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and published in *Control Engineering Practice*.

Cite this article as:

T. Glück, W. Kemmetmüller, C. Tump, and A. Kugi, "A novel robust position estimator for self-sensing magnetic levitation systems based on least squares identification", *Control Engineering Practice*, vol. 19, no. 2, pp. 146–157, 2011. DOI: 10.1016/j.conengprac.2010.11.003

BibTex entry:

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@Article{GlueckCEP2011,
  Title = {A novel robust position estimator for self-sensing magnetic levitation systems based on least
      squares identification},
  Author = {T. Glück and W. Kemmetmüller and C. Tump and A. Kugi},
  Journal = {Control Engineering Practice},
  Year = {2011},
  Number = {2},
  Pages = {146-157},
  Volume = {19},
  Doi = {10.1016/j.conengprac.2010.11.003},
  Url = {http://www.sciencedirect.com/science/article/pii/S0967066110002509}
}
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A novel robust position estimator for self-sensing magnetic levitation systems based on least squares identification

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Abstract

In this work a novel method is introduced for the estimation of the position of a self-sensing magnetic levitation system, based on a least squares identification strategy. In the first step, a detailed mathematical model of the magnetic levitation system is derived and the properties of this system are analyzed for the case of a pulse-width modulated control. Based on this model, an estimation algorithm for the inductance of the magnetic levitation system is introduced. In classical position estimation schemes known form the literature large estimation errors are typically induced by a deviation of the electric resistance from its nominal value or by a fast motion of the levitated object. In this work it is shown that these errors can be exactly compensated by means of a suitable estimation strategy. Furthermore, it is outlined that the chosen structure of the estimation scheme allows for a very efficient implementation in real-time hardware. Afterwards, the design of a cascaded position controller for the magnetic levitation system is briefly summarized. Finally, the excellent quality and the high robustness of the proposed position estimator is demonstrated by means of simulation studies and measurement results on a test bench.

Key words: magnetic levitation system, self-sensing, position estimation, switching control

1. Introduction

Magnetic levitation systems enable an almost frictionless suspension of objects. Only the magnetic force generated by the electromagnets of the magnetic levitation system support the levitated object. In addition to the low friction, further benefits are the possibility to actively change the position of the levitated object and to alter the characteristics (e.g. the stiffness) of the levitation system. Magnetic levitation systems are inherently unstable, so active position control of the levitated object is indispensable. Certainly, the determination of the position is necessary for the implementation of the controller, which makes magnetic levitation systems relatively expensive and causes problems in the case of a failure of the position sensor.

For this reason, so-called sensorless or self-sensing magnetic levitation systems have been developed in the recent years. The position sensor is replaced by an estimation algorithm which makes use of the voltage and current measurement of the magnetic levitation system. The basic idea of all estimation algorithms is to utilize the functional relationship between the inductance of the magnetic levitation system and the position of the levitated object. The numerous works dealing with the development of estimation algorithms for the magnetic levitation systems can basically be divided into two working principles: (i) state observer approach and (ii) parameter estimation approach.

The classical state observer approach uses a Luenberger state observer, which is designed on the basis of a linearized mathematical model of the magnetic levitation system, see, e.g., Vischer (1988). This approach, however, exhibits major shortcomings concerning the robustness with respect to changes of the parameters and external disturbances (Thibeault et al., 2002). Furthermore, the linearized treatment significantly limits the operating range of the magnetic levitation system. An improvement of the robustness in the case of a pulse-width modulation (PWM) controlled magnetic levitation system could be obtained in (Maslen et al., 2006; Montie, 2003) by formulating the system in form of a linear mathematical model with periodic parameters. However, no practical implementation of the algorithms have been reported in these works. To the author's knowledge, no attempts of applying the theory of nonlinear state observers to magnetic levitation systems have been made. The reason, of course, is the need of very high sampling rates and the resulting computational effort.

A by far larger number of works deals with the parameter estimation approach for the position estimation of the levitated object. These works can again be divided into

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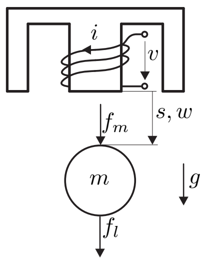
three categories: (a) The first approach is based on the injection of a high-frequency sinusoidal voltage test signal. The resulting changes in the amplitude of the current are a measure of the inductance and, therefore, of the position of the levitated object. The appropriate choice of the frequency of the test signal enables the decoupling of the control and the estimation (Sivadasan, 1996). The substantial disadvantage of this approach is the additional hardware effort for supplying and evaluating the test signal. Furthermore, the reported implementations of this approach make use of linear amplifiers with a low energy efficiency (Sivadasan, 1996). (b) The second subgroup makes use of hysteresis amplifiers, switching on and off the supply voltage such that the resulting amplitude of the current ripples is kept constant (Mizuno et al., 1998). The position of the levitated object is then inferred from the switching frequency of the hysteresis amplifier, where the frequency is typically measured by a phase-locked loop. This approach lacks in the ability to accurately estimate high-dynamic changes of the position signal.

Due to the increasing demands on energy efficiency nowadays switching amplifiers are almost exclusively used for the control of magnetic levitation systems. The third group (c) of the parameter estimation approaches utilizes pulse-width modulation (PWM) controlled switching amplifiers, where the average value of the voltage can be influenced by the duty ratio. Most of the contributions dealing with the position estimation for this configuration rely on a harmonic analysis of the voltage and the current signals (Kucera, 1997; Noh, 1997; Schammass, 2003; Schammass et al., 2005). Although several practical implementations have been reported in literature, the first harmonic is certainly only a rough approximation of the real current and voltage signals. It is well known that this approach yields inaccurate results if fast changes of the duty ratio or a fast motion of the levitated object do occur. For this reason, e.g. Kucera (1997) uses a look-up table to approximately account for these effects. Furthermore, the influence of the electric resistance of the coil is generally neglected. In contrast to the harmonic analysis a least squares estimation is performed in Pawelczak (2005) in order to obtain the actual value of the inductance. Although the dependence on the electric resistance is systematically included in this approach, a change of the duty ratio and a motion of the levitated object cause inaccuracies in the estimation results. Again a look-up table is used to approximately account for this effect.

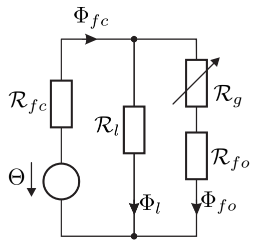
In this work a position estimation algorithm based on least squares identification is proposed, which, in contrast to existing works, is capable of systematically accounting for the influence of the electric resistance, the (rapid) change of the duty ratio, and for the motion of the levitated object. Section 2 is devoted to the deviation and the analysis of the mathematical model of the considered magnetic levitation system. The development of the position estimation scheme is outlined in Section 3, where first only a stationary object is considered. The position estimation algorithm is then generalized for the case of a moving levitated object. Here, it is also shown that the proposed position estimation algorithm is very robust to changes of the electric resistance. Furthermore, information on an efficient implementation of the algorithm is given in this section. Section 4 deals with the development of a basic position control algorithm. The results of simulation studies and measurements on a test bench are summarized in Section 5. The paper concludes with a short summary and an outlook to further research activities.

2. Mathematical Model

The mathematical model of the magnetic levitation system forms the basis of the subsequent position estimation algorithm and the design of a position controller. Clearly, that an exact model of the magnetic levitation system is necessary in order to achieve a good estimation and control performance. In Figure 1, a sketch of the considered magnetic suspension system is given. It basically comprises the levitated object, which, in the considered case, is a ball, and the magnetic core. Both, the levitated object and the magnetic core are made of highly permeable material with a relative permeability $\mu_r \gg 1$. The coil of the electromagnet is included in the core and has N turns. Applying a voltage v to the terminals of the coil results in a current i which in turn yields a magnetic field in the air gap between the core and the levitated object. By means of the resulting magnetic force f_m the position s of the levitated object can be controlled. The mathematical model of



(a) Schematic diagram.



(b) Equivalent magnetic circuit.

Figure 1: Schematic diagram of the magnetic levitation system and equivalent magnetic circuit.

the magnetic levitation system is based on the equivalent magnetic circuit given in Fig. 1b. It comprises the effective reluctance $\mathcal{R}_{fc}$ of the core, the effective reluctance $\mathcal{R}_{fo}$ of the levitated object, the effective reluctance $\mathcal{R}_g$ of the air gap between the core and the levitated object, and the reluctance $\mathcal{R}_l$, which accounts for the leakage fluxes.

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The reluctances are given as functions of the geometrical and magnetic parameters in the form

$$\mathcal{R}_{fc} = \frac{l_{fc}}{\mu_0 \mu_r A_{fc}} \tag{1a}$$

$$\mathcal{R}_{fo} = \frac{l_{fo}}{\mu_0 \mu_r A_{fo}} \tag{1b}$$

$$\mathcal{R}_{g} = \frac{s}{\mu_0 A_{g}} \tag{1c}$$

$$\mathcal{R}_{l} = \frac{l_{l}}{\mu_0 A_{l}}. \tag{1d}$$

Here, l_{fc}, l_{fo} and l_l are the effective lengths and A_{fc}, A_{fo}, A_l are the effective areas of the corresponding elements. The effective length of the air gap is given by the position s of the levitated object and the corresponding area is denoted by A_g. Furthermore, μ_0 denotes the permeability of air and μ_r is the relative permeability of the material of the core and the levitated object.

Using the electromotive force $\Theta = Ni$, the flux through the coil Φ_{fc} is given in the form

$$\Phi_{fc} = \frac{\Theta}{\mathcal{R}}, \tag{2}$$

with the equivalent reluctance $\mathcal{R}$ of the overall system given by

$$\mathcal{R} = \mathcal{R}_{fc} + \frac{\mathcal{R}_l \left(\mathcal{R}_g + \mathcal{R}_{fo}\right)}{\mathcal{R}_l + \mathcal{R}_g + \mathcal{R}_{fo}}. \tag{3}$$

Based on the flux linkage $\psi = N\Phi_{fc}$ of the coil, Faraday's law yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi = -Ri + v, \tag{4}$$

where R is the electric resistance, i denotes the current and v is the voltage applied to the coil. The flux linkage is a function of the current i and the position s of the levitated object. Using

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi = \frac{\partial \psi}{\partial i}\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{\partial \psi}{\partial s} w = L(s)\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{\partial L(s)}{\partial s} wi, \tag{5}$$

with the velocity $w = \dot{s}$ of the levitated object and the inductance $L(s)$,

$$L(s) = \frac{N^2}{\mathcal{R}(s)}, \tag{6}$$

(4) can be reformulated in the form

$$\frac{\mathrm{d}}{\mathrm{d}t} i = \frac{1}{L(s)} \left(-Ri - \frac{\partial L(s)}{\partial s} wi + v\right). \tag{7}$$

From the magnetic co-energy $\mathcal{W}_m^{co}$

$$\mathcal{W}_m^{co} = \frac{1}{2} L(s) i^2, \tag{8}$$

the magnetic force f_m of the magnetic levitation system can be calculated as

$$f_m(s,i) = \frac{\partial \mathcal{W}_m^{co}}{\partial s} = \frac{1}{2}\frac{\partial L(s)}{\partial s} i^2. \tag{9}$$

The overall mathematical model of the magnetic levitation system is completed by applying the balance of momentum to the levitated object.

$$\frac{\mathrm{d}}{\mathrm{d}t} s = w \tag{10a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w = \frac{1}{m}\left(f_m + f_l + mg\right) \tag{10b}$$

Here, m is the mass of the levitated object, f_l is an external load force and g denotes the gravitational constant.

In order to achieve high energy efficiency, magnetic levitation systems are usually driven by a switching amplifier. In the considered application an H-bridge comprising 4 MOSFETs is used, where the coil of the magnetic levitation is placed in the cross-path of the bridge, see Fig. 2. Using a suitable control strategy for the four transistors

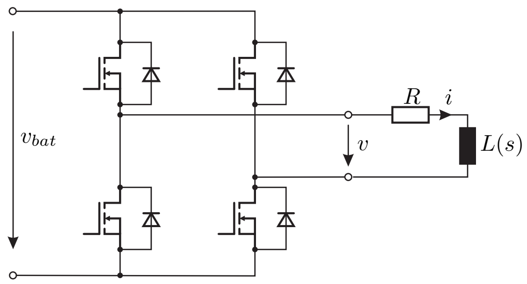

Figure 2: H-bridge switching amplifier.

of the H-bridge, either the supply voltage v_{bat} or the negative supply voltage $-v_{bat}$ is applied to the coil. In the considered application a pulse-width modulated voltage v of the form

$$v(t) = \begin{cases} v_{bat} & \text{for} \quad kT_{pwm} < t \leq (k+\chi)T_{pwm} \\ -v_{bat} & \text{for} \quad (k+\chi)T_{pwm} < t \leq (k+1)T_{pwm} \end{cases} \tag{11}$$

is used, where $0 \leq \chi \leq 1$ is the duty ratio and T_{pwm} is the fixed modulation period. Obviously, the average value $\bar{v} = 2v_{bat}\left(\chi - 1/2\right)$ of the voltage v can be directly adjusted by means of the duty ratio χ. The PWM voltage switching gives rise to a repeated charging and discharging of the coil, see Fig. 3. The amplitude of the resulting current ripple is significantly influenced by the inductance $L(s)$ of the coil, which is forms the basis of the estimation strategy for the inductance and thus for the position of the levitated object. Up to now it was assumed that the parameters of the reluctances are independent of the control input v. This assumption is no longer valid if the coil is actuated by means of a PWM voltage. It turns out that the relative permeability μ_r of the core and the levitated object strongly depends on the frequency of the excitation, i.e. the modulation period T_{pwm} of the voltage v. It is well known from literature that the relative permeability μ_r decreases with increasing frequency ω and tends to 1 if ω approaches infinity, i.e. $\mu_r \to 1$ for $\omega \to \infty$ (Boll, 1990).

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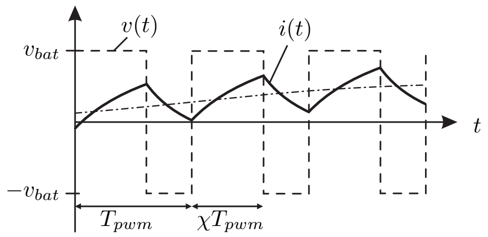

Figure 3: PWM switching waveform and resulting current ripple.

The PWM voltage actuation gives rise to a DC magnetic field superimposed by a small alternating magnetization (Boll, 1990). The change in the magnetization shifts the permeability and therefore the inductance, see Fig. 4. This, of course, has a strong impact on the (measurable) inductance L_o and the magnetic force f_m of the magnetic levitation system. Since the estimation algorithm

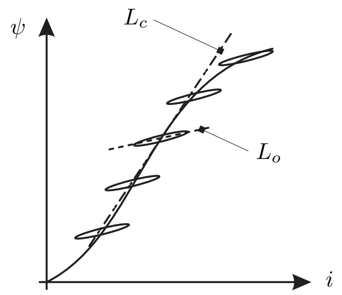

Figure 4: Alternating magnetization and DC magnetic field.

for the position of the levitated object to be designed in the next section fundamentally relies on an exact mathematical model of the magnetic system, this behavior must be thoroughly taken into account. A systematic description of the frequency dependent relative permeability μ_r is, however, rather difficult and thus not useful for the observer and controller design. For that reason, in this work two different mathematical models for the observer design and the controller design will be used. The mathematical models are based on the following considerations:

1. The goal of the controller design is to control the position s of the levitated object. It turns out that the dynamics of the mechanical system, i.e. the levitated object, are rather slow in comparison to the frequency $\omega_{pwm} = 2\pi/T_{pwm}$ of the pulse-width modulated voltage v. This implies that the position s of the levitated object is primarily influenced by the average value $\bar{v}$ of the voltage v and the oscillation of the position caused by the PWM excitation can be neglected. Thus, for the calculation of the magnetic force f_m the relative permeability $\mu_{r,0}$ for dc-excitation ($\omega = 0$) is used. This yields
$$f_m = \frac{1}{2}\frac{\partial L_c(s)}{\partial s} i^2, \tag{12}$$
where $L_c(s)$ denotes the effective inductance at $\omega = 0$, i.e. for $\mu_r = \mu_{r,0}$, cf. Fig. 4.
2. The situation is completely different for the observer design, since here the ripples in the current i due to the PWM voltage are exploited to estimate the inductance L and thus the position s of the levitated object. In this case, the main objective of the mathematical model is an exact description of the time evolution of the current i, thus the calculation of the inductance L_o is based on the relative permeability $\mu_{r,pwm}$ at $\omega = \omega_{pwm}$.

To summarize these considerations, the model Σ^o
$$\Sigma^o: \quad \frac{\mathrm{d}}{\mathrm{d}t} i = \frac{1}{L_o(s)}\left(-Ri - \frac{\partial L_o(s)}{\partial s} wi + v\right) \tag{13}$$
is used for the observer design, while the controller design is based on
$$\Sigma^c: \quad \frac{\mathrm{d}}{\mathrm{d}t} i = \frac{1}{L_c(s)}\left(-Ri - \frac{\partial L_c(s)}{\partial s} wi + v\right) \tag{14a}$$
$$\frac{\mathrm{d}}{\mathrm{d}t} s = w \tag{14b}$$
$$\frac{\mathrm{d}}{\mathrm{d}t} w = \frac{1}{m}\left(\frac{1}{2}\frac{\partial L_c(s)}{\partial s} i^2 + f_l + mg\right). \tag{14c}$$

The mathematical models (13) and (14) cannot be parametrized solely by geometrical data, since the value of some parameters as e.g. the length l_l and the area A_l of the leakage reluctance, the length l_{fo} and the area A_{fo} of the reluctance of the levitated object or the area A_g of the air gap, are not directly available. Moreover, the relative permeability μ_r as a function of the frequency ω is only roughly known. Therefore, an estimation of the unknown parameters was performed based on the following two measurements:

- In the first measurement, the inductance L_o was measured at ω_{pwm} for different (constant) positions s of the levitated object.
- The magnetic force f_m and thus $\partial L_c/\partial s$ was determined for different (constant) positions s of the levitated object in the second measurement.

Based on these two measurements, a (nonlinear) least squares estimation of the unknown parameters of the mathematical model was performed. A comparison of the resulting mathematical model with the measurement data is given in Fig. 5. Here, both the inductance and the partial derivative of the inductance with respect to the position s are depicted. Obviously, a very good approximation of the measurement data could be obtained.

One interesting result of the estimation of the parameters is the dependence of the relative permeability μ_r of the

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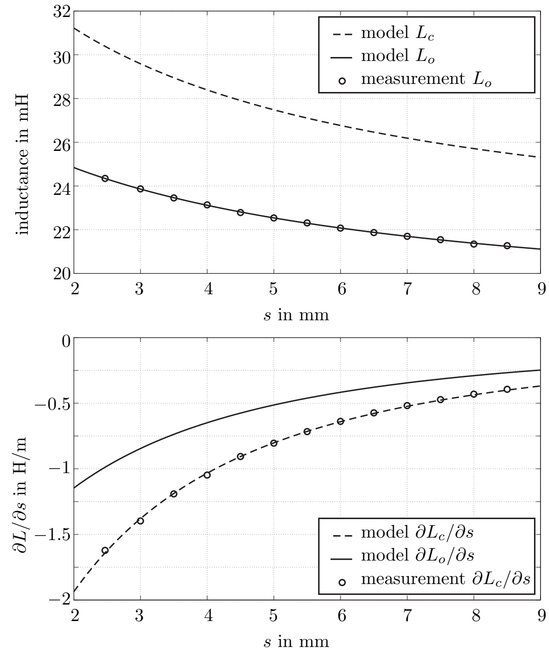

Figure 5: Comparison of measurement results with the mathematical model.

core and the levitated object on the frequency ω. Given (from data sheet (Kaschke, 2009)) the relative permeability $\mu_{r,0} = 4000$ of the material at $\omega = 0$, the relative permeability $\mu_{r,pwm}$ at $\omega_{pwm} \approx 2\pi \times 10^3$ rad/s reduces to $\mu_{r,pwm} = 3000$, which emphasizes the necessity of considering this effect in the mathematical model.

3. Least squares position estimation

In this section, a novel algorithm for the estimation of the position of the levitated object is presented. Apart from a high estimation accuracy the estimation algorithm to be developed should feature a numerically efficient implementation in order to be suitable for a cheap real-time hardware. In the first part of this section, the main estimation principle for the inductance of a static levitated object is outlined. Due to the shortcomings of this basic algorithm in the case of a moving levitated object and the poor robustness with respect to a deviation of the electric resistance from its nominal value, an extension of the basic algorithm is derived in the second part. It turns out that a motion of the levitated object has the same influence on the estimated inductance as a deviation of the electric resistance from its nominal value. This fact is advantageously utilized in the second part, where the influence of the motion of the object and of the deviation of the electric resistance is eliminated by means of a suitable averaging of the estimated values of the inductance. The model-based determination of the position and the velocity is given in the third part. A major challenge in the implementation of the estimation algorithm is the rather high PWM frequency and the associated even higher sampling frequency of the estimation algorithm. On that score, the last part of this section is concerned with the efficient implementation of the overall estimation algorithm on a real-time hardware. Here it is shown that the partitioning of the overall estimation algorithm in a fast but rather simple part and a slower but more complex part is advantageous for the implementation.

3.1. *Estimation of the inductance for a static levitated object*

The first step in the estimation of the position s of the levitated object is to estimate the actual value of the inductance L of the system. For this, again Faraday's law, cf. (4) is considered

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi(t) = -Ri(t) + v(t), \quad \psi(t_0) = \psi_0, \tag{15}$$

where ψ_0 denotes the initial condition of the flux linkage ψ and the electric resistance R is assumed to be constant. The flux linkage is given by $\psi(t) = L(t)i(t)$, where the inductance[1] L is a function of the position s which in turn is time dependent, i.e. $L(t) = L(s(t))$. The integration of (15) from time t_s to time t_e yields

$$\int_{t_s}^{t_e} \frac{\mathrm{d}\psi}{\mathrm{d}t}\mathrm{d}t = \int_{t_s}^{t_e} \frac{\mathrm{d}L}{\mathrm{d}t} i\,\mathrm{d}t + \int_{t_s}^{t_e} L\frac{\mathrm{d}i}{\mathrm{d}t}\mathrm{d}t = \int_{t_s}^{t_e} (-Ri + v)\,\mathrm{d}t. \tag{16}$$

For the time being it is assumed that the levitated object is not moving, i.e. $\dot{s} = w = 0$, and therefore the inductance L is constant, i.e. $\dot{L} = 0$. Then, (16) is given in the form

$$i(t_e) = i(t_s) + \frac{1}{L}\underbrace{\int_{t_s}^{t_e} (-Ri + v)\,\mathrm{d}t}_{\Delta\psi} \tag{17}$$

and the inductance L could in principle be estimated from the measurement of i and v and the calculation of $\Delta\psi$. There are, however, a number of drawbacks resulting from a direct implementation of (17):

- In order to derive L from (17) it is necessary to calculate the integral for $\Delta\psi$. If the integral is implemented in analog hardware, very high demands on the drift of the integrator and therefore on the circuit complexity have to be made.
- Although the value of $i(t_s)$ could be obtained by the first measurement value of the current, already a small measurement noise would result in large errors of the value of the inductance L.

[1]In order keep the notation short and readable, throughout Section 3 by L always the inductance L_o for $\omega = \omega_{pwm}$ is understood.

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- As already mentioned in Section 2, the considered magnetic levitation system is controlled by means of a pulse-width modulated voltage v. Here, the switching of the transistors of the H-bridge causes glitches in the measurement signals for the voltage and the current which also yield erroneous estimated values of L.

In order to circumvent these problems, the following strategy is proposed: First, a digital implementation of the estimation algorithm on a FPGA is used where the voltage v and the current i are sampled with a sampling time T_s. Furthermore, an estimated value of the inductance L is calculated only every T_{pwm} seconds, where T_{pwm} denotes the modulation period of the pulse-width modulated voltage. The influence of the switching glitches is avoided by ignoring some samples before and after every switching of the transitions of the H-bridge. Finally, the overall time period T_{pwm} is subdivided into the charging phase (index I) where $v \approx v_{bat}$ and the discharging phase (index II) where $v \approx -v_{bat}$. This strategy is illustrated in Fig. 6.

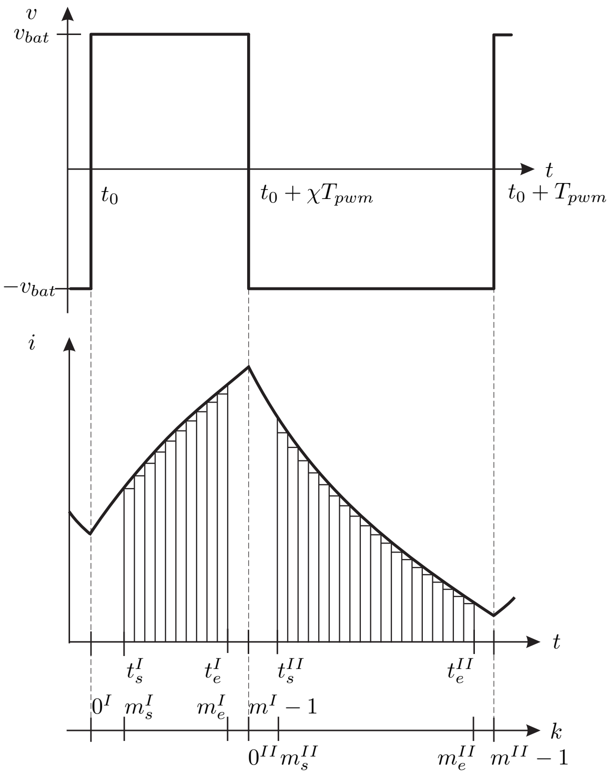

Figure 6: Charging I and discharging II phase of a single PWM-period.

With these prerequisites the estimation of the inductance L^I and L^{II} in the charging and the discharging phase, respectively, can be performed. In the subsequent derivations only the charging phase I is considered, since the results for the discharging phase II can be obtained in a similar way. As described before, in order to avoid the influence of the switching glitches only measurements in the time interval $\left[t_s^I, t_e^I\right]$, with the start time t_s^I and the end time t_e^I, of the overall charging phase $[t_0, t_0 + \chi T_{pwm}]$ are used for the estimation of the inductance L^I. This time interval corresponds to the sampled measurement data with indices $m_s^I, \ldots, m_e^I$. Thus, the change of the flux linkage $\Delta\psi_{k^I} = \Delta\psi(k^I T_s)$, with $k^I = m_s^I, \ldots, m_e^I$, results from (17) in the form

$$\Delta\psi_{m_s^I} = 0 \tag{18a}$$

$$\Delta\psi_{k^I} = T_s \sum_{j=m_s^I}^{k^I-1} \left(-Ri_j + v_j\right), \quad k^I = m_s^I + 1, \ldots, m_e^I \tag{18b}$$

and the equation (17) for the current i reads as

$$i_{k^I} = i_{m_s^I} + \left(L^I\right)^{-1} \Delta\psi_{k^I}, \quad k^I = m_s^I, \ldots, m_e^I. \tag{19}$$

As will be shown later, it turns out that the scaling

$$\Delta\psi_{k^I} = T_s \Delta\tilde{\psi}_{k^I} \tag{20a}$$

$$\left(L^I\right)^{-1} = \frac{1}{T_s}\left(\tilde{L}^I\right)^{-1}, \tag{20b}$$

with the sampling time T_s is useful for the implementation of the estimator. Then, (19) is given by

$$i_{k^I} = i_{m_s^I} + \left(\tilde{L}^I\right)^{-1} \Delta\tilde{\psi}_{k^I}, \quad k^I = m_s^I, \ldots, m_e^I. \tag{21}$$

From (21) it can be deduced that theoretically only one measurement suffices to calculate the unknown parameter, i.e. the inductance L. Of course, due to measurement noise this would lead to very imprecise estimation results and thus is not feasible in practical applications. However, by using $m_e^I - m_s^I + 1$ measurements of the current i and the voltage v, the resulting equations are over-determined and thus cannot be exactly solved. Therefore, a best approximation of the measurements in the sense of a quadratic measure (least squares approximation) is used in this work. For this purpose (21) is reformulated in vector-notation in the form

$$\underbrace{\begin{bmatrix} i_{m_s^I} \\ i_{m_s^I+1} \\ \vdots \\ i_{m_e^I} \end{bmatrix}}_{\mathbf{y}^I} = \underbrace{\begin{bmatrix} 1 & \Delta\tilde{\psi}_{m_s^I} \\ 1 & \Delta\tilde{\psi}_{m_s^I+1} \\ \vdots & \vdots \\ 1 & \Delta\tilde{\psi}_{m_e^I} \end{bmatrix}}_{\mathbf{S}^I} \underbrace{\begin{bmatrix} \tilde{i}_{m_s^I} \\ \left(\tilde{L}^I\right)^{-1} \end{bmatrix}}_{\boldsymbol{\theta}^I}, \tag{22}$$

where $\mathbf{y}^I \in \mathbb{R}^{m_e^I - m_s^I + 1}$ denotes the measurement vector, $\mathbf{S}^I \in \mathbb{R}^{(m_e^I - m_s^I + 1)\times 2}$ is the regression matrix and $\boldsymbol{\theta}^I \in \mathbb{R}^2$ is the parameter vector to be determined. It can be seen that in addition to the inverse normalized inductance $\left(\tilde{L}^I\right)^{-1}$ an estimation $\tilde{i}_{m_s^I}$ of the initial value of the current $i_{m_s^I}$

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in the time interval $\left[t_s^I, t_e^I\right]$ has been added as a second parameter. This is done in order to improve the robustness of the estimation with respect to measurement errors of $i_{m_s^I}$. The best approximation $\hat{\boldsymbol{\theta}}^I$ of the parameter vector $\boldsymbol{\theta}^I$ in the least squares sense is given by the pseudo-inverse

$$\hat{\boldsymbol{\theta}}^I = \left(\left(\mathbf{S}^I\right)^T \mathbf{S}^I\right)^{-1} \left(\mathbf{S}^I\right)^T \mathbf{y}^I. \tag{23}$$

With this, an estimation for the initial current value $\hat{\tilde{i}}_{m_s^I} = \hat{\theta}_1^I$ and for the inductance $\hat{L}^I = T_s/\hat{\theta}_2^I$ are obtained for the charging phase I. The same approach can be used for the discharging phase II yielding the estimations $\hat{\tilde{i}}_{m_s^{II}} = \hat{\theta}_1^{II}$ for the initial current value and $\hat{L}^{II} = T_s/\hat{\theta}_2^{II}$ for the inductance.

The two estimated values for the inductance, $\hat{L}^I$ and $\hat{L}^{II}$, are exactly equal only if (i) the measurements are exact, (ii) the electric resistance R is exactly known and (iii) the levitated object is at rest, i.e. $w = 0$. Of course, in practical applications none of these assumptions is fulfilled. Especially, the electric resistance R of the system changes during operation due to changing temperature. Furthermore, since it is desired to control the position of the levitated object, the last assumption, i.e. $w = 0$ or $\dot{L} = 0$, is also very limiting. Therefore, the influence of a moving levitated object and an inexact knowledge of the electric resistance R on the estimated values $\hat{L}^I$ and $\hat{L}^{II}$ will be examined in the next subsection.

3.2. Estimation of the inductance of a moving levitated object

In the previous subsection exact knowledge of the electric resistance R and a resting levitated object, i.e. $w = 0$, was assumed. In this section, the errors of the estimated values $\hat{L}^I$ and $\hat{L}^{II}$ of the inductance resulting from these assumptions are investigated in more detail. It is assumed that $w \neq 0$ and that the real value R of the electric resistance is given by

$$R = \hat{R} + \Delta R, \tag{24}$$

with the deviation ΔR of the electric resistance from its nominal value $\hat{R}$, which results e.g. from temperature variations.

According to (16) the relation including the motion of the levitated object and the real value of the electric resistance in the charging phase reads as

$$\int_{t_s^I}^{t_e^I} \frac{\mathrm{d}L}{\mathrm{d}t} i \mathrm{d}t + \int_{t_s^I}^{t_e^I} L \frac{\mathrm{d}i}{\mathrm{d}t} \mathrm{d}t = \int_{t_s^I}^{t_e^I} (-Ri + v)\, \mathrm{d}t, \tag{25}$$

whereas the estimation described in the last subsection is based on

$$\begin{aligned} \hat{L}^I \int_{t_s^I}^{t_e^I} \frac{\mathrm{d}i}{\mathrm{d}t} \mathrm{d}t &= \int_{t_s^I}^{t_e^I} \left(-\hat{R}i + v\right) \mathrm{d}t \\ &= \int_{t_s^I}^{t_e^I} (-(R - \Delta R)i + v)\, \mathrm{d}t. \end{aligned} \tag{26}$$

Substituting

$$\int_{t_s^I}^{t_e^I} (-Ri + v)\, \mathrm{d}t = \hat{L}^I \int_{t_s^I}^{t_e^I} \frac{\mathrm{d}i}{\mathrm{d}t} \mathrm{d}t - \int_{t_s^I}^{t_e^I} \Delta R i \mathrm{d}t \tag{27}$$

from (26) into (25) gives

$$\int_{t_s^I}^{t_e^I} \frac{\mathrm{d}L}{\mathrm{d}t} i \mathrm{d}t + \int_{t_s^I}^{t_e^I} L \frac{\mathrm{d}i}{\mathrm{d}t} \mathrm{d}t = \hat{L}^I \int_{t_s^I}^{t_e^I} \frac{\mathrm{d}i}{\mathrm{d}t} \mathrm{d}t - \Delta R \int_{t_s^I}^{t_e^I} i \mathrm{d}t, \tag{28}$$

where it is assumed that the deviation ΔR of the electric resistance is constant over the integration time $t_s^I - t_e^I$. This approximation is reasonable since the integration time $t_s^I - t_e^I$ is very short in comparison to the typical time constant of the change of the electric resistance R.

In order to further analyze (28) two additional assumptions are made:

1. It is presumed that the time derivative $\dot{L}$ of the inductance is constant over one PWM-period $t_s^I < t \leq t_s^I + T_{pwm}$. Note that this assumption is more general than the one used in the last section, i.e. $\dot{L} = 0$.
2. The current i is almost triangular[2], i.e.

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{i(t_e^I) - i(t_s^I)}{t_e^I - t_s^I} = \frac{\Delta i^I}{\Delta t^I} \tag{29}$$

and

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{i(t_e^{II}) - i(t_s^{II})}{t_e^{II} - t_s^{II}} = \frac{\Delta i^{II}}{\Delta t^{II}} \tag{30}$$

for the charging phase I and the discharging phase II, respectively.

With these assumptions (28) can be rewritten in the form

$$\frac{\Delta i^I}{\Delta t^I} \int_{t_s^I}^{t_e^I} L \,\mathrm{d}t = \hat{L}^I \frac{\Delta i^I}{\Delta t^I} \int_{t_s^I}^{t_e^I} 1 \mathrm{d}t - \left(\Delta R + \dot{L}\right) \int_{t_s^I}^{t_e^I} i \,\mathrm{d}t \tag{31}$$

or equivalently

$$\Delta i^I \frac{1}{\Delta t^I} \int_{t_s^I}^{t_e^I} L \mathrm{d}t = \hat{L}^I \Delta i^I - \left(\Delta R + \dot{L}\right) \int_{t_s^I}^{t_e^I} i \,\mathrm{d}t. \tag{32}$$

The definition of the average value $\bar{L}$ of the inductance L and the average value $\bar{i}^I$ of the current i

$$\bar{L} = \frac{1}{\Delta t^I} \int_{t_s^I}^{t_e^I} L \,\mathrm{d}t \tag{33a}$$

$$\bar{i}^I = \frac{1}{\Delta t^I} \int_{t_s^I}^{t_e^I} i \,\mathrm{d}t \tag{33b}$$

[2] This assumption is very well justified for small modulation periods T_{pwm}. However, when using larger modulation periods a degradation of the accuracy of the inductance estimation must be expected.

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finally yields

$$\bar{L} = \hat{L}^I - \left(\Delta R + \dot{L}\right) \frac{\bar{i}^I}{\Delta i^I} \Delta t^I. \tag{34}$$

With this, the average value $\bar{L}$ of the inductance is given by the estimation $\hat{L}^I$ and an additional term depending both on the error ΔR of the electric resistance and the change $\dot{L}$ of the inductance. Interestingly, ΔR and $\dot{L}$ identically influence the estimation error $\bar{L} - \hat{L}^I$ and thus cannot be distinguished. Additionally, (34) shows that rather large errors may result from ΔR and $\dot{L}$. Thus it is inevitable to find a better estimation which systematically accounts for errors of the electric resistance and for a motion of the levitated object.

For this purpose, the same approach as before is used for the discharge phase II to calculate

$$\bar{L} = \hat{L}^{II} - \left(\Delta R + \dot{L}\right) \frac{\bar{i}^{II}}{\Delta i^{II}} \Delta t^{II}. \tag{35}$$

Combing the results (34) and (35) it is possible to calculate the average value $\bar{L}$ in such a way that the influence of ΔR and $\dot{L}$ is canceled out, i.e.

$$\bar{L} = \frac{\hat{L}^I \Delta i^I \bar{i}^{II} \Delta t^{II} - \hat{L}^{II} \Delta i^{II} \bar{i}^I \Delta t^I}{\Delta i^I \bar{i}^{II} \Delta t^{II} - \Delta i^{II} \bar{i}^I \Delta t^I}. \tag{36}$$

In (36) the knowledge of Δi^I, Δi^{II}, $\bar{i}^I$ and $\bar{i}^{II}$ is necessary. These values can, of course, be estimated from the measurement of the current i. Anyway, for small changes of the current mean value $\bar{i}^I \approx \bar{i}^{II}$, the approximation $\Delta i^I = -\Delta i^{II}$ holds and (36) can be significantly simplified to

$$\bar{L} = \frac{\hat{L}^I \Delta t^{II} + \hat{L}^{II} \Delta t^I}{\Delta t^I + \Delta t^{II}}, \tag{37}$$

which is equal to a cross weighted averaging of the estimations $\hat{L}^I$ and $\hat{L}^{II}$. The drawback of this simple averaging is that in case of fast changes of the duty ratio of the voltage and therefore fast changes of the average value of the current rather imprecise estimations result from (37) while (36) still provides exact results.

3.3. Estimation of the position and velocity of a levitated object

The estimated average value $\bar{L}$ of the inductance has been calculated in the previous subsection by (36) or in its simplified form in (37). Given the reluctance model (6) with (1) and (3), a model-based estimation $\hat{s}$ of the position s of the levitated object can be found by inverting (6), i.e.

$$\hat{s} = s(\bar{L}). \tag{38}$$

Keeping in mind the equations of the mechanical part (10) it is obvious that an estimation $\hat{w}$ of the velocity w is required in order to stabilize the (undamped) levitated object. In this work, an approximate differentiation of the estimated position $\hat{s}$ is used in order to derive the estimated velocity $\hat{w}$,

$$\dot{x}_w = -\frac{1}{T_w} x_w + \hat{s} \tag{39a}$$

$$\hat{w} = -\frac{1}{(T_w)^2} x_w + \frac{1}{T_w} \hat{s}. \tag{39b}$$

Here, a proper choice of the time constant T_w of the approximate differentiation has to be made in order to guarantee a good trade-off between the suppression of the measurement noise and a sufficient dynamics of the estimated value $\hat{w}$.

3.4. Implementation of the algorithm

In this subsection, questions concerning the implementation of the estimation algorithm will be discussed. The main problem arising in the digital implementation of the proposed estimation algorithm results from the fast dynamics of the current and the resulting high frequency of the pulse-width modulated voltage, i.e. the small modulation period T_{pwm}. In order to suppress measurement noise and to obtain accurate estimations for the inductance L or equivalently for the position s of the levitated object, a very fast measurement of the current i and the voltage v is necessary. Hence, the sampling time T_s must be considerably smaller than the modulation period T_{pwm} of the pulse-width modulated voltage, i.e. $T_s \ll T_{pwm}$. On the one hand the modulation period T_{pwm} must be reasonable long in order to get enough measurements to attenuate noise and on the other hand it must be small enough to capture the time constant of the mechanical system. For the experimental setup the following typical values are used for T_{pwm} and T_s:

$$T_{pwm} = 1024\mu\text{s}, \quad T_s = 1\mu\text{s}. \tag{40}$$

Thus, a very efficient implementation of the estimation algorithm is required in order to be able to realize the very small sampling time T_s.

Recapitulating the estimation algorithm of the last subsections it turns out that some quantities, e.g. $\Delta\tilde{\psi}$, have to be calculated every sampling time T_s. The estimated values $\hat{L}^I$ and $\hat{L}^{II}$ as well as the average estimated value $\bar{L}$ and the position $\hat{s}$ or the velocity $\hat{w}$ only have to be calculated once every time period T_{pwm}. Therefore, a very efficient implementation of the first calculation is needed while computational efficiency is not crucial for the latter calculations. Hence, the following discussion on the efficient implementation will concentrate on the first part.

In order to calculate the estimated value of $\hat{L}^I$ the least squares estimate (23) has to be solved. Instead of setting up $\mathbf{S}^I$ and $\mathbf{y}^I$ it turns out to be more efficient to directly calculate the entries of $\left(\mathbf{S}^I\right)^T \mathbf{S}^I \in \mathbb{R}^{2\times 2}$ and $\left(\mathbf{S}^I\right)^T \mathbf{y}^I \in \mathbb{R}^2$. A short calculation shows that the entries of $\boldsymbol{\Xi} =$

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$\left(\mathbf{S}^I\right)^T \mathbf{S}^I$ are given in the form

$$\boldsymbol{\Xi} = \begin{bmatrix} m_e^I - m_s^I + 1 & \sum_{j=m_s^I}^{m_e^I} \Delta\tilde{\psi}_j \\ \sum_{j=m_s^I}^{m_e^I} \Delta\tilde{\psi}_j & \sum_{j=m_s^I}^{m_e^I} \left(\Delta\tilde{\psi}_j\right)^2 \end{bmatrix} \tag{41}$$

and the entries of $\boldsymbol{\xi} = \left(\mathbf{S}^I\right)^T \mathbf{y}^I$ read as

$$\boldsymbol{\xi} = \begin{bmatrix} \sum_{j=m_s^I}^{m_e^I} i_j \\ \sum_{j=m_s^I}^{m_e^I} i_j \Delta\tilde{\psi}_j \end{bmatrix}. \tag{42}$$

Obviously, only a few summations and multiplications but no divisions are necessary in every sampling time T_s in order to calculate the entries of $\boldsymbol{\Xi}^I$ and $\boldsymbol{\xi}^I$. This allows an easy implementation in fixed-point arithmetics e.g. on a FPGA without loss of accuracy. Furthermore, only the memory for the storage of 5 values of $\boldsymbol{\Xi}^I$ and $\boldsymbol{\xi}^I$ is necessary which further supports the hardware implementation.

Naturally, the calculation of the estimated parameter $\hat{\boldsymbol{\theta}}^I$ and therefore of the estimated inductance $\hat{L}^I$ is performed in floating point in order to achieve the necessary accuracy. However, since these calculations only have to be executed once every modulation period T_{pwm} this can be easily handled with a standard floating point processor.

For the calculation of the average inductance $\bar{L}$ by means of (36) additional knowledge of Δi^I, Δi^{II}, $\bar{i}^I$ and $\bar{i}^{II}$ is necessary. Here, Δi^I is defined in the form

$$\Delta i^I = i_{m_e^I} - i_{m_s^I} \tag{43}$$

which can be easily calculated from the measurement of the current. This way of calculating Δi^I, however, leads to very inaccurate results if the measurement is corrupted by noise. In this work a more robust determination of Δi^I based on a least squares approximation is used. Assuming, as before, that the current is an approximately triangular signal, the current i for every sampling time is given by

$$i_j = i^I_{m_s^I} + \frac{j - m_s^I}{m_e^I - m_s^I} \Delta i^I, \quad j = m_s^I, \ldots, m_e^I. \tag{44}$$

In order to obtain an optimal estimation of $i^I_{m_s^I}$ and Δi^I, the following least squares problem is formulated

$$\underbrace{\begin{bmatrix} i_{m_s^I} \\ i_{m_s^I+1} \\ \vdots \\ i_{m_e^I} \end{bmatrix}}_{\mathbf{h}^I} = \underbrace{\begin{bmatrix} 1 & \frac{m_s^I - m_s^I}{m_e^I - m_s^I} \\ 1 & \frac{m_s^I + 1 - m_s^I}{m_e^I - m_s^I} \\ \vdots & \vdots \\ 1 & \frac{m_e^I - m_s^I}{m_e^I - m_s^I} \end{bmatrix}}_{\mathbf{Q}^I} \underbrace{\begin{bmatrix} i^I_{m_s^I} \\ \Delta i^I \end{bmatrix}}_{\boldsymbol{\rho}^I}, \tag{45}$$

where $\mathbf{h}^I \in \mathbb{R}^{m_e^I - m_s^I + 1}$ denotes the measurement vector, $\mathbf{Q}^I \in \mathbb{R}^{(m_e^I - m_s^I + 1)\times 2}$ is the regression matrix and $\boldsymbol{\rho}^I \in \mathbb{R}^2$ is the parameter vector. The optimal solution $\hat{\boldsymbol{\rho}}^I$ of (45) in the least squares sense is given by (cf. (23))

$$\hat{\boldsymbol{\rho}}^I = \left(\left(\mathbf{Q}^I\right)^T \mathbf{Q}^I\right)^{-1} \left(\mathbf{Q}^I\right)^T \mathbf{h}^I. \tag{46}$$

As the entries of $\mathbf{h}^I$ and $\mathbf{Q}^I$ have to be determined every sampling time T_s, once again an efficient implementation is necessary. A simplification of (46) with (45) leads to

$$\left(\left(\mathbf{Q}^I\right)^T \mathbf{Q}^I\right)^{-1} = \begin{bmatrix} \frac{2\left(2\left(\Delta m^I\right)+1\right)}{\zeta} & \frac{-6\Delta m^I}{\zeta} \\ \frac{-6\Delta m^I}{\zeta} & \frac{12\Delta m^I}{\zeta} \end{bmatrix} \tag{47}$$

with $\zeta = \left(\Delta m^I + 2\right) + \left(\Delta m^I + 1\right)$ and

$$\left(\mathbf{Q}^I\right)^T \mathbf{h}^I = \begin{bmatrix} \sum_{j=m_s^I}^{m_e^I} i_j \\ \sum_{j=m_s^I}^{m_e^I} i_j \frac{j - m_s^I}{\Delta m^I} \end{bmatrix} \tag{48}$$

with $\Delta m^I = m_e^I - m_s^I$. Clearly, the entries of (47) can be easily calculated based on m_e^I and m_s^I only. Moreover, note that the first entry of (48) has already been calculated in (42).

With these considerations the practical implementation of the estimation algorithm can be summarized as follows:

1. The calculation of the entries of $\mathbf{S}^T\mathbf{S}$, $\mathbf{S}^T\mathbf{y}$ and $\mathbf{Q}^T\mathbf{h}$ according to (41), (42) and (48) both for the charging phase I and the discharging phase II is performed every fast sampling time T_s on a fixed-point FPGA.
2. The resulting data is transferred to a floating point processor (e.g. a soft-core processor emulated on the FPGA) once every modulation period T_{pwm}. Here, the estimated values of the inductance $\hat{L}^I$, $\hat{L}^{II}$ according to (23) and the values of $\Delta\hat{i}^I$, $\Delta\hat{i}^{II}$ according to (46) are determined. Based on these results the estimation of the average value of the inductance and the calculation of the estimated position s and the velocity w are performed.

4. Control strategy

A short analysis of the mathematical model (14) shows that the magnetic levitation system is unstable without control. Therefore, a suitable control strategy is necessary in order to validate the performance of the proposed estimation algorithm for the position s and the velocity w. In this work a rather simple cascaded control strategy is designed which comprises a controller for the current i in the inner control loop and a position controller in the outer control loop.

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4.1. Current controller

The current i of the magnetic levitation system is directly related to the magnetic force f_m via (12). By using a pulse-width modulated voltage v of the form (11) for the control of the magnetic levitation this results – as was outlined in detail in the last sections – in a triangular-like evolution of the current. Naturally, this also implies a changing magnetic force during the modulation period T_{pwm}. On the other hand, the duty ratio χ can only be set once at the beginning of every modulation period. This shows that only the average value $\bar{i}$ of the current i can be controlled by means of the duty ratio χ as the control input of the system. Given the average values $\bar{i}$ and $\bar{v}$ of the current and the voltage, respectively,

$$\bar{i} = \frac{1}{T_{pwm}} \int_{t-T_{pwm}}^{t} i(\tau)\mathrm{d}\tau \tag{49a}$$

$$\bar{v} = \frac{1}{T_{pwm}} \int_{t-T_{pwm}}^{t} v(\tau)\mathrm{d}\tau = 2v_{bat}\left(\chi - \frac{1}{2}\right), \tag{49b}$$

the following model for the average current results from (14a)

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{i} = -\frac{1}{L_c}\left(R\bar{i} - \bar{v}\right). \tag{50}$$

This equation, of course, only holds true if the position s of the levitated object and thus the inductance L_c is constant. In reality, the levitated object is moving and therefore this assumption is not fully justified. However, since the changes in the inductance L_c are rather small for the typical operating range of magnetic levitation systems and the current controller will be designed to be robust with respect to changing inductances, it is possible to use a nominal value L_{sp} which corresponds to the inductance at a nominal position setpoint of the levitated object.

In this work, a two degrees-of-freedom control strategy comprising a feedforward and a feedback control, is proposed for the control of the average value $\bar{i}$. If the sufficiently smooth desired trajectory of the average current is denoted by $\bar{i}_d$ then the feedforward control $\bar{v}_d$ is given by

$$\bar{v}_d = L_{sp}\frac{\mathrm{d}}{\mathrm{d}t}\bar{i}_d + R\bar{i}_d. \tag{51}$$

The error $e_i = \bar{i} - \bar{i}_d$ can then be stabilized by a PI-controller of the form

$$\bar{v}_c = -\lambda_{1,i}e_i - \lambda_{0,i}\int_0^t e_i\mathrm{d}\tau, \tag{52}$$

where $\lambda_{1,i}, \lambda_{0,i} > 0$ are the constant controller parameters. Using the control law $\bar{v} = \bar{v}_c + \bar{v}_d$, the overall closed-loop error system can be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}e_{i,i} = e_i \tag{53a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}e_i = -\frac{1}{L_{sp}}\left(\left(R + \lambda_{1,i}\right)e_i + \lambda_{0,i}e_{i,i}\right). \tag{53b}$$

Naturally, the dynamics of the (linear) error system can be arbitrarily assigned by means of the two controller parameters $\lambda_{1,i}$ and $\lambda_{0,i}$, where a suitable choice guarantees the robustness of the controller with respect to changes of the inductance L_c. The control law is implemented with a sampling time of T_{pwm}. The average value $\bar{i}$ of the current is determined from $\hat{i}^I$ and $\hat{i}^{II}$ (c.f. Section 3.4) by extrapolating to the current peak and calculating the mean value

$$\bar{i} = \frac{\Delta\hat{i}^I}{4}\left(\frac{m^I - 1 - 2m_s^I}{m_e^I - m_s^I}\right) + \frac{\Delta\hat{i}^{II}}{4}\left(\frac{m^{II} - 1 - 2m_s^{II}}{m_e^{II} - m_s^{II}}\right) + \frac{1}{2}\left(\hat{i}_{m_s^I} + \hat{i}_{m_s^{II}}\right). \tag{54}$$

4.2. Position controller

The position s of the levitated object is controlled in the outer control loop, where it is assumed that the inner current controller is sufficiently fast such that the assumption $\bar{i} = \bar{i}_d$ holds. Then, the average value $\bar{i}$ of the current can be used as the control input for the position controller. For zero load force $f_l = 0$ the equations for the mechanical system read as (cf. (14b), (14c))

$$\frac{\mathrm{d}}{\mathrm{d}t}s = w \tag{55a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w = \frac{1}{m}\left(mg + f_m(i,s)\right). \tag{55b}$$

As has been described in Section 2, the magnetic force f_m is a function of the current i and the position s, cf. (12). Since the dynamics of the mechanical system is considerably slower than the modulation period T_{pwm}, it can be assumed that only the average value $\bar{f}_m$ of the magnetic force f_m, i.e. $\bar{f}_m = f_m\left(\bar{i}, s\right)$, is responsible for a change in the position s. In the subsequent design of the position controller it is further assumed that the average value $\bar{f}_m$ of the magnetic force serves as the (virtual) control input of the system. This is justified by the fact that the desired average value $\bar{i}_d$ of the current can be easily calculated by solving (12) for a given position of the levitated object.

The position controller is again based on a two degrees-of-freedom control structure. Given the two times continuously differentiable desired trajectory s_d for the position of the levitated object, the feedforward control reads as

$$f_{m,d} = m\left(\ddot{s}_d - g\right). \tag{56}$$

Introducing the position error $e_s = s - s_d$ and the velocity error $e_w = w - w_d$, with the desired velocity $w_d = \dot{s}_d$, the PID feedback control can be written in the form

$$f_{m,c} = -\lambda_{2,s}e_w - \lambda_{1,s}e_s - \lambda_{0,s}\int_0^t e_s\mathrm{d}\tau, \tag{57}$$

with the constant controller parameters $\lambda_{2,s}, \lambda_{1,s}, \lambda_{0,s} > 0$. Using the overall position control input $f_m = f_{m,d} + f_{m,c}$

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in (55) the closed-loop error system reads

$$\frac{\mathrm{d}}{\mathrm{d}t} e_{i,s} = e_s \tag{58a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} e_s = e_w \tag{58b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} e_w = -\frac{1}{m}\left(\lambda_{2,s} e_w + \lambda_{1,s} e_s + \lambda_{0,s} e_{i,s}\right) \tag{58c}$$

and the closed-loop dynamics can be arbitrarily assigned.

4.3. Overall control and estimation strategy

Fig. 7 depicts the overall structure of the proposed control and estimation strategy. Here, the control strategy comprises the inner current control loop, the outer position control loop and the trajectory generation. The estimation of the position is divided into a fast part Σ^e_{fast} and a slow part Σ^e_{slow}. As outlined in Section 3.4, the fast part Σ^e_{fast} calculates the entries of $\mathbf{\Xi}$ (41), $\boldsymbol{\xi}$ (42) and $\mathbf{Q}^T\mathbf{h}$ (48). These values are updated every fast sampling time T_s and transferred to the slow part once every modulation period T_{pwm}. The calculation of the estimated position $\hat{s}$ and the estimated velocity $\hat{w}$ is performed in the slow part Σ^e_{slow} of the estimator. This part, as well as the control strategy, is evaluated once every modulation period T_{pwm}. This partitioning into a fast and a slow part of the overall control and estimation strategy allows for a very efficient (and cheap) implementation. In the considered application, the fast part is implemented in fixed-point arithmetics on a FPGA while the slow part is calculated in floating-point on a softcore processor emulated on the FPGA.

5. Simulation and experimental results

This section analyzes the properties of the proposed estimation and control strategy. First, simulation results are shown since they are more suitable to elaborate the characteristics of the self-sensing estimation algorithm. Afterwards, measurement results on a test bench are given in order to prove the practical feasibility of the proposed estimation scheme.

5.1. Simulation results

For testing and simulation, the estimation and control algorithm was implemented in MATLAB/SIMULINK. In order to simulate the behavior of the switching amplifier, a model of the pulse-width modulated voltage of the H-bridge according to Fig. 2 was built. The control and estimation strategy was tested for the nominal model with the model and controller parameters according to Table 1.

Fig. 8 shows the simulation results of a set point change of the position from 4 mm to 6 mm in 0.3 s. It is assumed that the real value of the electric resistance corresponds to the nominal one, namely $R = \hat{R} = 1.5\,\Omega$. In Fig. 8a, the estimated inductances $\hat{L}^I, \hat{L}^{II}$ and the mean value $\bar{L}$ are depicted. Moreover, Fig. 8b shows a detail of Fig. 8a. Due to the movement of the levitated object there is a spreading between the inductance $\hat{L}^I$ and $\hat{L}^{II}$. Nevertheless, this influence is canceled out by calculating the mean value $\bar{L}$ according to equation (54). The associated estimated position is shown in Fig. 8c and the resulting position error $s - \hat{s}$ in Fig. 8d, respectively. In Fig. 8e, the current i and the estimated mean value $\bar{i}$ according to equation (58) are shown. Finally, Fig. 8f depicts the duty ratio calculated by the current controller. These simulation results confirm that a very accurate position estimation is achieved by the proposed estimation algorithm, even in the case of a fast changing position of the levitated object. As already mentioned before, this represents the major advantage of the proposed estimation algorithm in comparison to others given in the literature (Noh, 1997; Pawelczak, 2005; Schammass et al., 2005).

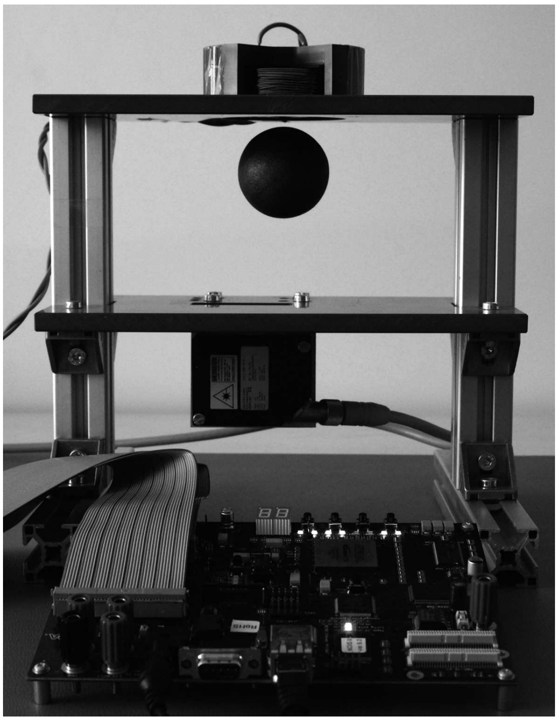

Figure 10: Picture of the experimental test bench.

For the simulation study in Fig. 9, a deviation of the electric resistance of $\Delta R = -0.5\,\Omega$ was assumed. The wrong value of the electric resistance causes a spreading between the estimated inductances $\hat{L}^I$ and $\hat{L}^{II}$, cf. Fig. 9a. However, in the mean value $\bar{L}$ of the inductance the error due to the motion of the levitated object as well as the error induced by the wrong value of the electric are suppressed. Certainly, the estimated position (cf. Fig. 9b) is also correct.

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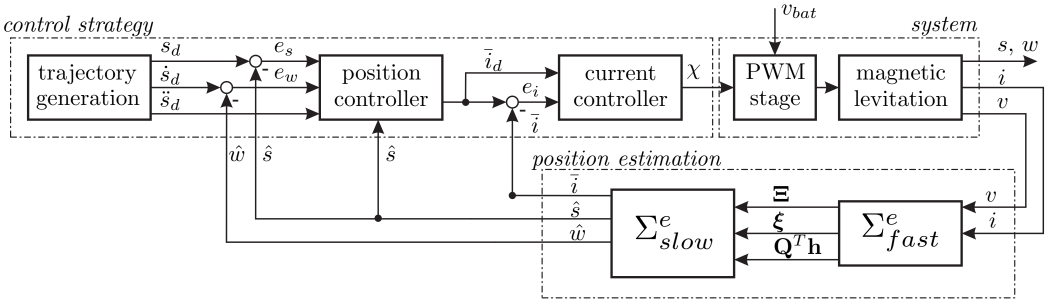
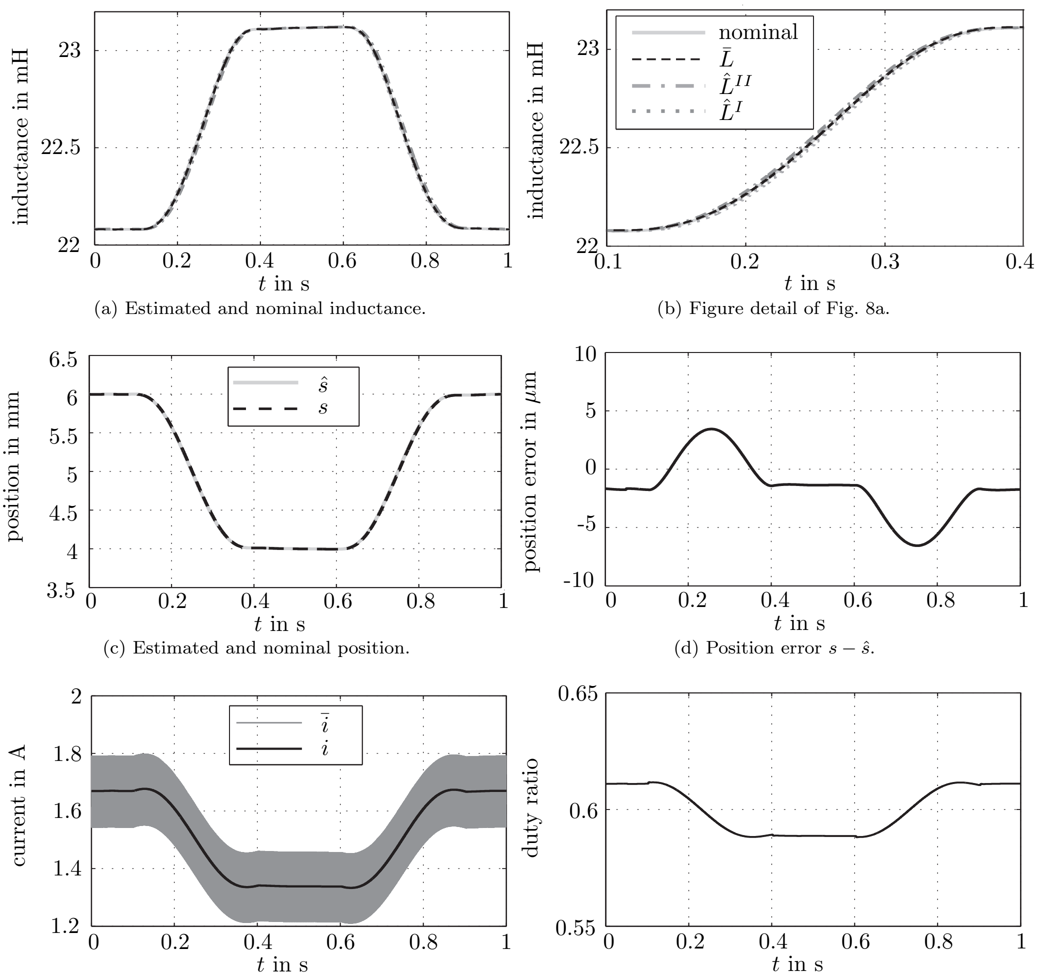


Figure 7: Overall structure of the proposed control and estimation strategy.



(a) Estimated and nominal inductance.

(b) Figure detail of Fig. 8a.

(c) Estimated and nominal position.

(d) Position error $s - \hat{s}$.

(e) Current and estimated current mean value.

(f) Duty ratio.

Figure 8: Simulation results for a setpoint change for $R = \hat{R} = 1.5\,\Omega$.

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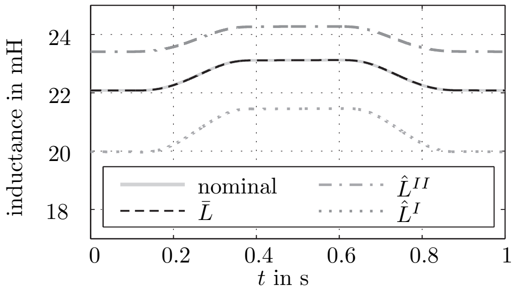
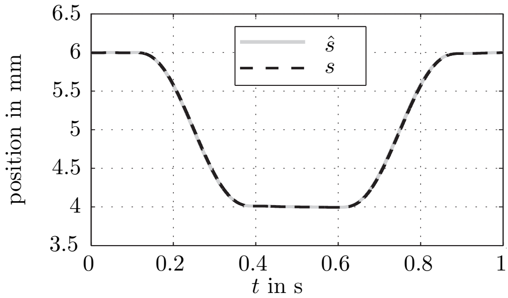


(a) Estimated and nominal inductance.



(b) Estimated and nominal position.

Figure 9: Simulation results for a setpoint change for a wrong electric resistance $R = 1.5\,\Omega$ and $\hat{R} = 2\,\Omega$.

5.2. Experimental results

After the successful testing in numerous simulations, the control and estimation scheme was implemented on a test bench, cf. Fig. 10. The test bench consists of an electromagnet comprising a ferrite cylindrical core with a plastic winding form wrapped in 452 turns of copper wire. The levitated object is a hollow ball with mass of 94.83 g and diameter of 40 mm fabricated of steel. Two 12 bit analog-digital converters are used to measure the current and the supply voltage at a sampling rate of 1 MSamples/s. The whole control and estimation algorithm was implemented on a Altera Stratix II FPGA test board with a connection to Matlab for debugging and initialising. Here, the fast summation part Σ^e_{fast} of the least squares estimator is implemented in fixed-point arithmetics using VHDL. For the computation of Σ^e_{slow} in the slow sampling time T_{pwm}, a softcore processor with a real-time operating system was generated with Altera software on the FPGA. This allows to implement the controller and the slow part Σ^e_{slow} in the programming language C and to use floating-point operations.

Fig. 11 shows a measured current ripple in the time interval $T_{pwm} = 1.024\,\mathrm{ms}$ for a current mean value of $\bar{i} = 1.5\,\mathrm{A}$ and a duty ratio of $\chi \approx 0.64$. Additionally, the waveform of the pulse-width modulated voltage is shown. Furthermore, the detail shows glitches at the beginning of the modulation period, which are due to the switching of the MOSFETs of the switching amplifier.

Finally, measurement results for a setpoint change are shown in Fig. 12 and Fig. 13. Fig. 12 shows measurement results for the nominal electric resistance $\hat{R} = 1.5\,\Omega$. In Fig. 12a, the desired position s_d and the estimated position $\hat{s}$ are depicted. Fig. 12b shows the resulting duty ratio of the current controller. The estimated inductances are shown in Fig. 12c. Finally, a comparison of the estimated position $\hat{s}$ and the measured position s_m is given in Fig. 12d. Here, a laser triangulation sensor was used in order to measure the actual position of the levitated object.

Additionally, in Fig. 13 measurement results for an error between the nominal ($\hat{R} = 2\,\Omega$) and the actual ($R = 1.5\,\Omega$) electric resistance are shown. As already discussed in detail before, this error causes a spreading of the estimated inductances $\hat{L}^I$ and $\hat{L}^{II}$. However, the mean value $\bar{L}$ according to (36) guarantees that this influence is exactly eliminated. Thus, the estimated position $\hat{s}$ is very close to the measured position s_m, cf. Fig. 13b. A movie of these experiments can be found in the Electronic Annex 1 in the online version of this article.

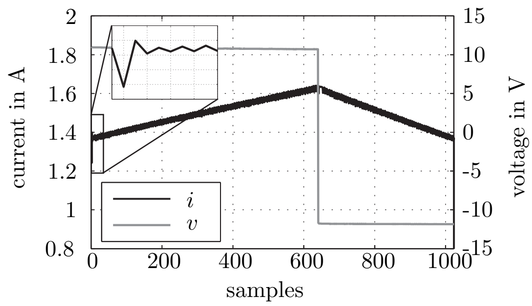


Figure 11: Measured current ripple at $\bar{i} = 1.5\,\mathrm{A}$ and switching waveform of the voltage.

6. Conclusion

In this work, a novel method for the sensorless estimation of the position of a PWM controlled magnetic levitation system was proposed. The design is based on a detailed mathematical model of the system, where a special emphasis was put on an accurate description of the influence of the switching amplifier on the time evolution of the current and the position of the levitated object. Based on a separate analysis of the charging and the discharging phase of the coil of the magnetic levitation system, the error induced by a wrong value of the electric resistance and by the motion of the levitated object could be exactly compensated. Furthermore, it was briefly outlined how the proposed estimation algorithm can be efficiently implemented in real-time hardware. Afterwards, a cascaded

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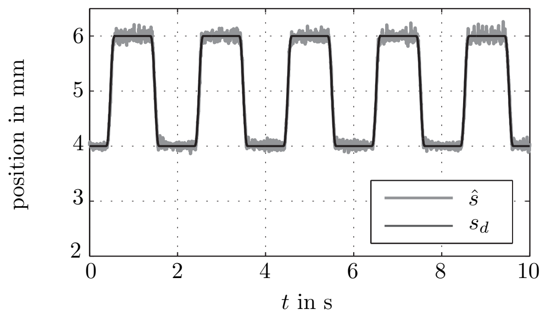
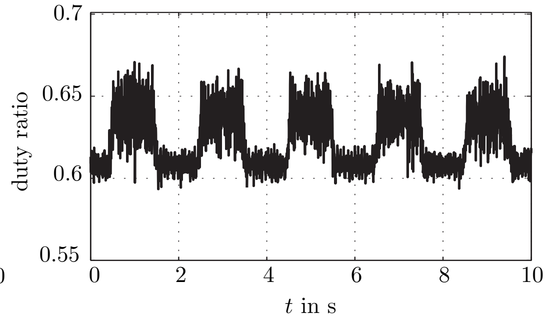
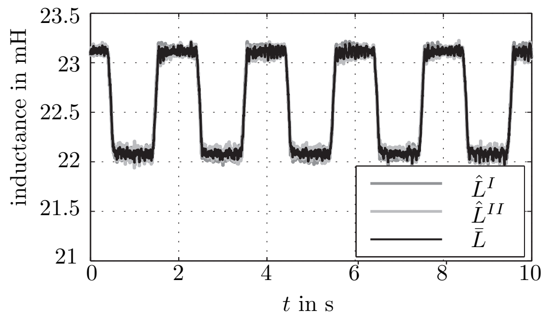


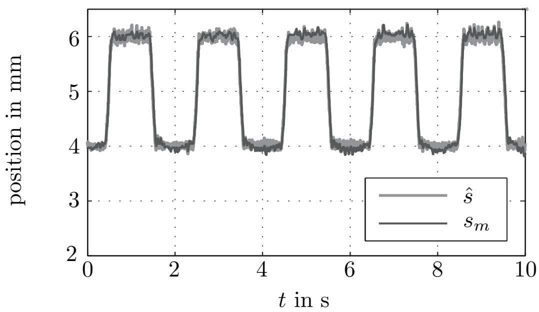
(a) Desired and the estimated position.



(b) Duty ratio.

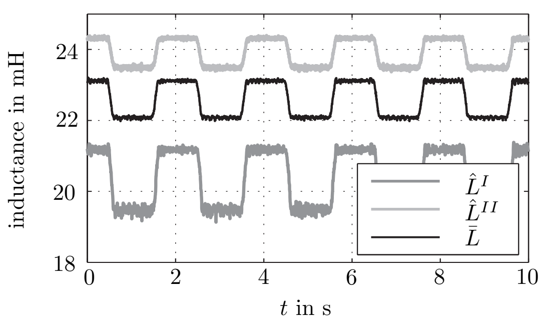


(c) Estimated inductance.

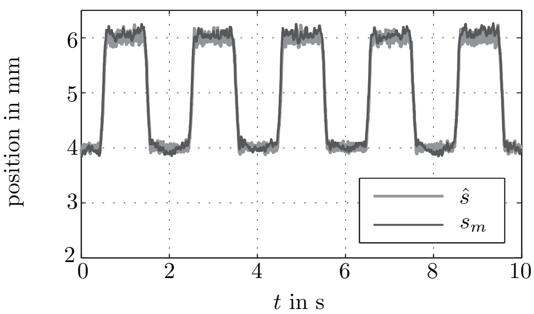


(d) Estimated and measured position by laser triangulation.

Figure 12: Experimental results for a setpoint change, a nominal electric resistance $\hat{R} \approx 1.5\,\Omega$ and a measured resistance $R = 1.5\,\Omega$.



(a) Estimated inductance.



(b) Estimated and measured position by laser triangulation.

Figure 13: Experimental results for a setpoint change, a nominal electric resistance $\hat{R} \approx 2\,\Omega$ and a measured resistance $R = 1.5\,\Omega$.

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position controller comprising a current controller in the inner control loop and a controller for the position in the outer control loop was designed. In the last part of this work, the accuracy of the position estimation and the high robustness with respect to a changing electric resistance and a moving levitated object was affirmed by means of simulation and measurement results.

Further research will focus on the application of this estimation algorithm to more complex configurations of magnetic levitation systems. Furthermore, actual work is concerned with the development of a robust estimation of the velocity of the levitated object avoiding numerical differentiation of the estimated position.

description	symb.	value	unit
mass of ball	m	94.83×10^{-3}	kg
coil turns	N	452	
supplied voltage	v_{bat}	11.4	V
reluct. ($\omega = 0$)	$\mathcal{R}_{fc}$	4.77×10^{6}	1/H
reluct. ($\omega = \omega_{pwm}$)	$\mathcal{R}_{fc}$	6.34×10^{6}	1/H
reluct. ($\omega = 0$)	$\mathcal{R}_{fo}$	8.07×10^{5}	1/H
reluct. ($\omega = \omega_{pwm}$)	$\mathcal{R}_{fo}$	1.07×10^{6}	1/H
reluct.	$\mathcal{R}_{l}$	5.08×10^{6}	1/H
effective area	A_g	8.32×10^{-4}	m^2
sampling time	T_s	1	μs
modulation period	T_{pwm}	1.024	ms
start sample	m_s^I, m_s^{II}	20	
par. pos. controller	$\lambda_{0,s}$	7	N/ms
par. pos. controller	$\lambda_{1,s}$	1750	N/m
par. pos. controller	$\lambda_{2,s}$	35	Ns/m
time constant filter	T_w	2.84×10^{-4}	s
par. cur. controller	$\lambda_{0,i}$	4560	V/As
par. cur. controller	$\lambda_{1,i}$	9.12	V/A

Table 1: Model and controller parameters.

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