Block diagram of the proposed flatness-based torque control structure, comprising the feedforward and feedback part.

control strategies are implemented on the DSPACE realtimeplatform DS1103 triggering an industrial standard VSI with insulated-gate bipolar transistors (IGBTs). A sampling time of $T_a = 50 \,\mu\text{s}$ of the controller and a PWM frequency of $f_p = 20 \,\text{kHz}$ were used in the experiments. The rotor angle φ is measured with a motor-shaft mounted encoder and the electric phase currents $\mathbf{i}_{tc} = [i_a, i_b]^T$ are measured using LEM current transducers. Furthermore, the static inverter losses of the VSI were identified and compensated in the voltage generation process.

A. Vector control of the PMSM

In a classical dq0-model, magnetic linearity (i.e. no magnetic saturation), negligible iron losses (including both eddy current and hysteresis losses) and fundamental waves are assumed. Application of the Blondel-Park transformation [2] in the form $\mathbf{v}_{dq0} = \mathbf{K}(\varphi)\mathbf{v}_{abc}$ and $\mathbf{i}_{dq0} = \mathbf{K}(\varphi)\mathbf{i}_{abc}$ with

$$\mathbf{K}(\varphi) = \frac{2}{3} \begin{bmatrix} \cos(\rho) & \cos(\rho + \frac{2\pi}{3}) & \cos(\rho - \frac{2\pi}{3}) \\ -\sin(\rho) & -\sin(\rho + \frac{2\pi}{3}) & -\sin(\rho - \frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$
(26)

and $\rho=p\varphi$ to a fundamental wave model of the PMSM directly results in the dq0-model expressed in rotor-fixed coordinates

$$\mathbf{L}_{dq}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_{dq} = -\mathbf{R}_{dq}\mathbf{i}_{dq} - p\omega\tilde{\mathbf{L}}_{dq}\mathbf{i}_{dq} - p\omega\tilde{\Psi}_{PM} + \mathbf{v}_{dq} \quad (27a)$$

$$\tau_{dq} = \frac{3}{2} p \left(\tilde{\Psi}_{PM}^T \mathbf{i}_{dq} + \mathbf{i}_{dq}^T \tilde{\mathbf{L}}_{dq} \mathbf{i}_{dq} \right),$$
(27b)

with the voltage vector $\mathbf{v}_{dq} = [v_d, v_q]^T$, the current vector $\mathbf{i}_{dq} = [i_d, i_q]^T$, the magnetic flux vector $\Psi_{PM} = [\psi_{PM}, 0]^T$ with $\tilde{\Psi}_{PM} = \mathbf{J}\Psi_{PM}$, the angular velocity ω , the number of pole pairs p, the electromagnetic torque τ_{dq} , the inductance matrix $\mathbf{L}_{dq} = \text{diag}([L_d, L_q])$ with $\tilde{\mathbf{L}}_{dq} = \mathbf{J}\mathbf{L}_{dq}$ and \mathbf{J} in the form

$$\mathbf{J} = \begin{bmatrix} 0 & -1\\ 1 & 0 \end{bmatrix}.$$
 (28)

The dynamics of the zero sequence components is decoupled and not presented in this work. Obviously, all electromagnetic quantities in (27) are independent of the rotor angle φ .

TABLE I Main Technical Data of the PMSM

Term	Value	Unit
Rated speed	3000	rpm
Rated torque	6.16	Nm
Rated current	3.78	A_{rms}
Torque constant k_t	1.63	Nm/A_{rms}
Voltage constant k_e	98.43	$V/1000min^{-1}$
Phase winding resistance R	1.245	Ω

Homogenous air gap geometries in surface-mounted PMSMs entail $L_d = L_q$. Application of the control law

$$\mathbf{v}_{dq} = \mathbf{R}_{dq} \mathbf{i}_{dq}^{d} + p\omega \tilde{\mathbf{L}}_{dq} \mathbf{i}_{dq} + p\omega \tilde{\Psi}_{PM} + \mathbf{L}_{dq} \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{i}_{dq}^{d} - \mathbf{k}_{p,dq} (\mathbf{i}_{dq} - \mathbf{i}_{dq}^{d}) - \mathbf{k}_{i,dq} \int (\mathbf{i}_{dq} - \mathbf{i}_{dq}^{d}) \mathrm{d}t$$
(29)

directly yields the second-order closed-loop error dynamics

$$\mathbf{L}_{dq}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta\mathbf{i}_{dq} + (\mathbf{R}_{dq} + \mathbf{k}_{p,dq})\frac{\mathrm{d}}{\mathrm{d}t}\Delta\mathbf{i}_{dq} + \mathbf{k}_{i,dq}\Delta\mathbf{i}_{dq} = \mathbf{0},$$
(30)

with the tracking error $\Delta \mathbf{i}_{dq} = \mathbf{i}_{dq} - \mathbf{i}_{dq}^d$ and the reference trajectories $\mathbf{i}_{dq}^d = [i_d^d, i_q^d]^T$. The positive controller gains $\mathbf{k}_{p,dq} = \text{diag}(k_{p,dq})$ and $\mathbf{k}_{i,dq} = \text{diag}(k_{i,dq})$ can be calculated e.g. by pole placement. For a surface-mounted PMSM (i.e. $L_d = L_q$), the torque τ_{dq} only depends on the q-axis current i_q , see (27b). Hence, the reference trajectory i_q^d has to be chosen according to the desired torque τ^d in the form $i_q^d = \tau^d/k_t$, with the torque constant $k_t = 3/2p\psi_{PM}$. The reference value for the d-axis current i_d is set to $i_d^d = 0$ A to minimize Joule losses. It should be noted that the control law (29) strongly depends on an accurate knowledge of the model parameters. Moreover, the exact compensation of the voltage terms on the right-hand side of (27a) is not assured in a practical implementation due to parameter variations and disturbances. Finally, the dq0-model with constant parameters is inaccurate when the PMSM is operated in the overload range, where magnetic saturation occurs.

For the practical implementation, typically a simplified version of (29) in the form

$$\mathbf{v}_{dq} = p\omega \tilde{\Psi}_{PM} - \mathbf{k}_{p,dq} (\mathbf{i}_{dq} - \mathbf{i}_{dq}^d) - \mathbf{k}_{i,dq} \int (\mathbf{i}_{dq} - \mathbf{i}_{dq}^d) \mathrm{d}t \quad (31)$$

is implemented. This control strategy results from neglecting the feedforward terms $\mathbf{R}_{dq}\mathbf{i}_{dq}^d$, $\mathbf{L}_{dq}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{i}_{dq}^d$ and the induced voltage term $p\omega \tilde{\mathbf{L}}_{dq}\mathbf{i}_{dq}$ due to the mutual coupling. Note that the remaining model parameters of the control law (31), also commonly known as voltage constant $k_e = p\psi_{PM}$, can be easily obtained from the technical data sheet of the motor. The main technical data of the PMSM are shown in TABLE I.

Eq. (27b) does not account for the nonlinear current-totorque behavior. To improve torque tracking accuracy, it is common industrial practice to use a (measured) current-totorque characteristic as given in Fig. 3 in order to calculate the desired current i_q^d as a function of the desired torque τ^d . This is also done in the experiments of the next subsection, which yields an improved torque tracking accuracy.

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Fig. 11. Test bench setup of the PMSM.

B. Benchmark Experiment

The experimental validation of the proposed flatness-based torque control strategy and the comparison with the vector control implementation (31) using the nonlinear current-totorque characteristic in Fig. 5 are performed on a test bench setup depicted in Fig. 11. The motor shaft is attached to a torque sensor and a load inertia $(J_{load} \approx 0.03 \, \text{kgm}^2)$ by means of torsionally stiff couplings. For the proposed flatness-based torque controller, both poles of the second-order closed-loop error dynamics (25) are placed at $\Lambda = [-1650, -1650]^T$ (the resulting controller gains k_p and k_i are, however, time-varying) and the controller gains of the vector control strategy (31) are chosen as $\mathbf{k}_{p,dq} = \text{diag}(52.33)$ and $\mathbf{k}_{i,dq} = \text{diag}(0.0024)$. Appropriate integrator anti-windup strategies are implemented for both control schemes. As a benchmark experiment, an accelerated run-up of the PMSM is performed. To do so, the motor is accelerated with the torque profile depicted in Fig. 12(a). According to Fig. 5, a desired torque of τ^d = $25\,\mathrm{N\,m}$ highly saturates the PMSM. As can be deduced from Fig. 12(a), the proposed flatness-based torque control strategy generates a sufficiently smooth torque even at high speeds. It should be noted that Fig. 12(a) illustrates the recalculated torque according to (5) on the basis of the measured currents $\mathbf{\bar{i}}_{tc} = [i_a, i_b]^T$ and the measured rotor angle φ . This makes sense, because the developed model is validated and calibrated with measurements. The torque measured by the torque sensor contains higher frequency oscillations caused by the elasticity of the drive-train and errors in the alignment of the mechanical setup. Thus, these measurements are not very illustrative and have therefore not been included in the present manuscript. A comparison of the averaged values, however, shows very good agreement with the recalculated torque.

Comparing the results of the proposed flatness-based torque control concept (τ) with the vector control strategy (τ_{dq}) shows significantly increased fluctuations of ~ 5 N m for the vector control concept in comparison to less than 1 N m of the proposed control concept, see Fig. 12(a). To analyze the reasons for this behavior, the desired and the actual currents i_q^d , i_q and i_d^d , i_d are depicted in Fig. 12(b) and Fig. 12(c), respectively. It can be seen that both currents show large variations around their desired values in case of the vector control strategy. This is due to the fact that in the considered experiment large magnetic saturation of the motor is present which is not covered by the simple dq0-model. It, however, seems that the higher frequency fluctuations do not



Fig. 12. Accelerated run-up of the PMSM and comparison of the proposed flatness-based torque control strategy with a common vector control implementation: (a) recalculated torque profile, (b) q-axis current, (c) d-axis current, (d) rotational speed. For all subplots, the abscissa is the time measured in s.

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Fig. 13. Sections of the accelerated run-up of the PMSM utilizing the proposed flatness-based torque control strategy for different rotational speeds, i.e. $n \approx 500$ rpm in (I), $n \approx 1000$ rpm in (II) and $n \approx 1500$ rpm in (III). For all subplots, the abscissa is the rotor angle measured in °.

have a large influence on the resulting rotational speed, see Fig. 12(d). In fact, the large inertia of the load suppresses these fluctuations. For applications with smaller inertia and in view of increased thermal losses of the VSI, the behavior of the vector control strategy turns out to be rather problematic in industrial applications.

In order to compare both control strategies with respect to their ability to track desired reference trajectories of the motor currents, the d-axis and q-axis currents \mathbf{i}_{dq} of the vector control strategy are transformed into phase currents $\mathbf{\bar{i}}_{tc,dq} = [i_{a,dq}, i_{b,dq}]^T$ by the inverse of the transformation matrix (26). First, the results of the proposed control strategy as depicted in Fig. 13 are discussed. In this figure, the quantities are plotted



Fig. 14. Sections of the accelerated run-up of the PMSM utilizing a common vector control implementation for different rotational speeds, i.e. $n \approx 500$ rpm in (I), $n \approx 1000$ rpm in (II) and $n \approx 1500$ rpm in (III). For both subplots, the abscissa is the rotor angle measured in °.

as a function of the rotor angle φ for different values of the rotational speed, i.e. $n \approx 500$ rpm in (I), $n \approx 1000$ rpm in (II) and $n \approx 1500$ rpm in (III). From Fig. 13(a) it is apparent that the motor current i_a and its reference trajectory i_a^d are in very good accordance. This is further confirmed by the current error Δi_a in Fig. 13(b). The required control inputs, converted into duty cycles, are depicted in Fig. 13(c). It can be seen that a large part of the control input (i.e. basically the fundamental wave component) is generated by the flatnessbased feedforward controller (17). One can further observe that the control input demand increases with increasing rotor speed. In (III) at $\varphi = 216^{\circ}$, however, the desired torque changes its sign, cf. Fig. 12(a) at $t \approx 250 \,\mathrm{ms}$. As a consequence, a reduction of the control inputs occurs. Please also note that the maximum rotor speed in Fig. 12(d) has been carefully chosen such that the maximum ratings of the VSI (i.e. duty cycles within the boundary 0 and 1) are met. In conclusion, very good control performance and practically feasible control inputs have been obtained with the proposed flatness-based torque control strategy.

The results of the vector control strategy, i.e. the phase current $i_{a,dq}$ and its reference trajectory $i_{a,dq}^d$, are depicted in Fig. 14(a). As can be seen, the vector control strategy fails to track the desired reference trajectory in case of high desired torques due to the high magnetic saturation. There are two

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main reasons for this undesired and unstable behavior:

- The magnetic saturation in the motor yields a loaddependent phase shift of the flux linkages with respect to the currents, and the influence of harmonics increases. Thus, the prerequisites of the Blondel-Park transformation are no longer fulfilled. Even worse, the frequency of the harmonics is increased by the transformation, and the additional phase shift results in a coupling of the d-axis and q-axis¹.
- II) Magnetic saturation yields a change in the effective inductances \bar{L}_m and \bar{L}_s , see Fig. 9(b). Typically, the parameters of vector control concepts are chosen such that a good control performance is obtained for nominal torque (this has also been done in the experiments of this section). This, however, results in too high controller gains in the saturated case, where \bar{L}_s and \bar{L}_m decrease, such that the closed loop system can become unstable. Of course, it would be possible to choose the controller gains such that stable operation is also guaranteed in the saturated case. This, in turn, would lead to an unsatisfactory control performance in the unsaturated case.

It is worth mentioning that, although the identified nonlinear current-to-torque characteristics given in Fig. 3 has already been considered in the classical vector control scheme used to validate the proposed flatness-based torque control strategy, the accuracy could be slightly increased if the nonlinear relationship for the inductances given in Fig. 9 is used in the feedback path of the vector controller to preserve a closed-loop dynamics, which is almost independent from the operating point. Such methods are frequently used in literature. However, the main reason for the fluctuations in the torque and the dqaxis currents is most likely related to the current-dependent phase shift in Fig. 8, which is incorrectly represented in a dq0-model (and extensions of it) in the presence of magnetic saturation of the motor. Moreover, the vector controller formulated in the rotor-fixed reference frame has to deal with disturbances of increased frequency in comparison to a current controller defined in the stator-fixed reference frame.

In conclusion, the experimental results show a very good control performance of the proposed flatness-based torque control concept in the whole operating range, while classical vector control strategies have significant drawbacks. The MEC model, which is able to systematically capture the magnetic saturation and the non-sinusoidal behavior, serves as a basis for the design of the flatness-based torque controller. These results also show that control strategies based on dq0-models or extensions of it (either to approximately account for harmonics or magnetic saturation) are not the best choice for applications where the motor is operated in regions with significant magnetic saturation.

VI. CONCLUSION

In this paper, a flatness-based torque control strategy for the whole operating range of saturated surface-mounted permanent

¹Note that one reason why this problem does not occur in the proposed flatness-based torque control concept is that the (physical) currents i_a and i_b are directly controlled.

magnet synchronous machines was presented. The proposed control strategy exhibits a two-degrees-of-freedom control structure with a flatness-based feedforward controller and a time-variant feedback controller for the trajectory error system. Different to existing works, magnetic saturation is considered in a thorough and physically consistent way. The proposed controller is formulated in the stator-fixed reference frame. Hence there is no need to utilize a coordinate transformation which depends on the rotor angle. Based on a calibrated magnetic equivalent circuit model of the PMSM, which has been proposed in a previous publication [35], a simplified model suitable for flatness-based control design was derived by current shape optimization and a comprehensive analysis of the corresponding flux linkages. The controller gains of the timevariant error controller, which is composed of a proportional and integral term, systematically accounts for the mutual coupling of the phase windings. As a benchmark experiment, the PMSM was accelerated with about four times the rated torque. The resulting currents generate a sufficiently smooth torque and are in very good accordance with their reference trajectories, even at higher speeds. It has been shown that the proposed control scheme outperforms the industrial state-ofthe-art vector control implementation, which was extended by a nonlinear current-to-torque characteristic.

To increase the speed range of the PMSM beyond rated values, future work is concerned with the extension of the proposed control strategy to systematically account for field weakening.

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