Attitude Estimation Using Redundant Inertial Measurement Units for the Control of a Camera Stabilization Platform

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Attitude Estimation Using Redundant Inertial Measurement Units for the Control of a Camera Stabilization Platform

F. Königseder, W. Kemmetmüller, and A. Kugi

Abstract— **In this brief, a three-axis gimbaled platform for the active stabilization of film and broadcast cameras is considered. To decouple the attitude of the camera from the motion of the operator, accurate estimation of the platform attitude is necessary. Attitude estimation strategies based on extended Kalman filter (EKF) and unscented Kalman filter (UKF) are developed, which fuse redundant measurements of inertial measurement units (IMUs) mounted at different positions of the platform. This brief extends the state of the art, where typically a single IMU or redundant IMUs placed at the same position are used. The performance of the EKF and UKF is analyzed and compared with an EKF using only a single IMU by measurement results of a prototype platform.**

Index Terms— **Attitude estimation, camera stabilization, sensor fusion.**

I. Introduction

In MANY movie scenes or video clips, the camera is dynamically moved to change the perspective or to make a more engaging film. Typically, camera cranes, steadicams, cable cams, or helicopter cameras are used in the production of such video sequences [1]–[3]. To avoid distracting changes of the line of sight induced by movements of the mobile carrier vehicle, the camera is inertially stabilized. For this task, inertially stabilized platforms (ISPs) with gimbaled assemblies are widely used [4]. Other applications of ISPs are pointing and target tracking devices [5] or control systems for antennas mounted on a movable carrier [6].

In this brief, an ISP utilized in the film and broadcast industry is considered, which uses a gimbal with 3 DOF to stabilize the attitude of a camera. The control tasks of the ISP are the decoupling of the camera from the motion of the carrier and adjusting it to a desired orientation. For this task, estimation of the camera attitude is necessary. In the literature, numerous works dealing with the estimation of the attitude have been published [7]–[13]. In recent contributions, attitude estimation methods use the measurements of a strapdown inertial measurement unit (IMU) that typically integrates gyroscopes, accelerometers, magnetometers, and hydrometers. While in [8]–[10], complementary filters are proposed for attitude estimation, in [11]–[13], Kalman filters are adopted for this problem. An extension to inertial navigation using GPS is described in [14]. The vast majority of the published literature uses the measurements of a single IMU. This brief discusses a configuration with two IMUs, one attached to the base and one to the platform of the ISP. This choice results from the desired attitude control strategy, which combines an indirect approach (feedforward compensation of the motion of the base [15]) with a feedback control of the camera motion. This configuration is characterized by a relative motion of the two IMUs due to the kinematic chain of the ISP, which constitutes a challenge for the design of an attitude estimation strategy fusing the measurements of both IMUs. In [16], a redundant inertial reference unit is introduced that is also equipped with two aligned gyroscopes and accelerometers, which are, however, located at the same place. A discussion of the optimal arrangement of redundant gyroscopes and accelerometers can be found in [17]. In this brief, the fact is exploited that the kinematic coupling of the two IMUs is known, and thus, the redundant measurements can be used to improve the estimation accuracy.

This brief is organized as follows. In Section II, the system is described and a mathematical model is derived. Section III shows the design of an attitude estimation based on an extended Kalman filter (EKF) and an unscented Kalman filter (UKF), respectively. Moreover, a method to reduce the influence of translational accelerations on the attitude estimation is discussed. The estimation accuracy of the proposed EKF and UKF is compared with that of an EKF using only a single IMU by measurement results on a prototype platform in Section IV.

II. System Description

The ISP is composed of four components p_n, $n = 0, \ldots, 3$ (see Fig. 1). The component p_0 refers to the base (handle) of the ISP, which is carried by the operator of the camera in the real application. The gimbals p_1, p_2, and p_3 are connected to each other via three rotational joints (angles φ_{01}, φ_{12}, and φ_{23}), which are actuated by brushless dc motors. The actuated degrees of freedom $\mathbf{q} = [\varphi_{01}, \varphi_{12}, \varphi_{23}]^T$ are measured by high-resolution encoders (SensiTec EBI7903CA-DA-IF, resolution of approximately 2×10^{-3} degree). The camera to be stabilized is mounted on component p_3. As already mentioned, the first IMU0 is placed at the handle p_0 of the platform to directly measure the motion induced by the operator of the platform. The second IMU3 is mounted on component p_3 of the ISP and measures the motion of the camera.

In the experimental setup, the handle p_0 is suspended in an additional gimbal with 2 DOF (angles ψ and ϕ), which are measured by encoders. This suspension enables one to

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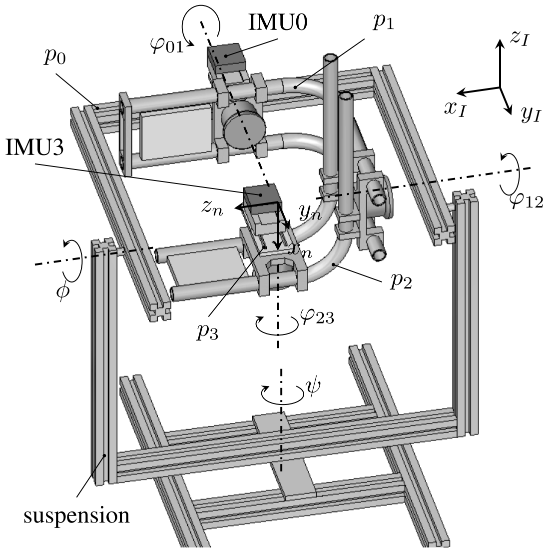

Fig. 1. Sketch of the platform.

apply defined rotations to the handle, which will be used in Section IV to assess the accuracy of the attitude estimation strategies.

A. Platform Kinematics

The orientation of a body rotated by the angle α around the rotation axis $\mathbf{n}$ with respect to a reference system is described using a unit quaternion $\mathbf{r} = [r_0, r_1, r_2, r_3]^T = [\cos(\alpha/2), \mathbf{n}^T \sin(\alpha/2)]^T$, $\|\mathbf{r}\|_2 = 1$ [18], [19]. The orientation $\mathbf{r}_I^0$ of the handle p_0 [body fixed frame $(0x_0y_0z_0)$] with respect to the inertial frame $(Ix_Iy_Iz_I)$ is described by Euler angles $\mathbf{q}_I = [\phi, \theta, \psi]^T$ in the form of three consecutive rotations

$$\mathbf{r}_I^0 = \begin{bmatrix} \cos\left(\frac{\psi}{2}\right) \\ 0 \\ 0 \\ \sin\left(\frac{\psi}{2}\right) \end{bmatrix} \otimes \begin{bmatrix} \cos\left(\frac{\phi}{2}\right) \\ \sin\left(\frac{\phi}{2}\right) \\ 0 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) \\ 0 \\ \sin\left(\frac{\theta}{2}\right) \\ 0 \end{bmatrix} \tag{1}$$

with the quaternion product $\otimes$. The rate of change of the orientation $\mathbf{r}_I^0$ of the handle is given by

$$\dot{\mathbf{r}}_I^0 = \frac{1}{2}\mathbf{r}_I^0 \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega}_0^{I0} \end{bmatrix} \tag{2}$$

where $\boldsymbol{\omega}_0^{I0}$ denotes the angular velocity of the rotation of the handle with respect to the inertial frame (superscript $I0$) expressed in the body fixed frame 0 (subscript 0). With the matrix $\boldsymbol{\Omega}(\boldsymbol{\omega})$

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{3}$$

where $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$, the time derivative of $\mathbf{r}_I^0$ can be rewritten in the form

$$\dot{\mathbf{r}}_I^0 = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega}_0^{I0})\mathbf{r}_I^0. \tag{4}$$

The relative orientation and angular velocities of the body fixed frames of the components p_n, $n = 1, 2, 3$, are described by (see Fig. 1)

$$\mathbf{r}_0^1 = \left[\cos\left(\frac{\varphi_{01}}{2}\right), 0, \sin\left(\frac{\varphi_{01}}{2}\right), 0\right]^T, \quad \boldsymbol{\omega}_0^{01} = [0, \dot{\varphi}_{01}, 0]^T \tag{5a}$$

$$\mathbf{r}_1^2 = \left[\cos\left(\frac{\varphi_{12}}{2}\right), 0, 0, \sin\left(\frac{\varphi_{12}}{2}\right)\right]^T, \quad \boldsymbol{\omega}_1^{12} = [0, 0, \dot{\varphi}_{12}]^T \tag{5b}$$

$$\mathbf{r}_2^3 = \left[\cos\left(\frac{\varphi_{23}}{2}\right), \sin\left(\frac{\varphi_{23}}{2}\right), 0, 0\right]^T, \quad \boldsymbol{\omega}_2^{23} = [\dot{\varphi}_{23}, 0, 0]^T. \tag{5c}$$

Then, the orientation $\mathbf{r}_I^3$ is given in the form

$$\mathbf{r}_I^3 = \mathbf{r}_I^0 \otimes \mathbf{r}_0^1 \otimes \mathbf{r}_1^2 \otimes \mathbf{r}_2^3 = \mathbf{r}_I^0(\mathbf{q}_I) \otimes \mathbf{r}_0^3(\mathbf{q}) \tag{6}$$

with the actuated degrees of freedom $\mathbf{q} = [\varphi_{01}, \varphi_{12}, \varphi_{23}]^T$. The corresponding rate of change reads as $\dot{\mathbf{r}}_I^3 = (1/2)\boldsymbol{\Omega}(\boldsymbol{\omega}_3^{I3})\mathbf{r}_I^3$ with

$$\boldsymbol{\omega}_3^{I3} = \mathbf{R}_3^0\boldsymbol{\omega}_0^{I0} + \mathbf{R}_3^0\boldsymbol{\omega}_0^{01} + \mathbf{R}_3^1\boldsymbol{\omega}_1^{12} + \mathbf{R}_3^2\boldsymbol{\omega}_2^{23}. \tag{7}$$

Therein, $\mathbf{R}_i^j$ describes the rotation matrix from coordinate frame j to i. From (6) and (7), it can be easily seen that the platform p_3 is stabilized if the actuated degrees of freedom $\mathbf{q}$ are assigned in a way that the orientation $\mathbf{r}_I^3$ is kept constant or, equivalently, the angular velocity $\boldsymbol{\omega}_3^{I3}$ vanishes.

The operator of the platform does not only change the orientation $\mathbf{r}_I^0$ of the handle but also its position $\mathbf{p}_I^{I0} = [p_{I,x}^{I0}, p_{I,y}^{I0}, p_{I,z}^{I0}]^T$. The translational accelerations $\mathbf{a}_{t,I}^{I0}$ and $\mathbf{a}_{t,I}^{I3}$ of the handle p_0 and the platform p_3, respectively, measured by IMU0 and IMU3 can be approximated by

$$\mathbf{a}_{t,I}^{I0} \approx \mathbf{a}_{t,I}^{I3} \approx \ddot{\mathbf{p}}_I^{I0} \tag{8}$$

since the contribution of the slow rotational motion of the platform to the translational accelerations is small.

B. Sensor Models

In the considered application, a light weight, small size, and low power consumption of the IMUs is inevitable. IMUs based on microelectromechanical systems (MEMSs) possess these features at a reasonable price and have shown increasing accuracy over the last years. The IMUs used in this brief (ADIS16480 and ADIS16485 [20], [21]) integrate a three-axial gyroscope, a three-axial accelerometer, and a three-axial magnetometer. The magnetometers, however, will not be used in the proposed estimation strategies since the measurements may be corrupted by the harsh electromagnetic environment induced by the power electronics of the motors and the rotating components of the platform.

The measurement $\tilde{\boldsymbol{\omega}}_0^{I0}$ of the gyroscope placed at the handle p_0 is modeled in the form [22]

$$\tilde{\boldsymbol{\omega}}_0^{I0} = \boldsymbol{\omega}_0^{I0} + \mathbf{b}_0 + \mathbf{v}_{\omega 0} \tag{9}$$

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with the true angular rate $\boldsymbol{\omega}_0^{I0}$ expressed in the body fixed frame $(0x_0y_0z_0)$ of p_0. The bias is denoted by $\mathbf{b}_0$ and $\mathbf{v}_{\omega 0}$ is white zero-mean Gaussian noise. The bias of a MEMS gyroscope shows a rate of change due to temperature variations and angular random walk [23]. However, since the slew rate of the bias is rather small, the assumption of a constant bias, i.e., $\dot{\mathbf{b}}_0 = \mathbf{0}$, is feasible for the subsequent considerations.

The measured angular rate $\tilde{\boldsymbol{\omega}}_3^{I3}$ is given in an equivalent form by

$$\tilde{\boldsymbol{\omega}}_3^{I3} = \mathbf{R}_3^0 \boldsymbol{\omega}_0^{I0} + \boldsymbol{\omega}_3^{03} + \mathbf{b}_3 + \mathbf{v}_{\omega 3} \tag{10}$$

where $\mathbf{R}_3^0 \boldsymbol{\omega}_0^{I0} + \boldsymbol{\omega}_3^{03}$ describes the true angular rate expressed in the body fixed frame $(3x_3y_3z_3)$. Again, the bias $\mathbf{b}_3$ is assumed to be constant ($\dot{\mathbf{b}}_3 = \mathbf{0}$) and $\mathbf{v}_{\omega 3}$ is white zero-mean Gaussian noise.

The accelerometer of the IMU placed at the handle p_0 is described by

$$\tilde{\mathbf{a}}_0^{I0} = \mathbf{R}_0^I \left(\mathbf{a}_{t,I}^{I0} - \mathbf{g}_I\right) + \mathbf{v}_{a0} \tag{11}$$

with the measurement $\tilde{\mathbf{a}}_0^{I0}$ expressed in the body fixed frame $(0x_0y_0z_0)$ and the true translational acceleration $\mathbf{a}_{t,I}^{I0}$ expressed in the inertial frame. Here, $\mathbf{g}_I = [0, 0, -9.81]^T$ is the constant gravitational acceleration expressed in the inertial frame and $\mathbf{v}_{a0}$ is white zero-mean Gaussian noise. The measurement $\tilde{\mathbf{a}}_3^{I3}$ of the second IMU can be expressed accordingly in the form

$$\tilde{\mathbf{a}}_3^{I3} = \mathbf{R}_3^0 \mathbf{R}_0^I \left(\mathbf{a}_{t,I}^{I3} - \mathbf{g}_I\right) + \mathbf{v}_{a3} \tag{12}$$

with white zero-mean noise $\mathbf{v}_{a3}$. The bias of the accelerometers is not considered since their influence on the attitude estimation accuracy is rather small.

III. Attitude Estimation Using Redundant IMUs

To reduce the attitude estimation errors caused by the bias and the unknown initial orientation of the gyroscopes, and the translational accelerations and the orientation around the axis of gravity of the accelerometers, a tailored sensor fusion concept will be developed [7], [24], [25]. Different multisensor fusion methods using Kalman filters are described in [26]–[28]. The attitude estimation strategy proposed in this brief is based on a modified Kalman filter using an augmented measurement vector with the data of the redundant IMUs, which performs bias estimation of both gyroscopes. Different from approaches reported in the literature, the ISP uses two IMUs located at different parts of the ISP (IMU0 at the handle p_0 and IMU3 at the platform p_3). This configuration is beneficial for the control of the ISP.

A. System Model

The task of the attitude estimation is to estimate the orientation $\mathbf{r}_I^0$, the bias $\mathbf{b}_0$ of IMU0, and the bias $\mathbf{b}_3$ of IMU3. Using the measurement $\tilde{\boldsymbol{\omega}}_0^{I0}$ of IMU0 (9) in (4) and assuming constant bias, the following system dynamics can be formulated:

$$\frac{\mathrm{d}}{\mathrm{dt}} \underbrace{\begin{bmatrix} \mathbf{r}_I^0 \\ \mathbf{b}_0 \\ \mathbf{b}_3 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \frac{1}{2}\boldsymbol{\Omega}\left(\tilde{\boldsymbol{\omega}}_0^{I0} - \mathbf{b}_0 - \mathbf{v}_{\omega 0}\right)\mathbf{r}_I^0 \\ \mathbf{v}_{b0} \\ \mathbf{v}_{b3} \end{bmatrix}}_{\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{w}_p)}. \tag{13}$$

Therein, $\mathbf{w}_p = [\mathbf{v}_{\omega 0}, \mathbf{v}_{b0}, \mathbf{v}_{b3}]^T$ denotes zero-mean Gaussian process noise due to modeling errors and external disturbances, and $\mathbf{u} = [\tilde{\boldsymbol{\omega}}_0^{I0}, \mathbf{q}, \dot{\mathbf{q}}]^T$ is the known system input.

For the time being, it is assumed that the translational accelerations vanish, i.e., $\mathbf{a}_{t,I}^{I0} = \mathbf{0}$. A discussion on the influence of nonvanishing accelerations is given in Section III-D. With this simplification, the measurement vector $\mathbf{y}$ can be formulated as

$$\underbrace{\begin{bmatrix} \tilde{\mathbf{a}}_0^{I0} \\ \tilde{\mathbf{a}}_3^{I3} \\ \tilde{\boldsymbol{\omega}}_3^{I3} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} -\mathbf{R}_0^I \mathbf{g}_I + \mathbf{v}_{a0} \\ -\mathbf{R}_3^0 \mathbf{R}_0^I \mathbf{g}_I + \mathbf{v}_{a3} \\ \mathbf{R}_3^0\left(\tilde{\boldsymbol{\omega}}_0^{I0} - \mathbf{b}_0 - \mathbf{v}_{\omega 0}\right) + \boldsymbol{\omega}_3^{03} + \mathbf{b}_3 + \mathbf{v}_{\omega 3} \end{bmatrix}}_{\mathbf{h}(\mathbf{x},\mathbf{u},\mathbf{w}_p,\mathbf{w}_m)} \tag{14}$$

where $\mathbf{w}_m = [\mathbf{v}_{a0}, \mathbf{v}_{a3}, \mathbf{v}_{\omega 3}]^T$ describes the zero-mean Gaussian measurement noise.

B. Extended Kalman Filter Design

In this brief, the ideas of a multiplicative EKF (MEKF) typically employed for the attitude estimation based on a single IMU [11]–[13] are extended for the proposed redundant IMU setup. The main idea of the MEKF is to split the true attitude $\mathbf{r}_0^I$ into the estimated attitude $\hat{\mathbf{r}}_I^0$ and the deviation $\delta\mathbf{r}$, $\mathbf{r}_I^0 = \hat{\mathbf{r}}_I^0 \otimes \delta\mathbf{r}$. Analogously, the true bias $\mathbf{b}_n$ is formulated as the sum of the estimated bias $\hat{\mathbf{b}}_n$ and the deviation $\delta\mathbf{b}_n$, $\mathbf{b}_n = \hat{\mathbf{b}}_n + \delta\mathbf{b}_n$, $n = 0, 3$. In the MEKF, the deviations $\delta\mathbf{r}$ and $\delta\mathbf{b}_n$ are estimated and the real values $\mathbf{r}_I^0$ and $\mathbf{b}_n$ are calculated by the previous equations. This approach ensures a correctly normalized quaternion $\mathbf{r}_I^0$.

The time derivative $\dot{\mathbf{r}}_I^0 = \dot{\hat{\mathbf{r}}}_I^0 \otimes \delta\mathbf{r} + \hat{\mathbf{r}}_I^0 \otimes \delta\dot{\mathbf{r}}$ can be rewritten with (2) in the form

$$\frac{1}{2}\mathbf{r}_I^0 \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega}_0^{I0} \end{bmatrix} = \frac{1}{2}\hat{\mathbf{r}}_I^0 \otimes \begin{bmatrix} 0 \\ \hat{\boldsymbol{\omega}}_0^{I0} \end{bmatrix} \otimes \delta\mathbf{r} + \hat{\mathbf{r}}_I^0 \otimes \delta\dot{\mathbf{r}} \tag{15}$$

with the estimated angular velocity $\hat{\boldsymbol{\omega}}_0^{I0} = \tilde{\boldsymbol{\omega}}_0^{I0} - \hat{\mathbf{b}}_0$. Rearranging (15), left multiplying by the inverse of $\hat{\mathbf{r}}_I^0$, and utilizing (9) result in

$$\delta\dot{\mathbf{r}} = \frac{1}{2}\left(\delta\mathbf{r} \otimes \begin{bmatrix} 0 \\ \hat{\boldsymbol{\omega}}_0^{I0} - \delta\mathbf{b}_0 - \mathbf{v}_{\omega 0} \end{bmatrix} - \begin{bmatrix} 0 \\ \hat{\boldsymbol{\omega}}_0^{I0} \end{bmatrix} \otimes \delta\mathbf{r}\right). \tag{16}$$

It can be assumed that the deviation $\delta\mathbf{r}$ is small such that it can be approximated by (see [13])

$$\delta\mathbf{r} \approx \begin{bmatrix} 1 \\ \frac{\boldsymbol{\alpha}}{2} \end{bmatrix}, \quad \|\boldsymbol{\alpha}\|_2 \ll 1. \tag{17}$$

Applying this approximation to (16), a reduced dynamic model

$$\dot{\boldsymbol{\alpha}} = \underbrace{\begin{bmatrix} 0 & \hat{\omega}_{0,3}^{I0} & -\hat{\omega}_{0,2}^{I0} \\ -\hat{\omega}_{0,3}^{I0} & 0 & \hat{\omega}_{0,1}^{I0} \\ \hat{\omega}_{0,2}^{I0} & -\hat{\omega}_{0,1}^{I0} & 0 \end{bmatrix}}_{\breve{\boldsymbol{\Omega}}\left(\hat{\boldsymbol{\omega}}_0^{I0}\right)} \boldsymbol{\alpha} - \delta\mathbf{b}_0 - \mathbf{v}_{\omega 0} \tag{18}$$

can be formulated. The system dynamics, which forms the basis of the MEKF, is finally obtained by adding the dynamics

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of the bias deviation $\delta\dot{\mathbf{b}}_n = \mathbf{v}_{bn}$

$$\underbrace{\begin{bmatrix}\dot{\boldsymbol{\alpha}}\\ \delta\dot{\mathbf{b}}_0\\ \delta\dot{\mathbf{b}}_3\end{bmatrix}}_{\dot{\boldsymbol{\xi}}} = \underbrace{\begin{bmatrix}\breve{\boldsymbol{\Omega}}(\hat{\boldsymbol{\omega}}_0^{I0}) & -\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}}_{\mathbf{A}(t)}\underbrace{\begin{bmatrix}\boldsymbol{\alpha}\\ \delta\mathbf{b}_0\\ \delta\mathbf{b}_3\end{bmatrix}}_{\boldsymbol{\xi}} + \underbrace{\begin{bmatrix}-\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}}_{\mathbf{G}}\underbrace{\begin{bmatrix}\mathbf{v}_{\omega 0}\\ \mathbf{v}_{b0}\\ \mathbf{v}_{b3}\end{bmatrix}}_{\mathbf{w}_p} \tag{19}$$

where $\mathbf{I}$ denotes the identity matrix of proper dimension. The corresponding output $\mathbf{y} = \breve{\mathbf{h}}(\boldsymbol{\xi}, \breve{\mathbf{u}}, \mathbf{w}_p, \mathbf{w}_m)$ of the system can be reformulated as

$$\mathbf{y} = \begin{bmatrix}-(\hat{\mathbf{R}}_I^0\delta\mathbf{R}_I^0)^T\mathbf{g}_I\\ -\mathbf{R}_3^0(\hat{\mathbf{R}}_I^0\delta\mathbf{R}_I^0)^T\mathbf{g}_I\\ \mathbf{R}_3^0(\tilde{\boldsymbol{\omega}}_0^{I0} - \hat{\mathbf{b}}_0 - \delta\mathbf{b}_0) + \boldsymbol{\omega}_3^{03} + \hat{\mathbf{b}}_3 + \delta\mathbf{b}_3\end{bmatrix} + \begin{bmatrix}\mathbf{v}_{a0}\\ \mathbf{v}_{a3}\\ -\mathbf{R}_3^0\mathbf{v}_{\omega 0} + \mathbf{v}_{\omega 3}\end{bmatrix}. \tag{20}$$

The augmented input vector $\breve{\mathbf{u}}^T = [\mathbf{u}^T, (\hat{\mathbf{r}}_I^0)^T, \hat{\mathbf{b}}_0^T, \hat{\mathbf{b}}_3^T]$ has been introduced to account for the known estimations.

In the MEKF, the model is used to characterize the deviations only over one sampling interval $kT_s \le t < (k+1)T_s$, with the sampling time T_s. It is reasonable to assume a constant angular velocity $\hat{\boldsymbol{\omega}}_{0,k}^{I0} = \tilde{\boldsymbol{\omega}}_{0,k}^{I0} - \hat{\mathbf{b}}_{0,k}$ over one sampling interval. Then, the system dynamics (19) can be formulated as the time-discrete system

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\Phi}_k\boldsymbol{\xi}_k + \mathbf{G}_k\mathbf{w}_{p,k} \tag{21}$$

with $\boldsymbol{\Phi}_k = \exp(\mathbf{A}(kT_s)T_s)$ and $\mathbf{G}_k = \int_0^{T_s}\exp(\mathbf{A}(kT_s)\tau)\mathrm{d}\tau\mathbf{G}$. The time-discrete output reads as $\mathbf{y}_k = \breve{\mathbf{h}}(\boldsymbol{\xi}_k, \breve{\mathbf{u}}_k, \mathbf{w}_{p,k}, \mathbf{w}_{m,k})$. It is further assumed that the measurement and process noise are zero-mean Gaussian noise, which fulfill $\mathrm{E}(\mathbf{w}_{m,i}\mathbf{w}_{m,j}^T) = \mathbf{Q}_m\delta_{ij} > 0$, $\mathrm{E}(\mathbf{w}_{p,i}\mathbf{w}_{p,j}^T) = \mathbf{Q}_p\delta_{ij} > 0$, and $\mathrm{E}(\mathbf{w}_{m,i}\mathbf{w}_{p,j}^T) = \mathbf{0}$, with $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ else.

With these prerequisites, one iteration of the MEKF proceeds as follows. In the first step, the measurement update of the estimated state $\hat{\boldsymbol{\xi}}_k^+ = [\hat{\boldsymbol{\alpha}}_k^+, \delta\mathbf{b}_{0,k}^+, \delta\mathbf{b}_{3,k}^+]^T$ and the covariance matrix $\mathbf{P}_k^+$ is calculated by

$$\hat{\boldsymbol{\xi}}_k^+ = \mathbf{K}_k(\tilde{\mathbf{y}}_k - \hat{\mathbf{y}}_k^-) \tag{22a}$$

$$\mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k\mathbf{C}_k\mathbf{P}_k^- \tag{22b}$$

with the measured output $\tilde{\mathbf{y}}_k$, the estimated output $\hat{\mathbf{y}}_k^- = \breve{\mathbf{h}}(\hat{\boldsymbol{\xi}}_k^-, \breve{\mathbf{u}}_k, \mathbf{0}, \mathbf{0})$, the matrices $\mathbf{C}_k = ((\partial/\partial\boldsymbol{\xi}_k)\breve{\mathbf{h}})(\hat{\boldsymbol{\xi}}_k^-, \breve{\mathbf{u}}_k, \mathbf{0}, \mathbf{0})$ and $\mathbf{H}_k = ((\partial/\partial\mathbf{w}_{p,k})\breve{\mathbf{h}})(\hat{\boldsymbol{\xi}}_k^-, \breve{\mathbf{u}}_k, \mathbf{0}, \mathbf{0})$, and the Kalman gain

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{C}_k^T(\mathbf{C}_k\mathbf{P}_k^-\mathbf{C}_k^T + \mathbf{H}_k\mathbf{Q}_p\mathbf{H}_k^T + \mathbf{Q}_m)^{-1}. \tag{23}$$

The *a priori* estimation of the state is zero in the measurement update step, i.e., $\hat{\boldsymbol{\xi}}_k^- = \mathbf{0}$.

In the second step, the attitude and bias are updated in the form

$$\hat{\mathbf{r}}_{I,k}^{0+} = \hat{\mathbf{r}}_{I,k}^{0-} \otimes \begin{bmatrix}1\\ \dfrac{\hat{\boldsymbol{\alpha}}_k^+}{2}\end{bmatrix} \tag{24a}$$

$$\hat{\mathbf{b}}_{0,k}^+ = \hat{\mathbf{b}}_{0,k}^- + \delta\mathbf{b}_{0,k}^+ \tag{24b}$$

$$\hat{\mathbf{b}}_{3,k}^+ = \hat{\mathbf{b}}_{3,k}^- + \delta\mathbf{b}_{3,k}^+. \tag{24c}$$

The time propagation of the state vector and the covariance matrix of the estimation error are given by

$$\hat{\boldsymbol{\xi}}_{k+1}^- = \mathbf{0} \tag{25a}$$

$$\mathbf{P}_{k+1}^- = \boldsymbol{\Phi}_k\mathbf{P}_k^+\boldsymbol{\Phi}_k^T + \mathbf{G}_k\mathbf{Q}_p\mathbf{G}_k^T \tag{25b}$$

with the initial conditions $\hat{\boldsymbol{\xi}}_0^- = \mathbf{0}$ and $\mathbf{P}_0^- > \mathbf{0}$. The reset $\hat{\boldsymbol{\xi}}_{k+1}^- = \mathbf{0}$ is necessary since the update information has been transferred to the estimated attitude and bias in (24a) [7], [13]. In the final step, the propagated values of the attitude and the bias are obtained from (13) as

$$\hat{\mathbf{r}}_{I,k+1}^{0-} = \exp\left(\frac{1}{2}T_s\boldsymbol{\Omega}(\tilde{\boldsymbol{\omega}}_{0,k}^{I0} - \hat{\mathbf{b}}_{0,k}^+)\right)\hat{\mathbf{r}}_{I,k}^{0+} \tag{26a}$$

$$\hat{\mathbf{b}}_{0,k+1}^- = \hat{\mathbf{b}}_{0,k}^+ \tag{26b}$$

$$\hat{\mathbf{b}}_{3,k+1}^- = \hat{\mathbf{b}}_{3,k}^+. \tag{26c}$$

C. Unscented Kalman Filter Design

A drawback of the EKF designed in the last section is that it relies on a linearization of the nonlinear system dynamics, which results in errors [7], [29]. To circumvent the linearization, unscented Kalman filtering has been proposed in the literature (see [30], [31]) and adapted to the attitude estimation task [29], [32]. In this section, these results are extended to the considered system with two redundant IMUs.

The first step of the UKF design is to calculate a time-discrete model from the time-continuous system (13), (14). Typically, this cannot be done analytically. Thus, in this brief, $\boldsymbol{\omega}_0^{I0} = \text{const.}$ is assumed within one sampling interval $kT_s \le t \le (k+1)T_s$. With this assumption, the following time-discrete model can be obtained from (13):

$$\mathbf{r}_{I,k+1}^0 = \boldsymbol{\Xi}(\tilde{\boldsymbol{\omega}}_{0,k}^{I0} - \mathbf{b}_{0,k} - \mathbf{v}_{\omega 0,k})\mathbf{r}_{I,k}^0 \tag{27a}$$

$$\mathbf{b}_{0,k+1} = \mathbf{b}_{0,k} + \mathbf{v}_{b0,k} \tag{27b}$$

$$\mathbf{b}_{3,k+1} = \mathbf{b}_{3,k} + \mathbf{v}_{b3,k} \tag{27c}$$

with

$$\boldsymbol{\Xi}(\boldsymbol{\omega}) = \begin{bmatrix}\mathrm{cs} & -\omega_1\mathrm{si} & -\omega_2\mathrm{si} & -\omega_3\mathrm{si}\\ \omega_1\mathrm{si} & \mathrm{cs} & \omega_3\mathrm{si} & -\omega_2\mathrm{si}\\ \omega_2\mathrm{si} & -\omega_3\mathrm{si} & \mathrm{cs} & \omega_1\mathrm{si}\\ \omega_3\mathrm{si} & \omega_2\mathrm{si} & -\omega_1\mathrm{si} & -\mathrm{cs}\end{bmatrix} \tag{28}$$

where the abbreviations $\mathrm{cs} = \cos((1/2)\|\boldsymbol{\omega}\|_2 T_s)$ and $\mathrm{si} = \sin((1/2)\|\boldsymbol{\omega}\|_2 T_s)/\|\boldsymbol{\omega}\|_2$ are used. The output $\mathbf{y}_k$ is given by (14) in the form $\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_{p,k}, \mathbf{w}_{m,k})$.

Analogously to the MEKF, the real attitude is split into an estimated part $\hat{\mathbf{r}}_{I,k}^0$ and an error quaternion $\delta\mathbf{r}_k$ [29]. Different from the EKF, the error quaternion $\delta\mathbf{r}_k$ does not have to be

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approximated but can be exactly parameterized by $\boldsymbol{\rho}_k$ in the form [29]

$$\begin{bmatrix}\delta r_{1,k}\\ \delta r_{2,k}\\ \delta r_{3,k}\end{bmatrix} = \frac{\delta r_{0,k}}{2}\boldsymbol{\rho}_k, \quad \delta r_{0,k} = \frac{2}{\sqrt{4+\rho_{1,k}^2+\rho_{2,k}^2+\rho_{3,k}^2}}. \tag{29}$$

With these preliminary considerations, one iteration of the UKF can be formulated in the following form [29]–[31].[1] In the first step, the augmented state of the system is defined in the form $\mathbf{x}_k^a = [\boldsymbol{\xi}_k^T, \mathbf{w}_{p,k}^T, \mathbf{w}_{m,k}^T]^T$ with $\boldsymbol{\xi}_k = [\boldsymbol{\rho}_k^T, \mathbf{b}_{0,k}^T, \mathbf{b}_{3,k}^T]^T$. The estimated (also expected) value $\hat{\mathbf{x}}_k^{a+}$ and its covariance $\mathbf{P}_k^{a+}$ are given by

$$\hat{\mathbf{x}}_k^{a+} = \mathrm{E}(\mathbf{x}_k^{a+}) = \left[\mathbf{0}, (\hat{\mathbf{b}}_{0,k}^+)^T, (\hat{\mathbf{b}}_{3,k}^+)^T, \mathbf{0}, \mathbf{0}\right]^T \tag{30a}$$

$$\mathbf{P}_k^{a+} = \mathrm{E}\left((\mathbf{x}_k^a - \hat{\mathbf{x}}_k^{a+})(\mathbf{x}_k^a - \hat{\mathbf{x}}_k^{a+})^T\right) = \begin{bmatrix}\mathbf{P}_k^+ & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{Q}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_m\end{bmatrix} \tag{30b}$$

where the estimation $\hat{\boldsymbol{\rho}}_k^+$ at time k is zero due to the reset after each iteration, see (25) and the subsequent discussion. The set $\boldsymbol{\mathcal{X}}_k^a$ of sigma points of the extended system is defined by [31]

$$\boldsymbol{\mathcal{X}}_k^a = \left\{\begin{bmatrix}\hat{\boldsymbol{\xi}}_k^+\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\hat{\boldsymbol{\xi}}_k^+\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \pm \boldsymbol{\sigma}_1, \ldots, \begin{bmatrix}\hat{\boldsymbol{\xi}}_k^+\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \pm \boldsymbol{\sigma}_N\right\} \tag{31}$$

with $[\boldsymbol{\sigma}_1, \ldots, \boldsymbol{\sigma}_N] = (N+\lambda)^{1/2}(\mathbf{P}_k^{a+})^{1/2}$, where $N = 27$ is the length of $\hat{\mathbf{x}}_k^a$ and λ is a constant scaling factor. The first three entries of $\boldsymbol{\mathcal{X}}_{k,i}^a$, $i = 0, \ldots, 2N$, correspond to the sigma points in $\hat{\boldsymbol{\rho}}_k^+$. Using (29), the $2N+1$ sigma points $\delta\hat{\mathbf{r}}_{k,i}^+$ for the attitude error can be calculated. In the next step, the sigma points are propagated through the system dynamics (27) in the form

$$\delta\hat{\mathbf{r}}_{k+1,i}^- = \left(\hat{\mathbf{r}}_{I,k}^{0+}\right)^{-1} \otimes \boldsymbol{\Xi}\left(\tilde{\boldsymbol{\omega}}_{0,k}^{I0} - \hat{\mathbf{b}}_{0,k,i}^+ - \hat{\mathbf{v}}_{\omega 0,k,i}\right)\hat{\mathbf{r}}_{I,k}^{0+} \otimes \delta\hat{\mathbf{r}}_{k,i}^+ \tag{32a}$$

$$\hat{\mathbf{b}}_{0,k+1,i}^- = \hat{\mathbf{b}}_{0,k,i}^+ + \hat{\mathbf{v}}_{b0,k,i} \tag{32b}$$

$$\hat{\mathbf{b}}_{3,k+1,i}^- = \hat{\mathbf{b}}_{3,k,i}^+ + \hat{\mathbf{v}}_{b3,k,i} \tag{32c}$$

where the corresponding entries of the sigma points $\boldsymbol{\mathcal{X}}_{k,i}^a$ are used for $\hat{\mathbf{b}}_{0,k,i}^+$, $\hat{\mathbf{b}}_{3,k,i}^+$, $\hat{\mathbf{v}}_{\omega 0,k,i}$, $\hat{\mathbf{v}}_{b0,k,i}$, and $\hat{\mathbf{v}}_{b3,k,i}$. Using the propagated sigma points in the output equation results in

$$\hat{\mathbf{y}}_{k+1,i}^- = \mathbf{h}\left(\hat{\mathbf{x}}_{k+1}^-, \mathbf{u}_{k+1}, \hat{\mathbf{w}}_{p,k,i}, \hat{\mathbf{w}}_{m,k,i}\right) \tag{33}$$

with $(\hat{\mathbf{x}}_{k+1}^-)^T = [(\hat{\mathbf{r}}_{I,k}^{0+} \otimes \delta\hat{\mathbf{r}}_{k+1,i}^-)^T, (\hat{\mathbf{b}}_{0,k+1,i}^-)^T, (\hat{\mathbf{b}}_{3,k+1,i}^-)^T]$, and the corresponding entries $\hat{\mathbf{w}}_{p,k,i}$ and $\hat{\mathbf{w}}_{m,k,i}$ of the sigma points.

The *a priori* estimations $\hat{\boldsymbol{\xi}}_{k+1}^-$, $\mathbf{P}_{k+1}^-$, and $\hat{\mathbf{y}}_{k+1}^-$ of the state, the covariance matrix, and the output, respectively, are given by the weighted sums

[1] Crassidis and Markley [29] assume that the process noise enters the system dynamics in a linear manner. Since the UKF can deal with the general nonlinear case, this approximation is not necessary and thus skipped in the following.

$$\hat{\boldsymbol{\xi}}_{k+1}^- = \sum_{i=0}^{2N} w_i \hat{\boldsymbol{\xi}}_{k+1,i}^-, \quad \hat{\mathbf{y}}_{k+1}^- = \sum_{i=0}^{2N} w_i \hat{\mathbf{y}}_{k+1,i}^- \tag{34a}$$

$$\mathbf{P}_{k+1}^- = \sum_{i=0}^{2N} w_i \left(\hat{\boldsymbol{\xi}}_{k+1,i}^- - \hat{\boldsymbol{\xi}}_{k+1}^-\right)\left(\hat{\boldsymbol{\xi}}_{k+1,i}^- - \hat{\boldsymbol{\xi}}_{k+1}^-\right)^T \tag{34b}$$

where the first three entries of $\hat{\boldsymbol{\xi}}_{k+1,i}^-$ are calculated by applying the transformation (29) to $\delta\hat{\mathbf{r}}_{k+1,i}^-$ and the weights are defined by $w_0 = \lambda/(N+\lambda)$ and $w_i = 1/(2(N+\lambda))$, $i = 1, \ldots, 2N$. The *a posteriori* estimations result from

$$\hat{\boldsymbol{\xi}}_{k+1}^+ = \hat{\boldsymbol{\xi}}_{k+1}^- + \mathbf{K}_k\left(\tilde{\mathbf{y}}_{k+1} - \hat{\mathbf{y}}_{k+1}^-\right) \tag{35a}$$

$$\mathbf{P}_{k+1}^+ = \mathbf{P}_k^- - \mathbf{K}_k \mathbf{P}_{yy} \mathbf{K}_k^T \tag{35b}$$

with the Kalman gain matrix $\mathbf{K}_k = \mathbf{P}_{\xi y}\mathbf{P}_{yy}^{-1}$ and

$$\mathbf{P}_{\xi y} = \sum_{i=0}^{2N} w_i \left(\hat{\boldsymbol{\xi}}_{k+1,i}^- - \hat{\boldsymbol{\xi}}_{k+1}^-\right)\left(\hat{\mathbf{y}}_{k+1,i}^- - \hat{\mathbf{y}}_{k+1}^-\right)^T \tag{36a}$$

$$\mathbf{P}_{yy} = \sum_{i=0}^{2N} w_i \left(\hat{\mathbf{y}}_{k+1,i}^- - \hat{\mathbf{y}}_{k+1}^-\right)\left(\hat{\mathbf{y}}_{k+1,i}^- - \hat{\mathbf{y}}_{k+1}^-\right)^T. \tag{36b}$$

In the final step, the estimated attitude at time $k+1$ is calculated by $\hat{\mathbf{r}}_{I,k+1}^{0+} = \hat{\mathbf{r}}_{I,k}^{0+} \otimes \delta\hat{\mathbf{r}}_{k+1}^+$ and the error quaternion, i.e., the corresponding parametrization $\hat{\boldsymbol{\rho}}_{k+1}^+$, is reset, $\hat{\boldsymbol{\rho}}_{k+1}^+ = \mathbf{0}$.

D. Presence of Translational Acceleration

Translational accelerations of the platform yield estimation errors, since both the EKF and the UKF have been designed assuming $\|\mathbf{a}_{I,I}^{I0}\|_2 = 0$. To reduce the influence of translational accelerations, the covariance matrix is adjusted in the following form [33], [34]: given the original covariance matrix $\mathbf{Q}_m = \mathrm{diag}(\mathbf{Q}_{a0}, \mathbf{Q}_{a3}, \mathbf{Q}_{\omega 3})$ of the measurement, the modified covariance matrix $\mathbf{Q}_m' = \mathrm{diag}(\mathbf{Q}_{a0}', \mathbf{Q}_{a3}', \mathbf{Q}_{\omega 3})$ is introduced, with

$$\mathbf{Q}_{ai}' = \mathbf{Q}_{ai}\exp\left(\gamma\left|\|\mathbf{g}_I\| - \|\tilde{\mathbf{a}}_0^{I0}\|\right|\right), \quad i = 0, 3. \tag{37}$$

With this measure, the covariances of the acceleration measurements are increased for nonvanishing translational accelerations. The parameter $\gamma > 0$ is a scaling factor. This approach brings along that the corresponding entries of the Kalman gain matrix $\mathbf{K}_k$ are reduced in the case of translational accelerations such that the influence of the erroneous measurements of the accelerometers is decreased.

IV. Measurements

The accuracy of the EKF and the UKF, as designed in Section III, is evaluated using measurements of the prototype system shown in Fig. 1. A dSPACE control unit running at a sampling time of 1 ms has been used to evaluate the sensor measurements and to control the motors. Two IMUs of Analog Devices Inc. were utilized (IMU0 ADIS16480 and IMU3 ADIS16485 [20], [21]).

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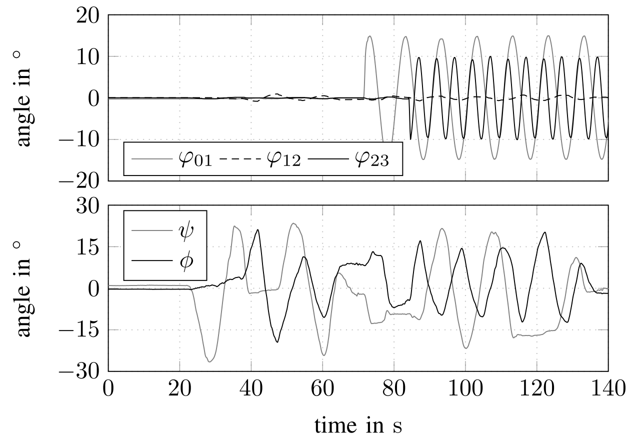

Fig. 2. Experiment 1: encoder measurements of the controlled degrees of freedom (top) and angles of the additional gimbal (bottom).

A. Experiment 1: No Translational Accelerations

In the first experiment, the estimation accuracy is evaluated for zero translational accelerations. Rotations of the handle p_0 are forced by the operator, using the additional gimbal of the experimental setup with the measured angles ψ and ϕ (see Fig. 1). Moreover, a relative rotation between the two IMUs of the handle p_0 and the platform p_3 is introduced by changing the actuated angles φ_{01}, φ_{12}, and φ_{23}. The measurements of the angles thus allow one to directly calculate the actual orientation of the handle p_0, which is used as a reference for the estimations. Fig. 2 shows the measurements of the actuated angles (top) and the angles of the additional gimbal (bottom). It can be seen that a sinusoidal combined rotation between p_0 and p_3, which accounts for a typical motion of the platform, is applied. The angle θ is fixed to $\theta = 90°$ for the whole time of the experiment (1).

The measurements of the three-axial gyroscope and three-axial accelerometer of IMU0 and IMU3 are presented in Fig. 3. Without relative rotation of the platform p_3 with respect to the handle p_0, the measurements of the two IMUs are identical. As it is expected, IMU3 shows a significantly different behavior if a relative rotation is induced in the platform.

In Fig. 4, the attitude estimation errors of the proposed EKF and UKF are compared with those of an EKF using the measurements of IMU0 only. The results of the single IMU EKF serve as a benchmark for comparison reasons. Instead of directly comparing the components of the estimated quaternions, the parameterizing angles ϕ, θ, and ψ are used. Basically, Fig. 4 shows that both estimation strategies proposed in Section III (EKF and UKF) exhibit a better accuracy compared with the single IMU EKF. A drift of the angle $\hat{\psi}$ can be seen for all estimation strategies. This drift can be attributed to the lack of being able to correct errors in this axis by means of the acceleration sensors. Nonetheless, the EKF and UKF perform better in this axis, which is mainly due to the redundant measurements of two IMUs.

A comparison of the results of the EKF and UKF reveals that their performance is pretty much the same. Thus, it is

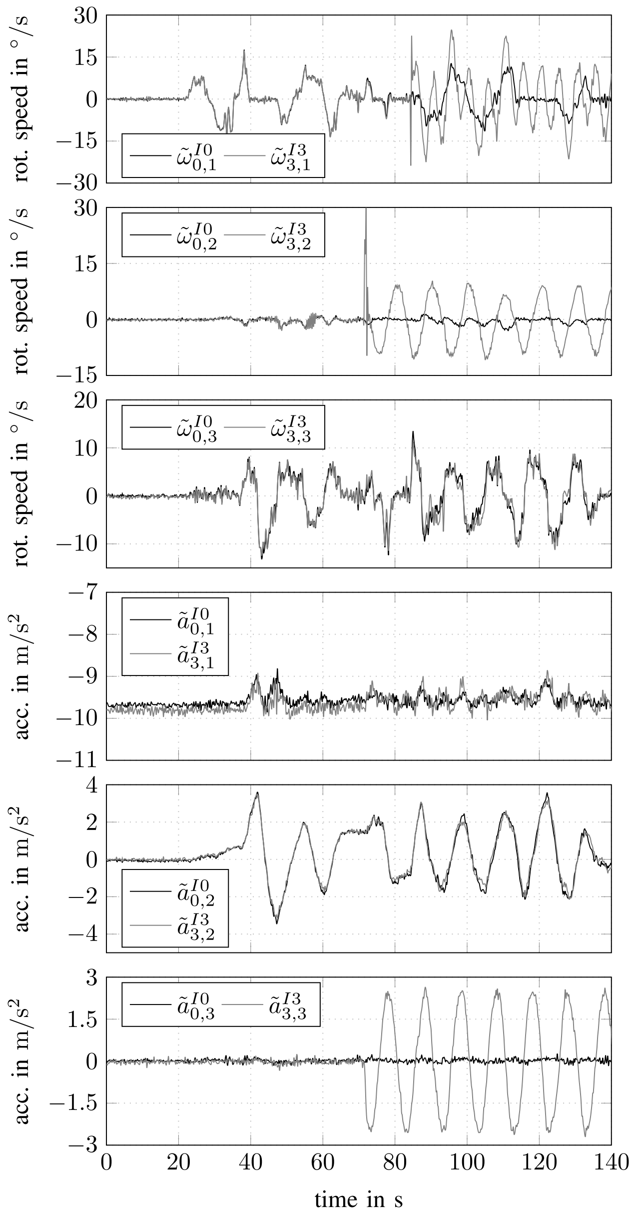

Fig. 3. Experiment 1: measurements of the gyroscopes and accelerometers.

reasonable to assume that the linearization inherently used in the EKF has only a negligible influence on the attitude estimation accuracy in this experiment.

B. Experiment 2: Influence of Translational Accelerations

The adaptation (37) of the covariance matrix $\mathbf{Q}_m$ was proposed in Section III-D to reduce the effect of nonvanishing translational accelerations $\mathbf{a}_{t,n}^{In} = \mathbf{0}$, $n = 0, 3$. In this second experiment, translational accelerations of the handle p_0 are introduced by the operator while the joints of the additional gimbal and the actuated joints are mechanically fixed. The resulting measurements of the translational acceleration $\tilde{\mathbf{a}}_0^{I0}$

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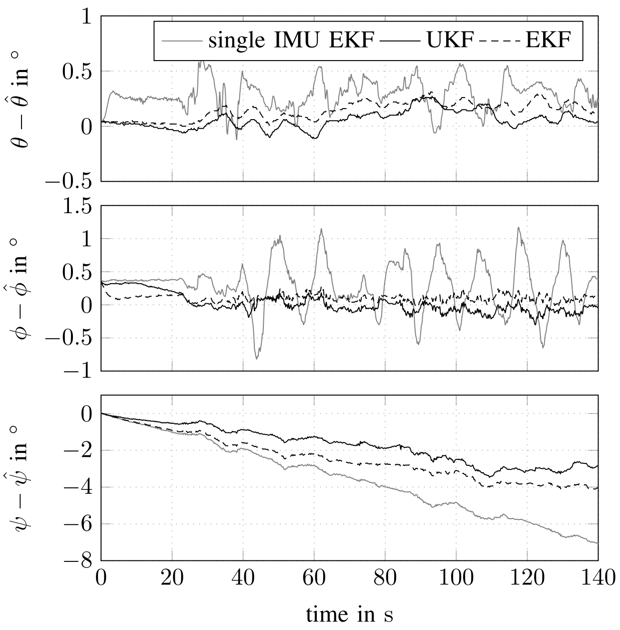

Fig. 4. Experiment 1: attitude estimation errors of the EKF, UKF, and the single IMU EKF.

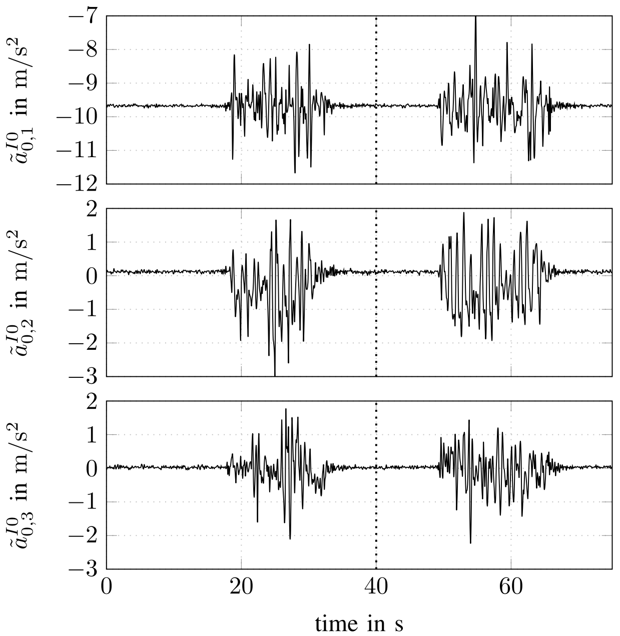

Fig. 5. Experiment 2: measurements of the accelerometer of IMU0.

of IMU0 are shown in Fig. 5. It can be seen that large accelerations occur in all three directions.

A comparison with the measured attitude of the handle p_0 is not possible in this experiment. Instead, the attitude is calculated by integration of the measured angular velocities $\tilde{\boldsymbol{\omega}}_0^{I0}$, where the average bias during the measurement campaign was obtained offline. Of course, the influence of the bias on the estimation accuracy cannot be shown by this, but it is useful to evaluate the effect of translational accelerations $\tilde{\mathbf{a}}_{t,0}^{I0}$ on the attitude estimation.

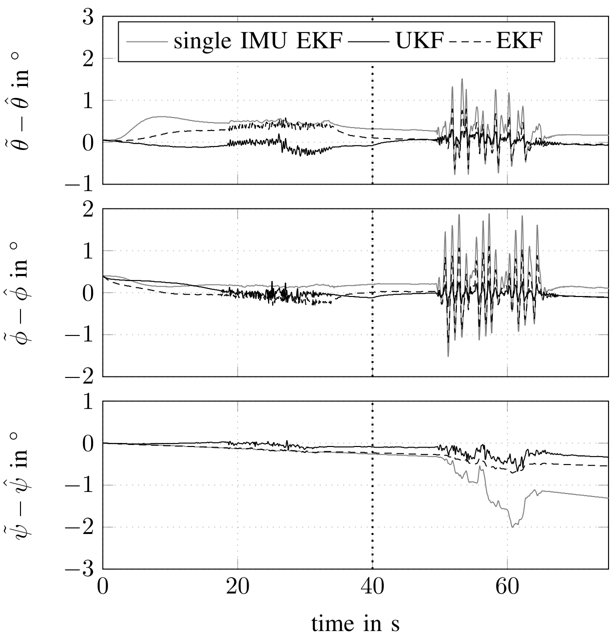

Fig. 6. Experiment 2: attitude estimation errors of the EKF, UKF, and the single IMU EKF in the case of translational accelerations.

Similar to experiment 1, the parameterizing angles θ, ψ, and ϕ of the quaternion $\tilde{\mathbf{r}}_I^0$ are used to evaluate the performance. To demonstrate the effect of the adapted covariance matrix $\mathbf{Q}_m$, a comparison without adaptation is also given. Fig. 6 shows the attitude errors of the EKF and UKF in comparison with those of an EKF using the measurements of IMU0 only. In the first 40 s, the adaptation of the covariance matrix $\mathbf{Q}_m$ is active. It can be seen that in this case, the EKF, the UKF and the single IMU EKF yield rather good results. For deactivated adaptation (after 40 s), a degradation of the estimation accuracy can be seen. This proves that the proposed adaptation algorithm is useful and significantly improves the estimation accuracy in the case of translational accelerations. Without the adaptation of $\mathbf{Q}_m$, the single IMU EKF produces the worst results and the UKF has the best performance.

V. Conclusion

In this brief, attitude estimation strategies of an ISP for film and broadcast cameras were developed, where the measurements of two IMUs mounted at different locations of the platform are fused by the proposed attitude estimation strategy. An EKF and an UKF were applied for the attitude estimation and an adaptation of the covariance matrix was proposed to reduce the influence of translational accelerations. The feasibility and accuracy of the EKF and UKF were shown by measurements of a prototype platform. As a result, the UKF and the EKF have nearly the same performance, which demonstrates that effects of the linearization used in the EKF do not play a major role. In comparison with attitude estimation strategies (EKF) using a single IMU, the proposed attitude estimation strategy achieves an improvement in accuracy. In a control strategy for the ISP, the results of the presented attitude estimation are utilized for feedforward disturbance rejection,

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whereas the measurements of the IMU fixed to the camera are used for feedback control.

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