Combined Path Following and Compliance Control with Application to a Biaxial Gantry Robot

Stefan Flixeder¹, Tobias Glück¹, Martin Böck¹, and Andreas Kugi¹

Abstract—This work presents an approach to simultaneously, but independently control the compliance of a robotic system transversal and tangential to a given curve. Therefore, a path following control concept known as transverse feedback linearization and a compliance control concept known as admittance control are combined. The subordinate path following controller transforms the nonlinear dynamics into a linear system with decoupled transversal and tangential dynamics via a coordinate and feedback transformation. The outer control loop utilizes admittance control to obtain the desired target compliance in the respective transformed coordinates. The proposed approach is applied to a biaxial gantry robot. Experimental results underline the feasibility of the proposed concept.

I. INTRODUCTION

The traditional way of planning an automated machining task, such as milling, grinding, deburring or saw cutting, is to define a specific path and a speed parameterization which is subsequently tracked using standard position control. Recent developments in robotized machining processes, see [1] for an overview, make also use of force control [2]. By controlling the material removal rate that depends on the counter force it is possible to react to material inhomogeneities and thus to improve the machining performance. Classical approaches either control the contact force perpendicular to the surface [3] or the feed force — and hence the material removal rate — along the machining path [4]. To the authors knowledge, no attempt has been made so far to independently control both, the force transversal and tangential to the machining path. For this purpose, the authors make use of recent advances in path following control, see, e.g., [5], [6], [7] and well known concepts of compliance control, see, e.g., [8], [9], [10]. By successfully combining these approaches, the proposed control strategy is capable of

- approaching the path from a given starting point,
- controlling the force perpendicular to the path while tracking a given trajectory along the path,
- controlling the force/position perpendicular to the path while manually guiding the position along the path by a human operator,
- controlling the feed force along the path while stabilizing and/or tracking a given trajectory perpendicular to the path and any combination of the previous.

This paper is organized as follows: Section II provides an introduction of the test setup of a two-degree-of-freedom (2-DOF) gantry robot. Moreover, an appropriate mathematical model is developed. Section III briefly summarizes the path following methodology, denoted as transverse feedback linearization, and applies it to the system under consideration. Section IV introduces admittance control, the utilized compliance control concept, and combines it with the transverse feedback linearization. Section V presents experimental results performed on the test setup.

II. EXPERIMENTAL SETUP AND MATHEMATICAL MODEL

Consider the schematic diagram of a linear 2-DOF gantry robot depicted in Fig. 1. Each linear drive consists of a servo motor and a ball screw drive. The ball screw drives convert the applied torques into a translational motion of the slides. The coupling between the motor, the ball screw and the slide as well as the mechanic connection of the y- and z-axis is considered stiff and backlash-free. Hence, the dynamics of the two linear axes are decoupled. The servo motors are equipped with fast current controllers allowing to use the actual motor torques \( \tau_i \in \{y,z\} \) as control inputs of the system. In order to measure the external load forces \( F_{L,i} \) acting on the end effector, the test setup is equipped with a triaxial force sensor based on strain gauge technology. Let \( s^T = [s_y \ s_z] \) be the position of the end effector with respect to the reference frame \( (y_0,z_0) \), \( m_{t,i} \) be the moved translational mass, \( k_i \) be the spindle pitch and \( m_{t,i} = (J_{sm,i} + J_{bs,i})/k_i^2 \) be the equivalent mass of the inertia of the servo motor.

---

*This work was supported by Festo AG & Co. KG
1The authors are with the Automation and Control Institute (ACIN), Vienna University of Technology, 1040 Vienna, Austria {flixeder;glueck;boeck;kugi}@acin.tuwien.ac.at

Fig. 1. Schematic diagram of the 2-DOF gantry robot.
J_{sm,i} and the ball screw J_{bs,i}, respectively. Newton’s second law yields the equations of motion (m_{s,i} + m_{J,i})\ddot{\mathbf{s}}_i = F_{M,i} - F_{F,i} - F_{G,i} - F_{L,i}, where F_{M,i} = τ_i/k_i is the equivalent motor force, F_{G,i} = 0, and F_{L,i} = g m_i z_i are the gravitational forces, with g = 9.81 m/s^2. The precision of the ball screw drives comes at the expense of high internal friction forces, which are approximated by F_{F,i}(\dot{s}_i) = c_{s,i} (\dot{s}_i/w) + c_{v,i} s_i , with w \ll 1 and the coefficients c_{v,i} > 0 and F_{G,i} > 0 of the viscous and Coulomb friction, respectively. By introducing the state vector \mathbf{x}_i^T = [\mathbf{x}_i^T \mathbf{x}_i^T] \in \mathbb{R}^4 with \mathbf{x}_i^T = [\dot{x}_i \dot{y}_i] and \mathbf{x}_i^T = [\dot{\xi}_i \dot{\eta}_i], the input vectors \mathbf{u}_i^T = [J_{bs,i} J_{sm,i}] \in \mathbb{R}^2 and \mathbf{d}_i^T = [F_{F,i} F_{L,i}] \in \mathbb{R}^2, the equations of motion can be written in the input affine form

\[ \mathbf{x}_i \in \mathbb{R}^2 \]

with

\[ \mathbf{f}_i(\mathbf{x}_i) = \begin{bmatrix} -F_{F,i}(s_i) \frac{m_z}{m_z + m_i} \\ -F_{L,i}(s_i) \frac{m_y}{m_z + m_i} \end{bmatrix} \mathbf{G}_i(\mathbf{x}_i) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \]

Moreover, the class of outputs y is restricted to smooth functions of the configuration states \mathbf{x}_i given by

\[ y = \mathbf{h}(\mathbf{x}_i) = \mathbf{x}_i, \quad y \in \mathbb{R}^2. \]

### III. PATH FOLLOWING CONTROL

The objective of the path following problem is to design a smooth feedback control law that makes the system output (3) approach and move along a path \gamma where no a priori time parametrisation is associated with the movement on the path. The approach utilized in this paper was introduced as transverse feedback linearization in, e.g., [5] and restricted to the class of mechanical systems in [7]. In the following section, this concept is revisited and applied to the considered 2-DOF gantry robot.

#### A. Path Assumptions

Suppose the path \gamma is given as a smooth parametrized curve \mathbf{σ}^i(\mathbf{t}) : \mathbb{R} \rightarrow \mathbb{R}^2 — not necessarily with a unit speed parametrization — \mathbf{σ}_1(\mathbf{t}) \parallel 1 — and the geometric restrictions on the class of curves imposed in [5] and [11]:

**Assumption 1:** The parametrization \mathbf{σ}(\mathbf{t}) is an embedded submanifold of \mathbb{R}^2, which implies that \gamma has no self-intersections.

**Assumption 2:** There exists a smooth map \mathbf{δ}(\mathbf{t}) : \mathbb{R}^2 \rightarrow \mathbb{R} such that the path \gamma can be represented as the zero-level set of \mathbf{δ} in the output space (3) of (1).

Applying and following the path \gamma is equivalent to stabilize the submanifold \Gamma := \{ \mathbf{x} \in \mathbb{R}^4 : \mathbf{δ} \circ \mathbf{h}(\mathbf{x}_i) = 0 \}. However, in [5], [7] it is shown that in general \Gamma is not invariant and thus only the stabilization of the largest controlled invariant subset \Gamma’ of \Gamma can be asked for the path following control design. \Gamma can be interpreted as the set of all trajectories for which the output \mathbf{y} according to (3) can be forced to stay on \gamma for all times by a suitable control input \mathbf{u}.

It can also be interpreted as the zero-dynamics manifold of (1) with the output \hat{\lambda}(\mathbf{x}_i) = \mathbf{δ} \circ \mathbf{h}(\mathbf{x}_i). For more details, the reader is kindly referred to [5], [6], [7].

#### B. Transverse and Tangential Feedback Linearization

The fundamental idea of the transverse feedback linearization concept is to find a coordinate transformation \mathbf{τ} : \mathbf{x} \mapsto (\mathbf{η}, \mathbf{ξ}) , defined in a neighborhood of \gamma, and a feedback transformation such that the system (1) transforms and decomposes into a tangential \mathbf{η} and a transversal \mathbf{ξ}-subsystem with respect to \gamma. Following Theorem 3.2 of [7], the system (1)-(3) is locally transverse feedback linearizable. Subsequently, the derivation of the coordinate transformation \mathbf{τ} : \mathbf{x} \mapsto (\mathbf{η}, \mathbf{ξ}) for a given curve \mathbf{δ}(\mathbf{t}) will be shown in detail.

Therefore, a projection operator \mathbf{δ} that maps each point \mathbf{y} \in \mathcal{Y} in a tubular neighborhood \mathcal{Y} of the closed curve \mathcal{C} to a unique parameter \hat{\theta} \in [0,\hat{L}], with \mathbf{δ}(\hat{\theta} + \hat{L}) = \mathbf{δ}(\hat{\theta}^*) is introduced. In order to calculate the point \mathbf{y}_* = \mathbf{δ}(\hat{\theta}^*) on the path \gamma that is closest to \mathbf{y} in the sense of a given metric, the minimization problem

\[ \hat{\theta}^* = \mathbf{δ}^*(\mathbf{y}) := \arg \min_{\hat{\theta} \in [0,\hat{L}]} \left\| \mathbf{y} - \mathbf{δ}(\hat{\theta}) \right\| \]

has to be solved [6]. The tangential state \mathbf{η}_1 = \pi(\mathbf{x}_i) is chosen as \mathbf{η}_1 = \mathbf{g} \circ \mathbf{h}(\mathbf{x}_i) with

\[ \mathbf{θ}^* = \mathbf{g}(\mathbf{y}) = \left( \int_0^{\hat{\theta}} \left\| \mathbf{d}^*(\mathbf{t}) \right\| \mathrm{d}\mathbf{t} \right) \Big|_{\hat{\theta} = \mathbf{δ}^*(\mathbf{y})}. \]

Clearly, the tangential unit vector on the path reads as

\[ \mathbf{σ}^*(\mathbf{θ}^*) = \frac{\mathbf{σ}^1(\mathbf{θ}^*)}{\left\| \mathbf{σ}^1(\mathbf{θ}^*) \right\|. \]

The transversal unit vector \mathbf{k}(\mathbf{θ}^*) is obtained by a counter clockwise rotation of \pi/2, i.e., \mathbf{k}(\mathbf{θ}^*) = R_{\pi/2} \mathbf{σ}^*(\mathbf{θ}^*) with the associated rotation matrix \mathbf{R}_{\pi/2}. See Fig. 2 for a graphic representation. Since \mathbf{y} - \mathbf{δ}(\hat{\theta}^*) is parallel to \mathbf{k}(\mathbf{θ}^*) = R_{\pi/2} \mathbf{σ}^*(\mathbf{θ}^*) \|\mathbf{σ}^1(\mathbf{θ}^*)\|, the output may be written in the form \mathbf{y} = \mathbf{σ}(\hat{\theta}^*) + \mathbf{k}(\mathbf{θ}^*) \mathbf{δ}(\mathbf{y}) and solved for

\[ \mathbf{δ}(\mathbf{y}) = \frac{1}{\left\| \mathbf{σ}^1(\mathbf{θ}^*) \right\|} \left( \mathbf{R}_{\pi/2} \mathbf{σ}^*(\mathbf{θ}^*) \right)^\top (\mathbf{y} - \mathbf{σ}(\hat{\theta}^*)) \mid_{\hat{\theta} = \mathbf{δ}^*(\mathbf{y})}. \]

The length of \mathbf{δ}(\mathbf{y}) is exactly the measure of the distance of the point \mathbf{y} to the corresponding point \mathbf{y}^* = \mathbf{σ}(\mathbf{θ}^*) on the curve \gamma. Thus, we choose \mathbf{ξ}_i = \lambda(\mathbf{x}_i) = \mathbf{δ} \circ \mathbf{h}(\mathbf{x}_i) as the first transversal state.

For the coordinate transformation $T: x \mapsto (\eta, \xi)$ the time derivatives of $\eta_1$ and $\xi_1$ are required. Although in general $\dot{\theta}^T$ from (4) can only be calculated numerically, an analytical expression can be found for its time derivative based on the necessary optimality condition of (4), i.e.,

$$ (y - \dot{\sigma}(\dot{\theta}^T))^{T} \dot{T}(\dot{\theta}^T) = 0. \tag{6} $$

Performing the time derivative of (6) yields

$$ \left( \frac{y - \dot{\sigma}'(\dot{\theta}^T)}{\varphi'(\dot{\theta}^T)} \right)^{T} \dot{T}(\dot{\theta}^T) + (y - \sigma'(\dot{\theta}^T))^{T} \dot{T}(\dot{\theta}^T) \dot{\theta}^T = 0 \tag{7} $$

and with $y - \dot{\sigma}(\dot{\theta}^T) = R_x^{T} \dot{\sigma}'(\dot{\theta}^T) / \| \dot{\sigma}'(\dot{\theta}^T) \| y(\dot{\theta})$ we get from (5)

$$ \eta_1 = \| \dot{\sigma}'(\dot{\theta}^T) \| y(\dot{\theta}) \quad \text{and} \quad \eta_2 = \left( \frac{R_x^{T} \dot{\sigma}'(\dot{\theta}^T) / \| \dot{\sigma}'(\dot{\theta}^T) \| y(\dot{\theta})} {\varphi'(\dot{\theta}^T) \varphi'(\dot{\theta}^T)} \right)^{T} \dot{T}(\dot{\theta}^T) y(\dot{\theta}) \quad \text{with} \quad \eta_1 = \eta_2 \text{ and} \quad \varphi'(\dot{\theta}^T) \varphi'(\dot{\theta}^T)^{T} = \frac{\partial h(x_c)}{\partial x_c}. \tag{8} $$

Thus, the coordinate transformation $T$ takes the form

$$ \begin{bmatrix} \eta_1 \\ \eta_2 \\ \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \pi(x_c) \\ d\pi(x_c) \\ \delta \cdot h(x_c) \\ d\delta \cdot h(x_c) \end{bmatrix}, \tag{9} $$

where $d\lambda = \partial \lambda(x_c) / \partial x_c$ and $d\pi = \partial \pi(x_c) / \partial x_c$. These results are also in accordance with [6]. Note that $d\lambda$ and $d\pi$ are given analytically and $\|d\eta\| = \|d\xi\| = 1$ holds for any point $y^*$ on the curve.

Along the lines of (7),

$$ \hat{y} = \begin{bmatrix} \lambda(x_c) \\ \pi(x_c) \end{bmatrix} = \begin{bmatrix} \delta \cdot h(x_c) \\ g \cdot h(x_c) \end{bmatrix}. \tag{10} $$

Inserting the feedback transformation (10) with (7) into (1) results in linear system dynamics of the form

$$ \dot{\xi}_1 = \xi_2, \quad \dot{\xi}_2 = v_\perp, \quad \eta_1 = \eta_2, \quad \eta_2 = v_\parallel. \tag{11} $$

It is worth noting that the dynamics of the transformed system are linear with respect to the nonlinear path $y$. Thus, the virtual input $v_\perp$ can effectively be used to stabilize the origin of the transverse $\xi$-subsystem and $v_\parallel$ can be utilized to control the motion along the path. The tracking errors $\tilde{e}_\parallel^p = \eta_1 - \eta_1^p$ and $\tilde{e}_\perp^p = \xi_1 - \xi_1^p$ are introduced with the sufficiently smooth path reference trajectories $\eta_1^p$ and $\xi_1^p$ along the path and transversal to the path, respectively. Eventually, the control laws

$$ v_\parallel = \tilde{e}_\parallel^p - a_\parallel \tilde{e}_\parallel^p - a_\parallel \tilde{e}_\parallel^p - a_\parallel 0 \int_0^\tau \tilde{e}_\parallel^p \, d\tau \tag{12} $$

yield the linear, exponentially stable error dynamics

$$ \begin{bmatrix} \tilde{e}_\parallel^p \\ \tilde{e}_\perp^p \end{bmatrix} + a_\parallel \tilde{e}_\parallel^p + a_\parallel \tilde{e}_\parallel^p + a_\parallel \tilde{e}_\parallel^p = 0 \tag{13} $$

that can be arbitrarily assigned by means of the constants $a_{l,k} > 0$ for $l \in \{\parallel, \perp\}$, $k = 0, 1, 2$. An anti-windup strategy can be used to account for input constraints.

**IV. COMPLIANCE CONTROL**

Compliance control addresses a classical problem in robotics of simultaneously controlling the position and the interaction force with the environment. A well-known approach in literature, see, e.g., [8], is referred to as impedance control. The fundamental idea of the concept is to design a controller such that the system establishes a desired dynamic relationship between an external (projected) force

$$ \tilde{F}_L = \begin{bmatrix} F_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} \sigma'(\theta)^T F_L \\ (\xi'(\theta))^T F_L \end{bmatrix} \tag{14} $$

and the state errors $\tilde{e}_\parallel^d = \eta_1 - \eta_1^d$ and $\tilde{e}_\perp^d = \xi_1 - \xi_1^d$ and their time derivatives. Herein, $\eta_1^d$ and $\xi_1^d$ denote the transversal and tangential reference trajectory, respectively. Typically, this relationship is specified in a linear form

$$ \tilde{F}_L = \begin{bmatrix} m_1 \, 0 \\ m_1 \, d_1 \end{bmatrix} + \begin{bmatrix} d_1 \, 0 \\ d_1 \, d_2 \end{bmatrix} \begin{bmatrix} \tilde{e}_\parallel^d \\ \tilde{e}_\perp^d \end{bmatrix} + \begin{bmatrix} k_1 \, 0 \\ k_1 \, k_2 \end{bmatrix} \begin{bmatrix} e_\parallel^d \\ e_\perp^d \end{bmatrix}, \tag{15} $$

where the positive definite matrices $\mathbf{M}^d, \mathbf{D}^d, \mathbf{K}^d$ represent the desired inertia, damping, and stiffness, respectively. If the matrices are diagonal, the compliances in tangential and transversal direction are decoupled. Thus, $m_1 > 0. d_1 > 0$ and $k_1 > 0$ for $j \in \{\parallel, \perp\}$ are the inertia, damping, and stiffness in tangential and transversal direction.
A. Admittance Control

In order to achieve a desired impedance behavior, the so-called admittance control [10], often denoted as position-based impedance control [13] or inner/outer impedance control [14], approach is utilized. The compliant behavior is realized by tracking the trajectory of a desired impedance model using the transverse and tangential feedback linearization in the inner loop and the admittance control in the outer loop, see Fig. 3. Assuming perfect tracking performance of the inner position controller, one may replace the actual path reference trajectories \( \eta_1^d \) and \( \xi_1^d \) in (14) with the path reference trajectories \( \eta_1^p \) and \( \xi_1^p \). Introducing the errors \( \epsilon_\eta^p = \eta_1^p - \eta_1^d \) and \( \epsilon_\xi^p = \xi_1^p - \xi_1^d \), the admittance control laws read as

\[
\begin{align*}
\eta_1^p &= \eta_1^d + F_1^m - \frac{d_\parallel}{d_\parallel} e_\eta^p - \frac{k_1}{m_1} e_\eta^p, \\
\xi_1^p &= \xi_1^d + F_1^m - \frac{d_\parallel}{d_\parallel} e_\xi^p - \frac{k_\perp}{m_\perp} e_\xi^p, \\
\eta_1^p &= \int_0^t \eta_1^d dt, \\
\xi_1^p &= \int_0^t \xi_1^d dt,
\end{align*}
\]

yielding the exponentially stable closed-loop dynamics in tangential and transversal direction of the form

\[
F_1^d = m_\parallel e_\eta^p + d_\parallel e_\eta^p + k_1 e_\eta^p \\
F_1^d = m_\perp e_\xi^p + d_\perp e_\xi^p + k_\perp e_\xi^p.
\]

Consequently, the compliance in transversal and tangential direction of the path are now decoupled, can be chosen independently, and do not depend on the position along the curve. Moreover, as the transformed states \((\eta_1, \xi_1)\) show the property \(|d\eta_1| = |d\xi_1| = 1\) for \(y \in \gamma\), no adjustment of the desired impedance parameters is required when choosing different paths.

In order to account for the velocity and the acceleration limits of the physical system, conditional execution and integration is performed to implement (15). Since system model uncertainties are compensated by the inner position control loop, this strategy provides high impedance tracking accuracy in the non-contact case and greatly facilitates the implementation on standard industrial manipulators with high and/or unknown friction forces [9].

B. Force Tracking Control

Many applications do not ask for a desired impedance behavior, but rather demand to maintain a desired interaction force with an environment while following a reference trajectory in the output space. Therefore, [16] suggests a control law that is inferred from (14) by subtracting the desired force \( F_1^d \) from the measured load force \( F_1 \) and setting the stiffness \( K_d \) to zero. Thus, the force tracking control law is equivalent to (14), except that \( F_1^d \) and \( F_1 \) are replaced by \( F_1 - F_1^d \) and \( F_1 - F_1^d \) and \( k_1 = k_\perp = 0 \). Under the assumption that the environment behaves like a nonlinear spring

\[
F_1 = -f_\parallel(e_\eta^p), \quad F_1 = -f_\perp(e_\xi^p),
\]

where \( e_\eta^p f_\parallel(e_\eta^p) > 0 \) and \( e_\eta^p f_\perp(e_\xi^p) > 0 \), and perfect tracking performance of the inner position controller, the equilibrium of the closed-loop dynamics

\[
-F_1^d = m_\parallel e_\eta^p + d_\parallel e_\eta^p + f_\parallel(e_\eta^p) \\
-F_1^d = m_\perp e_\xi^p + d_\perp e_\xi^p + f_\perp(e_\xi^p)
\]

is asymptotically stable. Thus, the desired forces \( F_1^d \) and \( F_1^d \) serve as driving forces to exert specific forces on the environment and in steady state condition, \( F_1 = F_1^d \) and \( F_1 = F_1^d \) holds.

V. EXPERIMENTAL RESULTS

Measurement results of the 2-DOF gantry robot are shown in this section. The servo motors as well as the ball screw drives are from Festo. The end effector position and velocity are calculated from the measurements of the integrated motor encoders. A triaxial force sensor form ME-MESSSYSTEME is used to measure the external forces acting on the end effector. Details on the components are listed in Table I. The model parameters are either extracted from data-sheets or measurements and are summarized in Table II. The presented control algorithms are implemented and executed using a real-time system DS1006 from DSPACE at

\begin{table}[h]
\centering
\caption{2-DOF Gantry Robot Components}
\begin{tabular}{|c|c|c|}
\hline
component & manufacturer & type \\
\hline
\hline
y-axis ball screw & Festo & EGC-80-BK-KF-1200 \\
\hline
z-axis ball screw & Festo & EGC-70-BK-KF-800 \\
\hline
servo motor & Festo & AS-55-M \\
\hline
force sensor & ME-MESSSYSTEME & K3D40 \\
\hline
\end{tabular}
\end{table}
TABLE II
2-DOF GANTRY ROBOT PARAMETERS

<table>
<thead>
<tr>
<th>Axis</th>
<th>( m_{x,i} )</th>
<th>( m_{y,i} )</th>
<th>( F_{c,i} )</th>
<th>( c_{x,i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )-axis</td>
<td>7.61 kg</td>
<td>29.26 kg</td>
<td>65.42 N s m (^{-1} )</td>
<td>175.93 N s m (^{-1} )</td>
</tr>
<tr>
<td>( y )-axis</td>
<td>4.41 kg</td>
<td>16.79 kg</td>
<td>37.69 N s m (^{-1} )</td>
<td>62.83 N s m (^{-1} )</td>
</tr>
</tbody>
</table>

TABLE III
DESIRED IMPEDANCE PARAMETERS

<table>
<thead>
<tr>
<th>Direction</th>
<th>( m_i )</th>
<th>( \zeta_i )</th>
<th>( k_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \perp )-direction</td>
<td>2.0 kg</td>
<td>0.3</td>
<td>200 N m(^{-1} )</td>
</tr>
<tr>
<td>( \parallel )-direction</td>
<td>0.5 kg</td>
<td>0.6</td>
<td>10 N m(^{-1} )</td>
</tr>
</tbody>
</table>

A sampling time of 1 ms. For illustration purpose, the desired path is chosen as an ellipse, which can be written as

\[
\dot{\mathbf{q}}(\hat{\theta}) = \begin{bmatrix} a \cos(\hat{\theta}) + s_{x,\hat{\theta}} \\ b \sin(\hat{\theta}) + s_{y,\hat{\theta}} \end{bmatrix},
\]

with the length of the major and minor axes \( a = 0.3 \) m and \( b = 0.15 \) m, and its center coordinates \( s_{x,0} = 0.5 \) m, \( s_{y,0} = 0.5 \) m with respect to the reference frame. Note that (16) is not in unit speed parametrization. Additionally, we want to emphasize that more complicated paths such as splines can be implemented with the same approach. In the following experiments, the controller parameters \( a_{i,k} > 0 \) for \( i \in \{\eta, \xi\} \), \( k = 0,1,2 \) were chosen such that the eigenvalues of the closed-loop error dynamics are \( p_{\perp,k} = -40\) s\(^{-1} \) and \( p_{\parallel,k} = -20\) s\(^{-1} \) for \( k = 0,1,2 \). The desired impedance parameters in tangential and transversal direction were selected according to Table III, where \( \zeta_l \) represents the damping ratio and the desired damping follows as \( d_j = 2\zeta_j\sqrt{m_j/k_j} \), for \( j \in \{\parallel, \perp\} \).

A. Experiment A - Path Following Control

In a first experiment, only the path following concept presented in Section III is investigated. Thus, solely the performance of the inner position control loop is examined. Fig. 4 shows the position of the end effector \( \mathbf{x}_e \) of the gantry robot in the output space and the corresponding transformed states \( \eta_l \) and \( \xi_l \). Starting from an initial position \( \mathbf{x}_e(0) \), tracking errors in transversal and tangential direction are quickly regulated to zero. Subsequently, set-point changes in the transversal and tangential direction are performed. Despite existing model inaccuracies, in particular in terms of friction, the tracking performance is clearly satisfactory.

B. Experiment B - Path Following and Compliance Control

The second experiment combines the path following approach of Section III and the compliance control strategy presented in Section IV. The path reference trajectories \( \eta_l^p \) and \( \xi_l^p \) of the inner path following controller are generated by the admittance controllers in the outer loop, see Fig. 3. The inner control loop tracks these generated path reference trajectories and realizes the impedance behavior according to (14) with the parameters listed in Table III. Starting from an initial position \( \mathbf{x}_e(0) \), a position change along the path by means of \( \eta_l^p \) is accomplished, see Fig. 5. Accordingly, the
In the second part, the human operator increases the force in normal direction and therefore causes desired deviations normal to the path. At last, forces are applied in transversal and tangential direction resulting in a transversal deviation and a guided motion along the path.

VI. CONCLUSIONS

This paper presents the design of a combined path following and compliance controller exemplarily applied to a biaxial gantry robot. Although the proposed controller is designed for a comparatively simple robotic manipulator, we emphasize that the proposed concept is applicable to any other industrial robot satisfying the assumptions of [7]. This approach may also greatly facilitate human machine interaction, i.e., as the workspace of the robot can be limited to a desired path, where the human operator controls the motion along the path.

REFERENCES