Immersion and invariance-based impedance control for electrohydraulic systems


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Immersion and Invariance based Impedance Control for Electrohydraulic Systems

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SUMMARY

This work deals with the impedance control of electrohydraulic systems based on the concept of immersion and invariance. In the first step, the impedance control task for a basic electrohydraulic system comprising a hydraulic cylinder and a 4/3 proportional valve is analyzed. In order to mitigate the problem of energetic inefficiency and the high demands on the dynamics of the valve, an extended electrohdydraulic system is proposed. By means of a suitable choice of the parameters and an immersion and invariance based controller strategy, a significant reduction of both energy consumption and required dynamics of the valve can be obtained. The feasibility of the proposed impedance control strategies is demonstrated by extensive simulation studies. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: electrohydraulic systems; impedance control; nonlinear control; immersion and invariance; energetic efficiency

1. Introduction

Electrohydraulic systems find a wide field of applications in many technical products, ranging from automotive and aeronautical applications over mobile hydraulics for excavators or cranes to industrial applications. Despite the compact design of the actuators electrohydraulic systems enable large forces or torques. A further advantage of electrohydraulic actuators is the possibility to realize large displacements at high velocities.

The most common control tasks for electrohydraulic systems are position or force control, which are the topic of numerous works, see, e.g., [5], [6], [7], [10], [16], [20] and [24]. However, there are applications where neither the force nor the position of the electrohydraulic actuator but the compliance of the system has to be actively controlled. Possible applications are given in the field of robotics, where a desired end-effector compliance should be achieved or in

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automotive applications, where e.g. the spring and damping characteristics of the suspension system should be continuously adjusted to the actual driving situation.

For active compliance control basically two approaches do exist [23]: (i) In the hybrid position/force control approach the overall task space is subdivided into two orthogonal position and force controlled subspaces (see, e.g., [21]). (ii) In the impedance control approach it is desired to establish a dynamic relationship between the position and the force of the system. Traditionally, the control strategies for the impedance control task are based on cascaded control loops. In the so-called position based impedance control, the position is controlled in the inner control loop while the desired force is controlled in the outer control loop. Contrariwise, the so-called force based impedance control uses a force controller in the inner control loop and a position controller in the outer loop.

Both, the hybrid position/force control and the impedance control approach are commonly used in robotics, in particular in combination with electric actuators. However, only few works are concerned with the impedance control of systems with electrohydraulic actuators. For instance, in [23] the authors propose linear controllers for the active impedance control of a teleoperated hydraulic excavator. The application of a robust sliding mode control strategy to the impedance control of an hydraulic excavator is discussed in [11]. The authors of [1] use a transfer function approach in combination with linear controllers to analyze the stability of an electrohydraulic impedance system. Up to the knowledge of the authors in this context no works can be found which directly account for the nonlinear characteristics of electrohydraulic systems. Therefore, this paper is dedicated to the active (nonlinear) impedance control of electrohydraulic systems, where the control task can be summarized as follows: The response of the closed-loop electrohydraulic control system to an external load force should equal that of a desired eventually nonlinear (mechanical) impedance system.

In [17] and [13] it was shown how an analysis of the flows of energy in the electrohydraulic system can be advantageously used to design an impedance controller which systematically takes into account the nonlinearities of the system. Thus, an energy-based control strategy was developed in [17]. In [13] it was demonstrated that a controller based on integrator backstepping exhibits a very good and robust performance.

In this work, we will pursuit a different approach to the impedance control task of electrohydraulic systems. Therefore, the impedance control task shall be formulated in a more conceptual form: In the impedance control task it is desired to control a nonlinear system of higher dimension (i.e. the electrohydraulic system) in such a way that its behavior is equal to that of a nonlinear system of lower dimension (i.e. the desired mechanical impedance system). For such control design tasks the framework of immersion and invariance proves to be very well suited, see, e.g., [2], [3], [4], [19]. Here, we will apply an immersion and invariance controller strategy to the impedance control of electrohydraulic systems. Therefore, the basic configuration of the system, its mathematical model and the controller task are introduced in Section 2. The first part of Section 3 is concerned with the design of an immersion and invariance based controller for the electrohydraulic system. The feasibility of this controller is demonstrated in the second part by extensive simulation studies using components and parameters of an industrial system. Even though a very good controller performance and robustness can be achieved by the proposed control structure, the electrohydraulic system lacks of energetic efficiency. Thus, the first part of Section 4 deals with the design of an extended electrohydraulic system in order to improve the energetic efficiency of the system. Afterwards, an immersion and invariance based impedance controller strategy is developed and

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tested for the extended electrohydraulic system. Here, especially the improvement of energetic efficiency is emphasized. The paper closes with a short summary and outlook to further research activities.

2. Control Task and Mathematical Modeling

First we will analyze the impedance control task for an electrohydraulic linear actuator as given in Fig. 1, which probably is the most common electrohydraulic system. This system comprises a double acting cylinder which is rigidly connected to a constant mass m and controlled by a four lands three ways (4/3) proportional directional valve. It is assumed that the system is supplied by a constant pressure supply with the supply pressure p_s and the tank pressure p_t.

The motion of the mass m is described by the balance of momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}s_p = w_p \tag{1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w_p = \frac{1}{m}\left(p_1A_1 - p_2A_2 - mg - \tau_l\right). \tag{1b}$$

Here, s_p denotes the position of the piston, w_p is the velocity of the piston, p_1 and p_2 denote the two chamber pressures and A_1 and A_2 are the corresponding effective areas of the two chambers of the piston. Furthermore, τ_l describes the load force which cannot be measured and thus is assumed to be unknown.

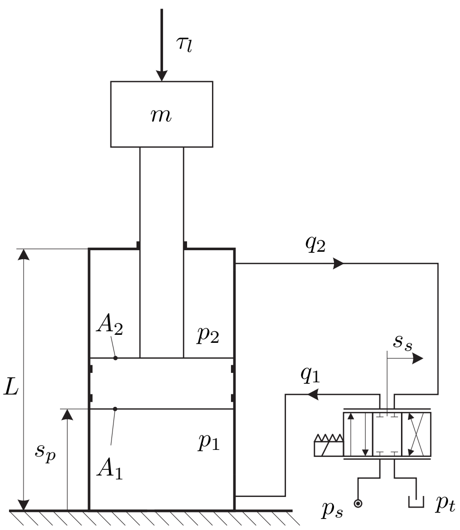

Figure 1. Basic electrohydraulic system used in the impedance control task.

Remark 1. *In many practical applications of an electrohydraulic impedance system a spring would be included in parallel to the hydraulic cylinder in order to provide a nominal spring*

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characteristics and to compensate for gravitational forces. The extension of the subsequent control strategy to include such a spring is rather straightforward and thus will not presented in this paper.

For the description of the material behavior of the fluid inside the chambers of the cylinder we assume a constant bulk modulus β of the oil (cf. [6], [9], [13]), i.e.

$$\beta = \rho \frac{\partial p}{\partial \rho} = \text{const.}, \tag{2}$$

with the density of the oil ρ. Using the constitutive equation (2) in the continuity equation for the two chambers of the cylinder we obtain the following equations for the description of the chamber pressures p_1 and p_2.

$$\frac{\mathrm{d}}{\mathrm{d}t} p_1 = \frac{\beta}{A_1 s_p} \left(-A_1 w_p + q_1 \right) \tag{3a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} p_2 = \frac{\beta}{A_2 \left(L - s_p \right)} \left(A_2 w_p - q_2 \right) \tag{3b}$$

Thereby, leakages either between the chambers or out of the chambers are neglected and the effective length of the cylinder is denoted by L.

The hydraulic actuator is controlled by a 4/3 proportional directional valve which provides the volume flows q_1 and q_2. Without loss of generality we assume a symmetrical critical center valve with rectangular ports, see, e.g., [6], [9]. This gives the mathematical model in the form

$$q_1 = \Gamma_1 s_s = \begin{cases} k_v s_s \sqrt{p_s - p_1} & \text{for} \quad s_s > 0 \\ k_v s_s \sqrt{p_1 - p_t} & \text{for} \quad s_s \leq 0 \end{cases} \tag{4a}$$

$$q_2 = \Gamma_2 s_s = \begin{cases} k_v s_s \sqrt{p_2 - p_t} & \text{for} \quad s_s > 0 \\ k_v s_s \sqrt{p_s - p_2} & \text{for} \quad s_s \leq 0, \end{cases} \tag{4b}$$

where the valve coefficient k_v for a proportional directional valve with a main spool of diameter d_s reads as

$$k_v = \alpha \pi d_s \sqrt{\frac{2}{\rho}}. \tag{5}$$

Here, α denotes the constant discharge coefficient (typically $\alpha \approx 0.7$). The dynamics of the valve is basically given by the dynamics of the spool position. For fast dual or multi-stage proportional directional valves as considered in this work, it can be assumed that the dynamics of the valve is much faster than the dynamics of the remaining system. Thus, the spool position s_s can be regarded as the control input of the system.

Remark 2. *Of course the subsequent control design is not limited to proportional directional valves with rectangular ports. If e.g. a symmetrical valve with circular ports is used, the volume flows q_1 and q_2 of the valve can be described by*

$$q_1 = \begin{cases} \alpha A_s \left(|s_s| \right) \sqrt{\frac{2}{\rho}} \sqrt{p_s - p_1} & \textit{for} \quad s_s > 0 \\ -\alpha A_t \left(|s_s| \right) \sqrt{\frac{2}{\rho}} \sqrt{p_1 - p_t} & \textit{for} \quad s_s \leq 0 \end{cases} \tag{6a}$$

$$q_2 = \begin{cases} \alpha A_t \left(|s_s| \right) \sqrt{\frac{2}{\rho}} \sqrt{p_2 - p_t} & \textit{for} \quad s_s > 0 \\ -\alpha A_s \left(|s_s| \right) \sqrt{\frac{2}{\rho}} \sqrt{p_s - p_2} & \textit{for} \quad s_s \leq 0, \end{cases} \tag{6b}$$

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with the opening area A_s from supply to port 1 or 2 and the opening area A_t from port 1 or 2 to tank. Since an inversion of the (strictly increasing) functions A_s and A_t is always possible, the valve spool position s_s can again be regarded as the control input.

In order to define the impedance control task, the desired mechanical impedance system Σ^I as depicted in Fig. 2 is introduced. The system comprises a mass m which is supported by a (nonlinear) spring and a (nonlinear) damper. The mathematical model of the desired mechanical impedance system Σ^I is given by

$$\Sigma^I: \quad \frac{\mathrm{d}}{\mathrm{d}t} s_p^I = w_p^I \tag{7a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w_p^I = \frac{1}{m}\left(-\chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right) - \tau_l\right), \tag{7b}$$

where $\chi_s^I\left(s_p^I\right)$ denotes the desired nonlinear spring characteristics and $\chi_d^I\left(w_p^I\right)$ is the desired nonlinear damping characteristics. For simplicity it is assumed that both the damping and the spring characteristics are strictly increasing functions in their respective arguments, with $\chi_d^I(0) = 0$ and $\chi_s^I\left(s_{p0}^I\right) = 0$, where s_{p0}^I denotes the relaxed length of the spring.

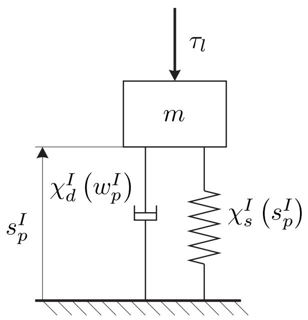

Figure 2. Desired mechanical impedance system Σ^I.

Thus, the impedance control task can be formulated as follows:

Definition 1. ***(Impedance Control Task)*** *The electrohydraulic system (1), (3) and (4) has to be controlled by means of the valve spool position s_s as the control input in such a way that the dynamic behavior of the piston position s_p in response to an external load force τ_l is equal to that of the desired mechanical impedance system Σ^I according to (7).*

3. Impedance Control based on Immersion and Invariance

3.1. Control Design

For the subsequent control design a reformulation of the mathematical model of the electrohydraulic system (1), (3) and (4) proves to be useful. Therefore, consider (3) in combination with (4) describing the pressures p_1 and p_2 in the two chambers of the piston.

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As a matter of fact, it is not possible to independently control the pressures p_1 and p_2 with only one control input s_s. Furthermore, the mechanical subsystem (1) is only influenced by the pressure force $\tau_p = p_1 A_1 - p_2 A_2$. Thus, a change of coordinates in the form $[s_p, w_p, p_1, p_2] \rightarrow [s_p, w_p, \tau_p, p_\Sigma]$ with the sum pressure $p_\Sigma = p_1 + p_2$ yielding

$$\frac{\mathrm{d}}{\mathrm{d}t} s_p = w_p \tag{8a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w_p = \frac{1}{m}\left(\tau_p - mg - \tau_l\right) \tag{8b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \tau_p = -\Upsilon_1 w_p + \Xi_1 s_s \tag{8c}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t} p_\Sigma = -\Upsilon_2 w_p + \Xi_2 s_s \tag{9}$$

is meaningful. Here, the abbreviations

$$\Upsilon_1 = \left(\frac{\beta A_1}{s_p} + \frac{\beta A_2}{L - s_p}\right), \quad \Upsilon_2 = \left(\frac{\beta}{s_p} - \frac{\beta}{L - s_p}\right) \tag{10a}$$

$$\Xi_1 = \left(\frac{\beta}{s_p}\Gamma_1 + \frac{\beta}{L - s_p}\Gamma_2\right), \quad \Xi_2 = \left(\frac{\beta}{A_1 s_p}\Gamma_1 - \frac{\beta}{A_2\left(L - s_p\right)}\Gamma_2\right) \tag{10b}$$

with Γ_1 and Γ_2 according to (4) are used.

For the solution of the impedance control task only the subsystem (8) with the state $x = [s_p, w_p, \tau_p]^T$ and the input s_s has to be considered, since the influence of the sum pressure p_Σ (9) due to the valve characteristics Γ_1 and Γ_2 can be exactly compensated if the supply pressure p_s, the tank pressure p_t and the chamber pressures p_1 and p_2 are measured. Of course, the stability of the overall system including the sum pressure p_Σ has to be proven separately in the final step of the controller design.

In the following the subsystem (8) with $\tau_l = 0$ is denoted as Σ,

$$\Sigma: \quad \frac{\mathrm{d}}{\mathrm{d}t} x = f\left(x\right) + g\left(x\right) u \tag{11}$$

with $x = [s_p, w_p, \tau_p]^T$, $u = s_s$ and

$$f\left(x\right) = \begin{bmatrix} w_p \\ \frac{1}{m}\left(\tau_p - mg\right) \\ -\Upsilon_1 w_p \end{bmatrix}, \quad g\left(x\right) = \begin{bmatrix} 0 \\ 0 \\ \Xi_1 \end{bmatrix}. \tag{12}$$

Definition 1 of the impedance control task states that the higher order system Σ with $x \in \mathbb{R}^3$ should be controlled by means of the control input $u = s_s \in \mathbb{R}$ in such a way that, in a certain sense, its behavior is equal to those of a lower order system, i.e. the desired impedance system Σ^I with $x^I \in \mathbb{R}^2$. Certainly, this control problem can be interpreted as an immersion and invariance (I&I) stabilization problem which is well known from the literature [2]. Following the ideas of this controller design strategy, the impedance control task can be reformulated in the form:

Definition 2. ***(Impedance Control Task with I&I)*** *Given the system Σ (11), (12) and the desired mechanical impedance system Σ^I ((7) with $\tau_l = 0$), the goal of the impedance control*

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task is to find a manifold $\mathcal{M}$, described implicitly by $\{x \in \mathbb{R}^3 | \phi(x) = 0\}$ or in parametrized form $\{x \in \mathbb{R}^3 | x = \pi(x^I), x^I \in \mathbb{R}^2\}$, which can be rendered invariant and asymptotically stable by means of the control input u, such that the restriction of the closed-loop system to $\mathcal{M}$ is described by Σ^I.

The design of the manifold $\mathcal{M}$ and the control input u basically follows Theorem 2.1 of [2]. For the sake of clarity this theorem is given in a form which fits the impedance control task considered in this work.

Theorem 1. ***(I&I Stabilization)****[2] Consider the following system (i.e. the electrohydraulic system Σ)*

$$\frac{\mathrm{d}}{\mathrm{d}t}x = f(x) + g(x)u, \tag{13}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and an equilibrium point $x_s \in \mathbb{R}^n$ to be stabilized. Assume that there exist smooth mappings $f^I : \mathbb{R}^p \to \mathbb{R}^p$, $\pi : \mathbb{R}^p \to \mathbb{R}^n$, $\phi : \mathbb{R}^n \to \mathbb{R}^{n-p}$, $c : \mathbb{R}^p \to \mathbb{R}^m$ and $v : \mathbb{R}^n \times \mathbb{R}^{n-p} \to \mathbb{R}^m$, with $p < n$, such that the following holds.
(A1) The target system, i.e. the desired impedance system Σ^I,

$$\frac{\mathrm{d}}{\mathrm{d}t}x^I = f^I(x^I), \tag{14}$$

with $x^I \in \mathbb{R}^p$, has a globally asymptotically stable equilibrium at $x_s^I \in \mathbb{R}^p$ and

$$x_s = \pi(x_s^I). \tag{15}$$

(A2) For all $x^I \in \mathbb{R}^p$

$$f(\pi(x^I)) + g(\pi(x^I))c(\pi(x^I)) = \frac{\partial \pi}{\partial x^I} f^I(x^I). \tag{16}$$

(A3) The set identity

$$\{x \in \mathbb{R}^n | \phi(x) = 0\} = \{x \in \mathbb{R}^n | x = \pi(x^I), x^I \in \mathbb{R}^p\} \tag{17}$$

holds.
(A4) All trajectories of the system

$$\frac{\mathrm{d}}{\mathrm{d}t}z = \frac{\partial \phi}{\partial x}(f(x) + g(x)v(x,z)), \tag{18a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}x = f(x) + g(x)v(x,z), \tag{18b}$$

are bounded and (18a) has a uniformly globally asymptotically stable equilibrium at $z = 0$. Then, x_s is a globally asymptotically stable equilibrium of the closed-loop system

$$\frac{\mathrm{d}}{\mathrm{d}t}x = f(x) + g(x)v(x,\phi(x)). \tag{19}$$

On the previous assumptions that both the spring characteristics $\chi_s^I(s_p^I)$ and the damping characteristics $\chi_d^I(w_p^I)$ are strictly increasing, the unique equilibrium point x_s^I of Σ^I for $\tau_l = 0$

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is given by $x_s^I = [s_{p0}^I, 0]^T$. Using the total energy stored in the desired mechanical impedance system consisting of the kinetic energy $\mathfrak{E}_k$ of the mass m

$$\mathfrak{E}_k = \frac{1}{2} m \left(w_p^I\right)^2 \tag{20}$$

and the potential energy $\mathfrak{E}_s$ of the spring

$$\mathfrak{E}_s = \int_{\xi = s_{p0}^I}^{s_p^I} \chi_s^I(\xi) \, \mathrm{d}\xi \tag{21}$$

as a positive definite Lyapunov function candidate, $V = \mathfrak{E}_k + \mathfrak{E}_s > 0$, it can be easily seen that the change of V along a solution curve of the system Σ^I is negative semi-definite,

$$\frac{\mathrm{d}}{\mathrm{d}t} V = -\chi_d\left(w_p^I\right) w_p^I \leq 0. \tag{22}$$

This already proves the stability of the desired impedance system. The asymptotic stability can be shown by means of the invariance principle of Krassovskii-LaSalle [14]. Finally, the global asymptotic stability of Σ^I results from the fact that the Lyapunov function V is radially unbounded [14]. Consequently, (A1) of Theorem 1 is fulfilled.

In the next step the manifold $\mathcal{M}$ is calculated by means of (16). For the system Σ (cf. (11), (12)) under consideration and the desired mechanical impedance system Σ^I due to (7) this results in three nonlinear partial differential equations, which, without previous knowledge, cannot be easily solved for $\pi = [\pi_1, \pi_2, \pi_3]$ and $c(\pi)$. However, an analysis of the physical meaning of the impedance control task advises to choose

$$s_p = \pi_1\left(s_p^I, w_p^I\right) = s_p^I \tag{23a}$$

$$w_p = \pi_2\left(s_p^I, w_p^I\right) = w_p^I. \tag{23b}$$

With this assumption used in (16), the first PDE is trivially fulfilled and the second and third (partial differential) equations for $\tau_p = \pi_3$ and $u = c(\pi)$ read as

$$\pi_3\left(s_p^I, w_p^I\right) - mg = -\chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right) \tag{24a}$$

$$-\Upsilon_1 w_p^I + \Xi_1 c(\pi) = \frac{\partial \pi_3}{\partial s_p^I} w_p^I + \frac{\partial \pi_3}{\partial w_p^I} \frac{1}{m}\left(-\chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right)\right) \tag{24b}$$

Solving (24a) for $\pi_3\left(s_p^I, w_p^I\right)$ gives

$$\pi_3\left(s_p^I, w_p^I\right) = mg - \chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right) \tag{25}$$

and using this solution in (24b) yields

$$c(\pi) = \frac{\Upsilon_1 w_p^I - \frac{\partial \chi_s^I}{\partial s_p^I} w_p^I - \frac{\partial \chi_d^I}{\partial w_p^I} \frac{1}{m}\left(-\chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right)\right)}{\Xi_1}. \tag{26}$$

The manifold $\mathcal{M}$ is defined in its explicit form by $x = \pi\left(x^I\right) = [\pi_1, \pi_2, \pi_3]^T$ with $x^I \in \mathbb{R}^2$, π_1 and π_2 from (23) and π_3 from (25), or in its implicit form as

$$\phi(x) = \tau_p - mg + \chi_s^I(s_p) + \chi_d^I(w_p) = 0. \tag{27}$$

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Then, the off-manifold dynamics (18a) is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}z = \frac{\partial \chi_s^I}{\partial s_p} w_p + \frac{\partial \chi_d^I}{\partial w_p} \frac{1}{m} (\tau_p - mg) - \Upsilon_1 w_p + \Xi_1 v(x,z), \tag{28}$$

where $u = v(x,z)$ is used. In order to asymptotically stabilize the off-manifold dynamics the control law

$$v(x,z) = \frac{\Upsilon_1 w_p - \frac{\partial \chi_s^I}{\partial s_p} w_p - \frac{\partial \chi_d^I}{\partial w_p} \frac{1}{m} (\tau_p - mg) - \delta_1^f z}{\Xi_1} \tag{29}$$

is chosen, with the constant and positive controller parameter $\delta_1^f \in \mathbb{R}^+$ which is used as a tuning parameter for the off-manifold dynamics.

In order to show that all trajectories of the closed-loop system (18) with $v(x,z)$ from (29) are bounded, the transformation $[s_p, w_p, \tau_p] \to [s_p, w_p, \eta]$ with $\eta = \tau_p - mg + \chi_s^I(s_p) + \chi_d^I(w_p)$ is used. This gives the transformed system

$$\frac{\mathrm{d}}{\mathrm{d}t}z = -\delta_1^f z \tag{30a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}s_p = w_p \tag{30b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w_p = \frac{1}{m}\left(\eta - \chi_s^I(s_p) - \chi_d^I(w_p)\right) \tag{30c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\eta = -\delta_1^f z. \tag{30d}$$

By construction of the control law $v(x,z)$, cf. (29), the equilibrium $z = 0$ is globally exponentially stable. Thus, η in (30d) is clearly bounded. Furthermore, it has been shown that the equilibrium $s_p = s_{p0}^I$ and $w_p = 0$ of the subsystem (30b), (30c), describing the position s_p and the velocity w_p of the piston, is asymptotically stable for $\eta = 0$ (see the discussion for the stability of the desired mechanical impedance system). Thus, the fact that η is bounded also implies that the trajectories of s_p and w_p are bounded, cf. [22]. Now, Theorem 1 states that the equilibrium $x_s = [s_{p0}^I, 0, mg]^T$ of the closed-loop system (8) with the control law $s_s = v(x, \phi(x))$ according to (29) is asymptotically stable. In the last step of the controller design the boundedness of the sum pressure p_Σ has to proven. Since this proof is completely equivalent to the proof presented in [18], it is omitted in this paper and the interested reader is referred to this article.

Remark 3. *A similar control strategy can be obtained using the method of integrator backstepping, cf. [13]. Thereby, it is taken advantage of the fact that the system (11) with (12) is given in strict feedback form. In the first step of the backstepping controller design, the pressure force* τ_p *is considered a virtual control input. Then, the matching condition yields* $\tau_p = mg - \chi_s^I(s_p) - \chi_d^I(w_p)$*. The stability of the corresponding subsystem can be shown by using the total energy of the desired impedance system as a suitable Lyapunov function candidate. In the next step, the Lyapunov function is extended by a quadratic term in the pressure force error* $e_{\tau_p} = \tau_p - mg + \chi_s^I(s_p) + \chi_d^I(w_p)$ *and a suitable control law is calculated such that the closed-loop system is asymptotically stable. It can be easily seen that* $z = e_{f_p}$ *holds and that* $z = 0$ *is equal to the matching condition used in the controller design via integrator backstepping. Nonetheless, the major advantage of using I&I for the design of the impedance controller is*

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that the I&I stabilization implies the invariance and the asymptotic stability of the desired manifold $\mathcal{M}$.

3.2. Simulation Results

For testing the I&I based impedance controller (29) a number of simulation studies were carried out using MATLAB/SIMULINK. Here, the results for an electrohydraulic system comprising a standard industrial single rod cylinder (Hänchen 300 15000-0140-25 [12]), an industrial dual stage 4/3 proportional directional valve (Bosch Rexroth 4WS2EM 6-2X [8]) and a mass $m = 250$ kg, cf. Table I, are presented. In the simulation studies the dynamics of the valve is approximated by a linear second order low pass filter with cut-off frequency ω_s and damping ratio ξ_s.

rod diameter	D_r	25	mm
piston diameter	D_p	40	mm
effective area 1 of cylinder	A_1	125.6	mm^2
effective area 2 of cylinder	A_2	76.6	mm^2
effective length of cylinder	L	200	mm
rated flow of the valve at 70 bar	q_{nom}	20	l/min
maximum stroke of valve	$s_{s,\max}$	0.6	mm
valve natural frequency	ω_s	$2\pi 300$	1/s
valve damping	ξ_s	$\sqrt{2}/2$	
mass	m	250	kg
bulk modulus of the oil	β	$1.6 \cdot 10^9$	Pa
density of the oil	ρ	860	kg/m^3

Table I. Parameters of the electrohydraulic system given in Fig. 1.

For the desired impedance system a nonlinear spring characteristics of the form

$$\chi_s^I\left(s_p^I\right) = c_1^I\left(s_p^I - s_{p0}^I\right) + c_3^I\left(s_p^I - s_{p0}^I\right)^3, \tag{31}$$

with the parameters c_1^I and c_3^I given in Table II is chosen. This spring characteristics exhibits progressive stiffness for deflections from the relaxed length $s_{p0}^I = L/2$, cf. Fig. 3. The desired damping characteristic is given by

$$\chi_{di}^I\left(w_p^I\right) = d_{1i}^I w_p^I + d_{3i}^I\left(w_p^I\right)^3, \quad i = \{1, 2, 3\}. \tag{32}$$

The corresponding coefficients provided in Table II show that χ_{d1}^I relates to a medium damping, χ_{d2}^I to a small damping and χ_{d3}^I to a high damping, cf. Fig. 3. Naturally, the mass of the electrohydraulic system Σ cannot be changed by the proposed control strategy, which is why the mass m of the desired impedance system Σ^I is assumed to be equal to that of the electrohydraulic system Σ.

In the first simulation, the performance of the closed-loop system Σ with the control law (29) is verified using nominal parameters and an infinitely fast 4/3 proportional directional valve. Figure 4 depicts the results for a desired spring characteristics χ_s^I given in (31) and a desired damping characteristics χ_{d1}^I from (32). A staircase like external force τ_l was used (cf. Fig. 4) and a value of $\delta_1^f = 8 \cdot 10^3$ was chosen for the controller parameter.

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linear stiffness coefficient	c_1^I	$50 \cdot 10^3$	N/m
cubic stiffness coefficient	c_3^I	$12.5 \cdot 10^6$	N/m^3
viscous damping coefficient 1	d_{11}^I	$2 \cdot 10^3$	Ns/m
cubic damping coefficient 1	d_{31}^I	$1 \cdot 10^4$	Ns^3/m^3
viscous damping coefficient 2	d_{12}^I	500	Ns/m
cubic damping coefficient 2	d_{32}^I	0	Ns^3/m^3
viscous damping coefficient 3	d_{13}^I	$4 \cdot 10^3$	Ns/m
cubic damping coefficient 3	d_{33}^I	$2 \cdot 10^5$	Ns^3/m^3

Table II. Parameters of the spring and damping characteristics of the desired mechanical impedance system.

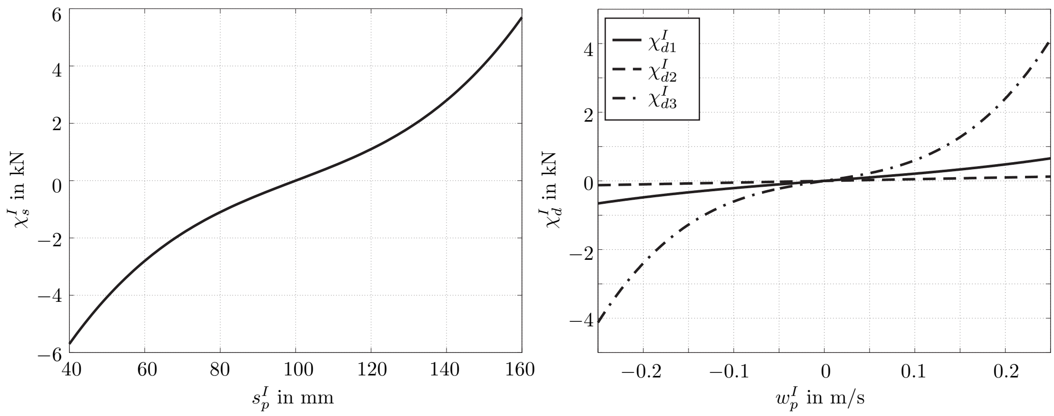

Figure 3. Desired nonlinear stiffness characteristics $\chi_s^I\left(s_p^I\right)$ and damping characteristics $\chi_{di}^I\left(w_p^I\right)$ according to (31) and (32) with parameters given in Table II.

In the topmost graphs of Fig. 4 it can be seen that an almost perfect tracking of the desired impedance characteristics is achieved by the proposed control concept. The graph for the chamber pressures p_1 and p_2 reveals that the initial value of p_1 is larger than the initial value of p_2 which is necessary to compensate for the gravitational force mg. On the right hand side of Fig. 4 the valve spool position s_s and the corresponding volume flows q_1 and q_2 are depicted. Here, the different values of q_1 and q_2 are mainly due to the different effective areas A_1 and A_2 of the single rod cylinder. Finally, it can be seen that relatively large volume flows are necessary to track the desired impedance system.

One of the main reasons for using an electrohydraulic impedance control approach is the possibility of actively changing the impedance behavior during operation without constructive changes. On the left hand side of Fig. 5 simulation results for different desired damping characteristics χ_{di}^I, $i \in \{1, 2, 3\}$ with parameters given in Table II are presented. The system is excited by an external load force τ_l as shown in Fig. 4. Obviously, a change of the system behavior from nearly undamped to strongly damped is easily possible. Of course it has to be pointed out that the minimum damping of the system is limited by the friction of the hydraulic

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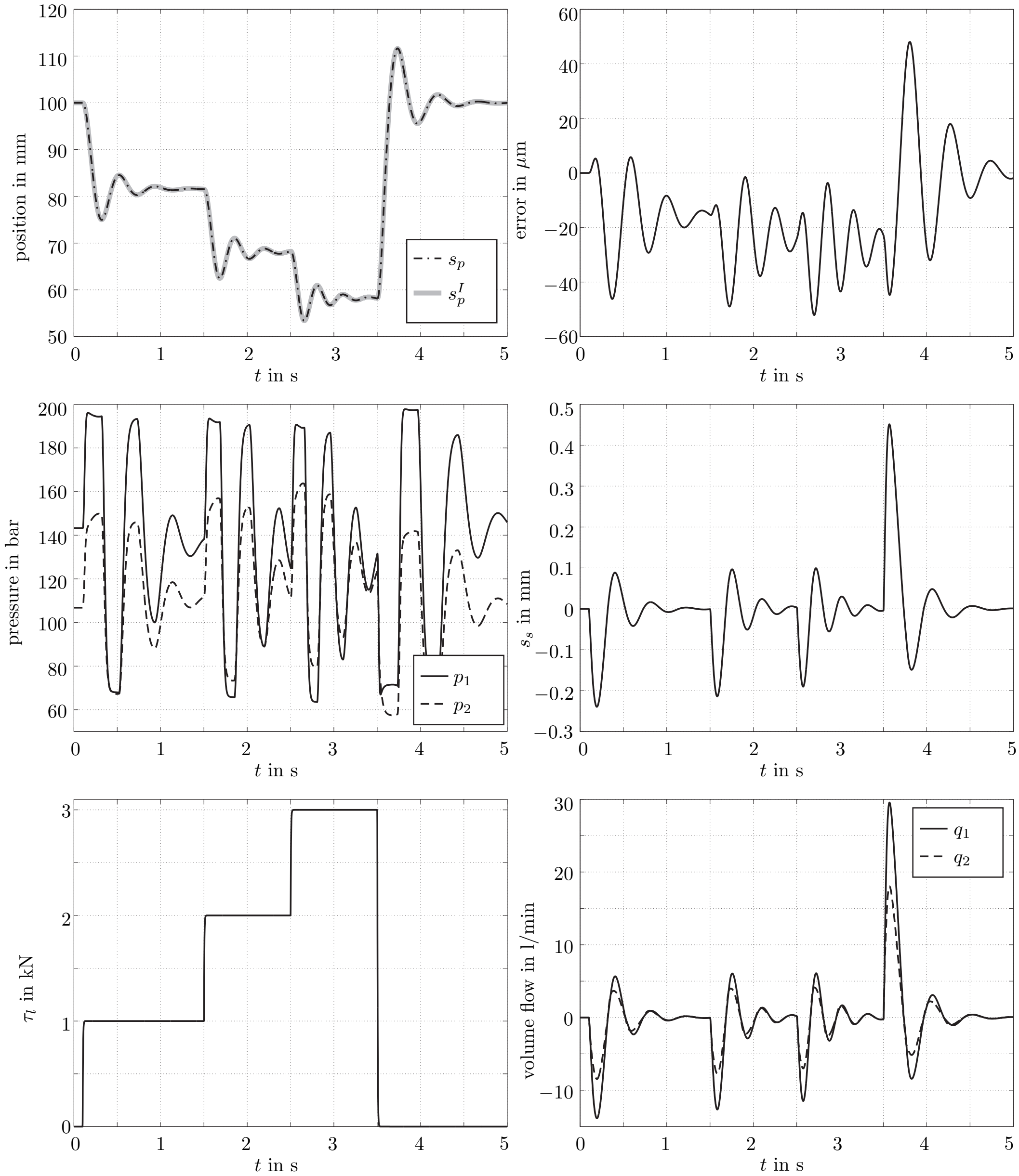

Figure 4. Simulation results for the nominal electrohydraulic system Σ with the I&I based controller (29), the desired spring characteristics χ_s^I (31) and the desired damping characteristics χ_{d1}^I (32) with parameters given in Table II.

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cylinder. On the right hand side of Fig. 5 a change of the linear stiffness c_1^I to $2c_1^I$ at time $t = 5$ s is examined. In this simulation a rectangular external load force τ_l of amplitude 1 kN, a cycle time of 2 s and a duty cycle of 50 % is used to excite the system. As expected, the increase of the linear stiffness coefficient c_1^I yields a significantly smaller deflection of the mass m. As can be seen from Fig. 5, the tracking of the desired impedance system is excellent also in this case.

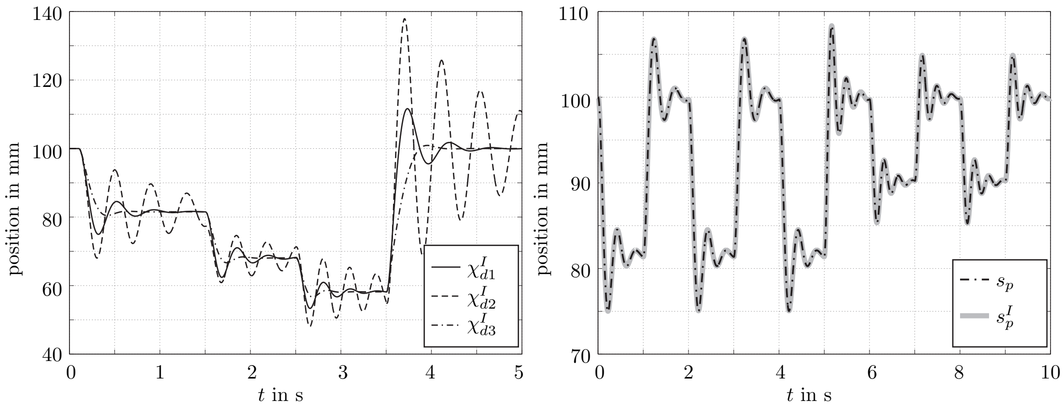

Figure 5. Change of the desired damping characteristics χ_d^I and of the desired spring characteristics χ_s^I.

Up to now, an ideal hydraulic system, neglecting leakages, variations of the bulk modulus of the oil and of the valve coefficient, an infinitely fast valve and no measurement noise have been assumed. For the proposed impedance controller (29) to be practically feasible, the robustness of the closed-loop system with respect to such imperfections has to be analyzed. For conciseness the results of the numerous simulations, which were carried out to study these effects, are only summarized. First of all, the proposed control concept is relatively insensitive to measurement noise. Even rather large measurement noise on the pressures signals p_s, p_1 and p_2 of amplitude 1 bar and in the position s_p of amplitude 50 μm (the velocity w_p was calculated by approximate numerical differentiation of the perturbed piston position) does not significantly deteriorate the performance of the system. Secondly, the closed-loop system also turns out to be robust to changes in the system parameters. Here, especially a change of the bulk modulus β in the range $0.1\beta_{nom} < \beta < 2\beta_{nom}$, with the nominal bulk modulus β_{nom} given in Table I, and a change of the valve coefficient k_v in the range $0.9k_{v,nom} < k_v < 1.1k_{v,nom}$, with the nominal valve coefficient $k_{v,nom}$, result in only a small and thus tolerable decrease in the performance. Also internal or external leakages typically occurring in the cylinder or the valve do not reduce the quality of the impedance control.

Finally, the influence of the dynamics of the 4/3 proportional directional valve is analyzed. Here, it turns out that the assumption of an ideally fast valve is crucial in order to obtain the perfect matching behavior of the electrohydraulic system with the desired impedance system as it has been presented before. If the dynamics of a fast two stage 4/3 proportional directional valve, represented by a second order LTI system with valve natural frequency ω_s and valve

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damping ξ_s (cf. Table I) is included in the simulation, it turns out that this, in combination with a controller parameter $\delta_f^1 = 8 \cdot 10^3$, yields an unstable system. By means of a significant reduction of the controller parameter to $\delta_f^1 = 1.6 \cdot 10^3$ a stable system can be obtained with the drawback of an increased tracking error between actual and desired impedance characteristics, see Fig. 6.

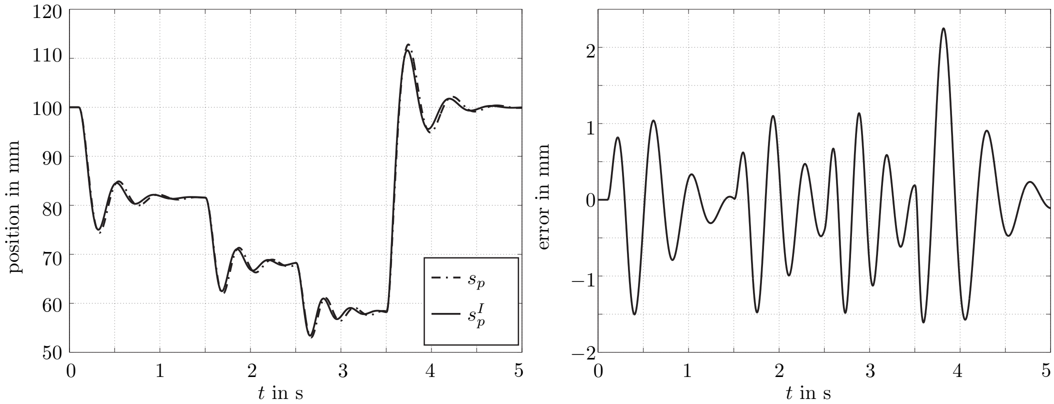

Figure 6. Simulation result of the electrohydraulic system Σ with the I&I based controller (29) and an industrial valve with parameters given in Table I.

For many practical problems this tracking error is still tolerable. Nonetheless, there are two main obstacles for the practical implementation of the proposed control concept. (i) As has been pointed out in the previous analysis, very fast valves are necessary for the implementation of the control concept. Naturally, these valves entail high costs rendering them unsuitable for many applications. (ii) The energetic efficiency of the overall system is rather poor, especially from the perspective that the desired mechanical impedance system only dissipates energy but does not need any external power supply.

Thus, the next section is concerned with an extension of the construction of the electrohydraulic system of Fig. 1 in order to improve these issues.

4. Design and Control of an Extended Electrohydraulic System

4.1. Constructional Setup and Mathematical Modeling

For the improvement of the energetic efficiency of the electrohydraulic system let us first analyze the desired mechanical impedance system given in Fig. 2. Assuming that the spring and damping characteristics are fixed, it can be easily seen that energy is either transformed from kinetic energy of the mass m into potential energy of the spring and vice versa or it is dissipated by the damper. Obviously, no external supply of energy is necessary to achieve the desired impedance characteristics. In comparison to this, no efficient possibility of storing potential energy is included in the basic electrohydraulic system according to Fig. 1. Thus, in order to improve the energetic efficiency of the electrohydraulic impedance system, the

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inclusion of an appropriate energy storage device is inevitable.

Secondly, the natural stiffness of the basic electrohydraulic impedance system is very high compared to the desired stiffness of the mechanical impedance system. This results in a very high dynamics of the chamber pressures p_1 and p_2 and therefore of the pressure force τ_p. This fact also explains the high demands on the dynamics of the valve used for the control of the basic electrohydraulic impedance system.

Summarizing, an improvement of the basic electrohydraulic system relies on the efficient storage of energy and on the reduction of the stiffness of the system. It is well known that (hydraulic) energy can be efficiently stored in hydraulic accumulators. Thus, the connection of hydraulic accumulators to the chambers of the cylinder as depicted in Fig. 7 enables the efficient storage of energy. Naturally, this also yields a significant reduction of the stiffness of the system. Furthermore, the hydraulic impedance system is extended by two laminar damping orifices which provide a certain nominal damping characteristics. Finally, the 4/3 proportional directional valve used in the basic electrohydraulic impedance system is replaced by two 3/3 proportional directional valves which obviously provide more degrees-of-freedom for the controller design. Of course, the usage of two 3/3 valves instead of one 4/3 valve increases the complexity of the system. Nonetheless, the overall cost of the system might be significantly lower for the extended electrohydraulic system according to Fig. 7 since, as it will be demonstrated later, rather slow and therefore low-cost valves can be used.

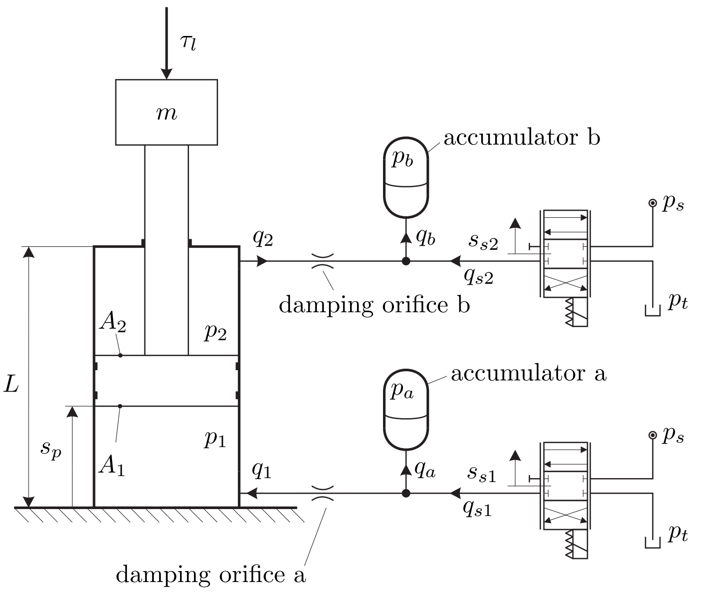

Figure 7. Extended electrohydraulic impedance system.

The mathematical model for the description of the motion of the mass m of the extended

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electrohydraulic system is equal to (1) and reads as

$$\frac{\mathrm{d}}{\mathrm{d}t}s_p = w_p \tag{33a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w_p = \frac{1}{m}\left(p_1 A_1 - p_2 A_2 - mg - \tau_l\right). \tag{33b}$$

The pressures p_1 and p_2 in the chambers of the cylinder are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}p_1 = \frac{\beta}{A_1 s_p}\left(-A_1 w_p + q_1\right) \tag{34a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_2 = \frac{\beta}{A_2\left(L - s_p\right)}\left(A_2 w_p - q_2\right). \tag{34b}$$

Here, the volume flows q_1 and q_2 through the laminar damping orifices a and b can be described by

$$q_1 = k_{da}\left(p_a - p_1\right) \tag{35a}$$

$$q_2 = k_{db}\left(p_2 - p_b\right), \tag{35b}$$

where k_{da} and k_{db} denote the laminar flow coefficients of the damping orifices. For calculating the pressures p_a and p_b in the hydraulic accumulators, the gas inside the accumulator is assumed to satisfy the isentropic equation

$$pV_g^{\kappa} = p_0 V_{g0}^{\kappa} = \zeta_0. \tag{36}$$

Thereby, p denotes the pressure, V_g is the volume, p_0 and V_{g0} are the pressure and the volume of the gas at precharge condition ζ_0, and κ denotes the constant isentropic coefficient of the gas. Using the constitutive equation of the gas (36) and the oil (2), the pressures in the accumulators follow as (cf. [6], [9], [13])

$$\frac{\mathrm{d}}{\mathrm{d}t}p_a = \frac{\kappa\beta p_a q_a}{\kappa p_a V_a + (\beta - \kappa p_a)\left(\frac{\zeta_{0a}}{p_a}\right)^{\frac{1}{\kappa}}} \tag{37a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_b = \frac{\kappa\beta p_b q_b}{\kappa p_b V_b + (\beta - \kappa p_b)\left(\frac{\zeta_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}, \tag{37b}$$

where V_a and V_b denote the overall volume of the hydraulic accumulators. The volume flows q_a and q_b into the accumulators are described by

$$q_a = q_{s1} - q_1 \tag{38a}$$

$$q_b = q_{s2} + q_2, \tag{38b}$$

with the valve volume flows q_{s1} and q_{s2} (cf. (4))

$$q_{s1} = \Gamma_1 s_{s1} = \begin{cases} k_{v1} s_{s1}\sqrt{p_s - p_a} & \text{for} \quad s_{s1} > 0 \\ k_{v1} s_{s1}\sqrt{p_a - p_t} & \text{for} \quad s_{s1} \leq 0 \end{cases} \tag{39a}$$

$$q_{s2} = \Gamma_2 s_{s2} = \begin{cases} k_{v2} s_{s2}\sqrt{p_s - p_b} & \text{for} \quad s_{s2} > 0 \\ k_{v2} s_{s2}\sqrt{p_b - p_t} & \text{for} \quad s_{s2} \leq 0. \end{cases} \tag{39b}$$

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Thereby, k_{v1} and k_{v2} denote the valve coefficients, p_s is the supply pressure, p_t the tank pressure and s_{s1} and s_{s2} denote the valve spool position of valve 1 and 2, respectively.

Analyzing the overall mathematical model (33)–(35) and (37)–(39) of the extended electrohydraulic impedance system due to Fig. 7, it can be seen that the dynamics of the chamber pressures p_1 and p_2 is significantly faster compared to the dynamics of the remaining system. This fact becomes more evident when rewriting the mathematical model in the form of a singularly perturbed system. Therefore, (34) with (35) is replaced by

$$\varepsilon \frac{\mathrm{d}}{\mathrm{d}t} p_1 = \frac{1}{A_1 s_p} \left(-A_1 w_p + k_{da} \left(p_a - p_1\right)\right) \tag{40a}$$

$$\varepsilon \frac{\mathrm{d}}{\mathrm{d}t} p_2 = \frac{1}{A_2 \left(L - s_p\right)} \left(A_2 w_p - k_{db} \left(p_2 - p_b\right)\right), \tag{40b}$$

where $\varepsilon = 1/\beta$ serves as an appropriate singular perturbation parameter. Considering the limit $\varepsilon \to 0$, which refers to an incompressible fluid, yields the quasi-stationary solution of the fast subsystem, see, e.g. [15]

$$p_1 = p_a - \frac{A_1 w_p}{k_{da}}, \quad p_2 = p_b + \frac{A_2 w_p}{k_{db}}. \tag{41}$$

The reduced slow dynamics (33), (37) is then given by

$$\frac{\mathrm{d}}{\mathrm{d}t} s_p = w_p \tag{42a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w_p = \frac{1}{m} \left(p_a A_1 - p_b A_2 - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}} \right) w_p - mg - \tau_l \right) \tag{42b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} p_a = \frac{\kappa p_a \left(\Gamma_1 s_{s1} - A_1 w_p\right)}{\left(\frac{\zeta_{0a}}{p_a}\right)^{\frac{1}{\kappa}}} \tag{42c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} p_b = \frac{\kappa p_b \left(\Gamma_2 s_{s2} + A_2 w_p\right)}{\left(\frac{\zeta_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}. \tag{42d}$$

Naturally, an appropriate choice of the parameters of the extended electrohydraulic system is of particular importance in order to achieve the desired reduction in the energy consumption. Thereby, the parameters of the cylinder (i.e. the length L and the areas A_1 and A_2) are chosen such that the demands on pressure force and travel are met. The parameters of the accumulators and the damping orifices are typically designed in such a way that the extended electrohydraulic system already shows a desired spring and damping characteristics without control, i.e. $s_{s1} = s_{s2} = 0$.

Remark 4. *For the dimensioning of the accumulators the pressure force $\tau_p = p_a A_1 - p_b A_2$ in the stationary case $w_p = 0$ for closed valves, i.e. $s_{s1} = s_{s2} = 0$, is calculated. The pressure force is a function of the position s_p and the parameters of the system only, see, e.g., [13].*

$$\tau_p = \frac{\zeta_{0a} A_1}{\left(V_{ga0} + A_1 \left(s_p - s_{p0}\right)\right)^{\kappa}} - \frac{\zeta_{0b} A_2}{\left(V_{gb0} - A_2 \left(s_p - s_{p0}\right)\right)^{\kappa}} \tag{43}$$

Clearly, in the stationary case the pressure force τ_p has to be equal to the sum of the spring force χ_s^I and the gravitational force mg. Let us assume that the desired spring characteristics

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$\chi_s^I(s_p)$ is given in the form

$$\chi_s^I(s_p) = c_1^I\left(s_p - s_{p0}^I\right) + \tilde{\chi}_s^I(s_p)\,, \tag{44}$$

with the desired linear stiffness c_1^I and the nonlinear part $\tilde{\chi}_s^I(s_p)$ which meets the following assumption

$$\left.\frac{\partial\tilde{\chi}_s^I(s_p)}{\partial s_p}\right|_{s_p=s_{p0}^I} = 0. \tag{45}$$

If the linear part $c_1^I\left(s_p - s_{p0}^I\right)$ of the spring characteristics is dominating at least in the vicinity of s_{p0}^I, it is meaningful to choose the parameters of the accumulator such that the stationary pressure force gradient σ_1 of the extended electrohydraulic system

$$\begin{aligned}\sigma_1 = -\frac{\partial\tau_p}{\partial s_p} &= \frac{A_1^2\kappa}{\zeta_{0a}^{\frac{1}{\kappa}}}p_a^{\frac{\kappa+1}{\kappa}} + \frac{A_2^2\kappa}{\zeta_{0b}^{\frac{1}{\kappa}}}p_b^{\frac{\kappa+1}{\kappa}} \\ &= \frac{\zeta_{0a}A_1^2\kappa}{\left(V_{ga0} + A_1\left(s_p - s_{p0}\right)\right)^{\kappa+1}} + \frac{\zeta_{0b}A_2^2\kappa}{\left(V_{gb0} - A_2\left(s_p - s_{p0}\right)\right)^{\kappa+1}}\end{aligned} \tag{46}$$

at $s_p = s_{p0}^I$ corresponds to the desired linear stiffness c_1^I from (44). With this assumption a systematic determination of the parameters of the accumulators is possible [13].
The choice of the parameters of the damping orifice is rather trivial if the desired damping characteristic $\chi_d^I(w_p)$ is supposed to be written in the form

$$\chi_d^I(w_p) = d_{11}^I w_p + \tilde{\chi}_d^I(w_p)\,, \tag{47}$$

where d_{11}^I is the linear (viscous) damping coefficient and $\tilde{\chi}_d^I(w_p)$ denotes the nonlinear part. Considering (42b) it is obvious to choose

$$\left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) = d_{11}^I \tag{48}$$

for the desired viscous part of the damping characteristics.

In order to be able to change the damping and spring characteristics, a control input has to be added to the system. If it is only intended to control the pressure force τ_p then one control input, e.g. valve 1 or 2 in Fig. 7, is sufficient. Otherwise, with only one valve it is not possible to actively influence the natural stiffness of the system, cf. (46). It can be easily shown that the extended electrohydraulic system with only one control input is energetically very inefficient in case large changes of the stiffness are demanded [13]. Therefore, it is useful to add a second valve as a second control input. Then, both pressures p_a and p_b can be controlled independently and thus also the pressure force $\tau_p = p_a A_1 - p_b A_2$ and also the quantity σ_1 corresponding to the stiffness of the system (46) can be controlled independently. This contributes to increase the overall energetic efficiency of the system.

For the subsequent design of an impedance controller the system (42) is transformed into the new coordinates $\xi = [s_p, w_p, \tau_p, \sigma_1]^T$. Note that the coordinate transformation $\xi = t(x)$ with $x = [s_p, w_p, p_a, p_b]^T$ locally defines a diffeomorphism and thus serves as an admissible

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change of coordinates. The transformed system reads as

$$\Sigma^p: \quad \frac{\mathrm{d}}{\mathrm{d}t}s_p = w_p \tag{49a}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}w_p = \frac{1}{m}\left(\tau_p - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p - mg - \tau_l\right) \tag{49b}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\tau_p = -\chi_f w_p + \psi_f \tag{49c}$$
$$\Sigma^c: \quad \frac{\mathrm{d}}{\mathrm{d}t}\sigma_1 = -\chi_c w_p + \psi_c \tag{49d}$$

with the abbreviations

$$\chi_p = \left.\left(\frac{A_1^2\kappa p_a}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}} + \frac{A_2^2\kappa p_b}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\right)\right|_{x=t^{-1}(\xi)} \tag{50a}$$

$$\psi_p = \left.\left(\frac{A_1\kappa p_a}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}}\Gamma_1 s_{s1} - \frac{A_2\kappa p_b}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\Gamma_2 s_{s2}\right)\right|_{x=t^{-1}(\xi)} \tag{50b}$$

$$\chi_c = \left.\left(\frac{A_1^3\kappa(\kappa+1)}{\varsigma_{0a}^{\frac{2}{\kappa}}} p_a^{\frac{\kappa+2}{\kappa}} - \frac{A_2^3\kappa(\kappa+1)}{\varsigma_{0b}^{\frac{2}{\kappa}}} p_b^{\frac{\kappa+2}{\kappa}}\right)\right|_{x=t^{-1}(\xi)} \tag{50c}$$

$$\psi_c = \left.\left(\frac{A_1^2\kappa(\kappa+1)}{\varsigma_{0a}^{\frac{2}{\kappa}}} p_a^{\frac{\kappa+2}{\kappa}}\Gamma_1 s_{s1} + \frac{A_2^2\kappa(\kappa+1)}{\varsigma_{0b}^{\frac{2}{\kappa}}} p_b^{\frac{\kappa+2}{\kappa}}\Gamma_2 s_{s2}\right)\right|_{x=t^{-1}(\xi)}. \tag{50d}$$

Since (50b) and (50d) can be solved for given ψ_p and ψ_c w.r.t s_{s1} and s_{s2}, henceforth ψ_p and ψ_c may be considered as new control inputs. A closer inspection of the system (49) reveals that the subsystem Σ^p (49a), (49b) and (49c) can be decoupled from the subsystem Σ^c (49d) by applying the input transformation

$$\psi_p = \chi_p w_p + \tilde{\psi}_p \tag{51a}$$
$$\psi_c = \chi_c w_p + \tilde{\psi}_c, \tag{51b}$$

where $\tilde{\psi}_p$ and $\tilde{\psi}_c$ serve as new control inputs. The real input signals s_{s1} and s_{s2} of the system can be calculated from the virtual control inputs $\tilde{\psi}_p$ and $\tilde{\psi}_c$ using (50) and (51)

$$s_{s1} = \frac{\tilde{\psi}_c + \chi_c w_p + \frac{A_2(\kappa+1)}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\left(\tilde{\psi}_p + \chi_p w_p\right)}{\frac{\Gamma_1\kappa(\kappa+1)A_1 p_a}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}}\left(\frac{A_1}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}} + \frac{A_2}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\right)} \tag{52a}$$

$$s_{s2} = \frac{\tilde{\psi}_c + \chi_c w_p - \frac{A_1(\kappa+1)}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}}\left(\tilde{\psi}_p + \chi_p w_p\right)}{\frac{\Gamma_2\kappa(\kappa+1)A_2 p_b}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\left(\frac{A_1}{\left(\frac{\varsigma_{0a}}{p_a}\right)^{\frac{1}{\kappa}}} + \frac{A_2}{\left(\frac{\varsigma_{0b}}{p_b}\right)^{\frac{1}{\kappa}}}\right)}. \tag{52b}$$

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The subsequent control design task can then be subdivided into two parts: (i) the control of the pressure force f_p such that the closed-loop system exhibits the desired impedance characteristics and (ii) the control of the pressure force gradient σ_1 of the system to achieve a good energetic efficiency.

4.2. Control Design

Basically, the first control design task is similar to the one already formulated in Definition 2. That is, the subsystem Σ^p with $x^p = [s_p, w_p, \tau_p]^T$ of (49), (50) has to be controlled in such a way that its response to an external load force τ_l is equal to that of the desired mechanical impedance system Σ^I due to (7). Thus, this control task can once again be tackled as an I&I-stabilization problem. Since the derivations are basically equal to those in Section 3, here only the results of the controller design are summarized.

The transformation $x^p = \pi\left(x^I\right)$ is given by, cf. (23), (24a)

$$s_p = \pi_1\left(s_p^I, w_p^I\right) = s_p^I \tag{53a}$$
$$w_p = \pi_2\left(s_p^I, w_p^I\right) = w_p^I \tag{53b}$$
$$\tau_p = \pi_3\left(s_p^I, w_p^I\right) = mg + \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p^I - \chi_s^I\left(s_p^I\right) - \chi_d^I\left(w_p^I\right) \tag{53c}$$

and $c(\pi)$ results in

$$c(\pi) = -\frac{\partial \chi_s^I}{\partial s_p^I} w_p^I + \left(\frac{\partial \chi_d^I}{\partial w_p^I} - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right)\right) \frac{1}{m_p}\left(\chi_s^I\left(s_p^I\right) + \chi_d^I\left(w_p^I\right)\right). \tag{54}$$

With the definition of $\phi\left(x^p\right)$

$$\phi\left(x^p\right) = \tau_p - mg - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p^I + \chi_s^I\left(s_p\right) + \chi_d^I\left(w_p\right), \tag{55}$$

the off-manifold dynamics yields

$$\frac{\mathrm{d}}{\mathrm{dt}} z = \left(\frac{\partial \chi_d^I}{\partial w_p} - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right)\right) \frac{1}{m}\left(\tau_p - mg - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p\right) + \frac{\partial \chi_s^I}{\partial s_p} w_p + v\left(x^p, z\right), \tag{56}$$

where $v(x^p, z) = \tilde{\psi}_p$, which can be rendered asymptotically stable by the choice

$$v\left(x^p, z\right) = -\left(\frac{\partial \chi_d^I}{\partial w_p} - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right)\right) \frac{1}{m}\left(\tau_p - mg - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p\right) - \frac{\partial \chi_s^I}{\partial s_p} w_p - \delta_1^f z. \tag{57}$$

Therein, $\delta_1^f > 0$ denotes the controller parameter. Along the lines of the previous section it can be shown that the requirements of (A4) of Theorem 1 are met. Thus the control law

$$\begin{aligned}\tilde{\psi}_f\left(x^p\right) = &-\left(\frac{\partial \chi_d^I}{\partial w_p} - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right)\right) \frac{1}{m}\left(\tau_p - mg - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p\right) \\ &- \frac{\partial \chi_s^I}{\partial s_p} w_p - \delta_1^f\left(\tau_p - mg - \left(\frac{A_1^2}{k_{da}} + \frac{A_2^2}{k_{db}}\right) w_p + \chi_s^I\left(s_p\right) + \chi_d^I\left(w_p\right)\right)\end{aligned} \tag{58}$$

yields an asymptotically stable closed-loop system.

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For the second control task let us consider the subsystem Σ^c of (49) with $x^c = \sigma_1$ from (46). Henceforth, the desired linear stiffness $c_1^I(t)$ is supposed to be sufficiently smooth and is commanded by a superordinate system. Introducing the stiffness error $e_c = \sigma_1 - c_1^I$ and the integral of the stiffness error e_c^i yields

$$\frac{\mathrm{d}}{\mathrm{d}t}e_c^i = e_c \tag{59a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}e_c = \tilde{\psi}_c - \dot{c}_1^I.$$

It can be easily verified that the control law

$$\tilde{\psi}_c = -\delta_0^c e_c^i - \delta_1^c e_c + \dot{c}_1^I \tag{60}$$

with the positive controller parameters $\delta_0^c, \delta_1^c > 0$ entails an exponentially stable dynamics for the stiffness error. Furthermore, the error dynamics can be arbitrarily assigned by means of the controller parameters.

4.3. Simulation Results

For verifying the proposed control strategy of the extended electrohydraulic system according to Fig. 7 numerous simulations were carried out. Thereby, the parameters of the piston and the desired impedance system were chosen identically to Section 3.2. Furthermore, the parameters of the accumulators and the laminar damping orifices were calculated such that the stationary pressure force gradient σ_1 of the extended electrohydraulic system is equal to c_1^I at $s_{p0}^I = L/2$ and the damping is equal to d_{11}^I. The resulting parameters of the extended electrohydraulic system are summarized in Table III.

volume of accumulator a	V_a	1.0	l
volume of accumulator b	V_b	0.79	l
precharge pressure a at $V_{0a} = V_a$	p_{0a}	58.2	bar
precharge pressure b at $V_{0b} = V_b$	p_{0b}	75.1	bar
coefficient of damping orifice a	k_{da}	$1.58 \cdot 10^{-9}$	$\frac{\mathrm{m}^5}{\mathrm{Ns}}$
coefficient of damping orifice b	k_{db}	$5.86 \cdot 10^{-10}$	$\frac{\mathrm{m}^5}{\mathrm{Ns}}$
isentropic coefficient	κ	1.6	
rated flow of the valves at 70 bar	q_{nom}	20	l/min
valve natural frequency	ω_s	$2\pi 30$	1/s
valve damping	ξ_s	$\sqrt{2}/2$	

Table III. Parameters of the extended electrohydraulic system given in Fig. 7.

In the first simulation the closed-loop behavior of the nominal extended electrohydraulic system with a desired impedance characteristics due to (7), (31), (32) and the parameters c_1^I, c_3^I, d_{11}^I and d_{31}^I given in Table II is investigated. As it can be seen in Fig. 8 a very good matching of the desired impedance characteristics can be obtained. This is remarkable since the rather slow dynamics of the valves (approximately 1/10 of the dynamics of the valve used in the basic electrohydraulic system due to Fig. 1) has already been included in the simulation model. Since these valves are significantly cheaper than the fast valve used in the basic system, a reduction of costs can be achieved despite the increased complexity of the system.

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Another interesting point is the influence of the damping orifices which can be seen in the plots for the chamber and accumulator pressures in Fig. 8. Obviously, the differences between the pressure p_a and p_1 or p_b and p_2, respectively, correspond to the dissipation of energy.

Of course the robustness of the extended electrohydraulic system with respect to parameter variations and model uncertainties has been examined in extensive simulation studies. Summarizing, the results obtained by the extended electrohydraulic system are similar to those of the basic system. The dynamics of the valves can further be lowered without causing instability problems.

The active change of the desired stiffness c_1^I of the system is presented in Fig. 9. There, σ_1 is doubled from $\sigma_1 = c_1^I$ to $\sigma_1 = 2c_1^I$ at time $t = 5$ s. As it has already been pointed out before, the stiffness of the system is basically equal to the sum of the pressures in the cylinder chambers. Thus, in order to increase the stiffness of the extended electrohydraulic system, the pressures have to be increased, see Fig. 9. For this, rather large volume flows are necessary. However, the maximum value can be decreased if the dynamics of the stiffness change is limited. It should be emphasized that the change of the stiffness of the system by means of the change of the pressures is mainly done in order to increase the energetic efficiency of the system. The desired impedance characteristics is already obtained by the controller for the pressure force τ_p which is not influenced by the stiffness controller.

In the last simulation it is shown that the extended electrohydraulic system indeed yields a significant improvement of the energetic efficiency. Therefore, both the basic and the extended electrohydraulic system are excited by a rectangular load force τ_l of amplitude 1 kN and frequency 0.5 Hz. The desired stiffness was chosen to $\sigma_1 = c_1^I$ ($c_3 = 0$) and doubled at time $t = 20$ s. The desired damping is given by $d_1 = d_{11}^I$ with $d_{31} = 0$. Then, the hydraulic energy consumption of both systems is depicted in Fig. 10. As can be seen, the energetic efficiency is considerably improved for the extended electrohydraulic system in comparison to the basic electrohydraulic system for both values of the desired stiffness of the system.

Remark 5. *In [13] it has been shown that the substitution of a laminar damping orifice by a proportional valve can be used to even further increase the energetic efficiency. This is especially the case when large changes of the desired damping behavior are demanded while the stiffness of the system is basically kept constant.*

5. Conclusion

In this paper the impedance control task for electrohydraulic systems was systematically analyzed and a new control strategy was proposed. First, an I&I (immersion and invariance) based impedance controller was developed for a basic electrohydraulic system consisting of a hydraulic cylinder controlled by a 4/3 proportional directional valve. It was shown that the proposed control concept yields a very good and robust performance of the closed-loop system. Nonetheless, two main problems for a practical implementation were identified: (i) the required dynamics of the valve has to be very high and (ii) the system is energetically inefficient. Therefore, an extended electrohydraulic system was designed yielding a significant increase in the energetic efficiency. For this extended system a control concept consisting of an I&I-based impedance controller and a controller for the stiffness of the system was proposed. By means of extensive simulation studies using components and parameters of an industrial system it was

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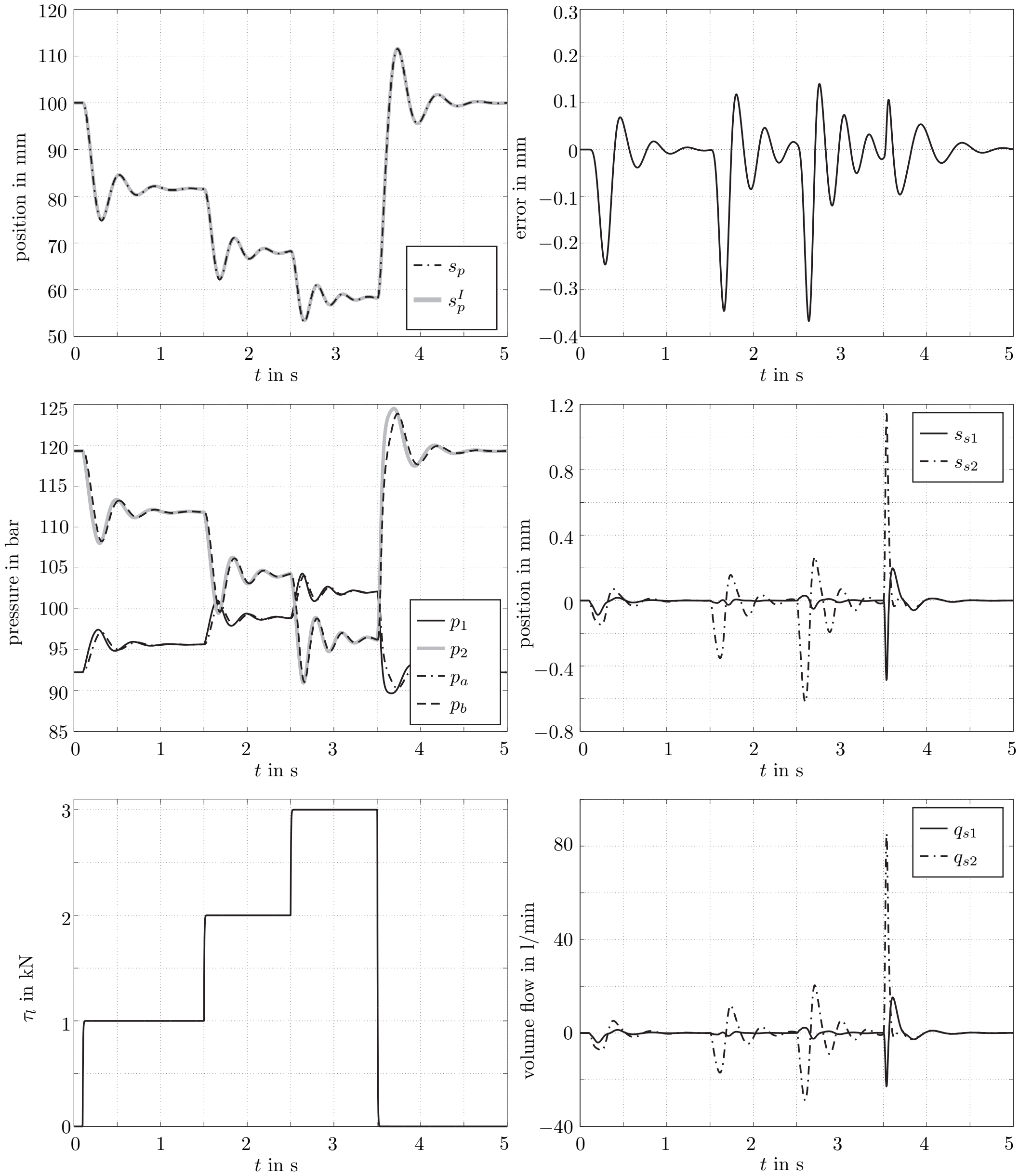

Figure 8. Simulation results of the extended electrohydraulic system in combination with the I&I based impedance control strategy.

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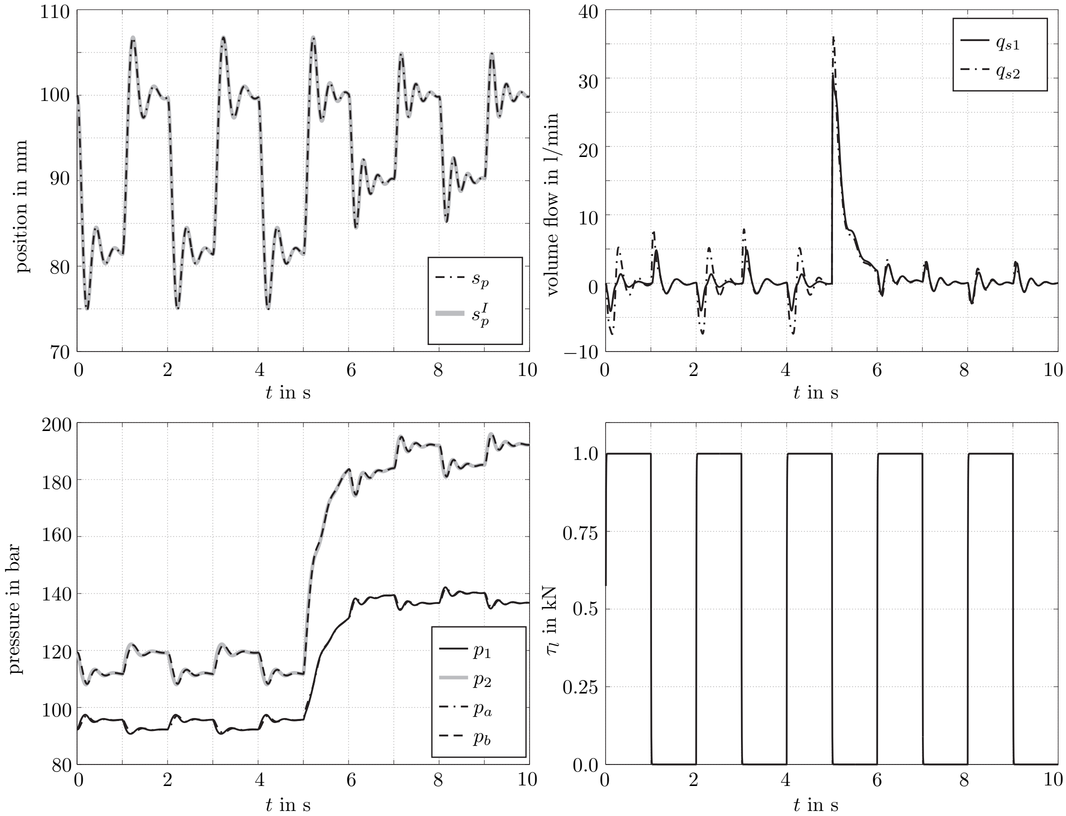

Figure 9. Change of σ_1 from $\sigma_1 = c_1^I$ to $\sigma_1 = 2c_1^I$ at time $t = 5$ s for the extended electrohydraulic system.

shown that a very good agreement between the desired and the realized impedance behavior can be achieved even when using rather slow low-cost valves.

Future work will deal with a generalization of the proposed impedance control strategy for electrohydraulic systems with several degrees-of-freedom.

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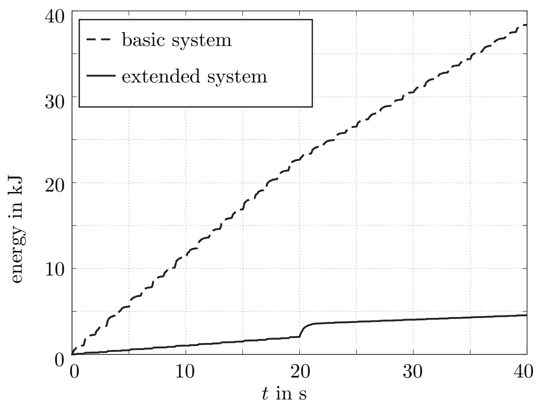

Figure 10. Comparison of the consumed energy of the basic (cf. Fig. 1) and the extended electrohydraulic impedance system (cf. Fig. 7).

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