Flatness-Based Feedforward Control of a Two-Stage Turbocharged Diesel Air System with EGR

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***Abstract*— In this work, a nonlinear inversion-based feedforward controller for a two-stage turbocharged diesel air system with exhaust-gas recirculation is developed. A nonlinear mathematical model is derived for the controller design on the basis of a high-order reference model by applying the singular perturbation theory. In this context, simplified models of different air system components are formulated in such a way that the resulting reduced-order model is differentially flat. Thereafter, the differential flatness property of the design model is exploited to derive the feedforward controller. The approximation performance of the reduced-order model as well as the suitability of the developed feedforward control scheme are finally evaluated by means of simulation studies.**

I. Introduction

Today, the efficiency of Diesel engines is often increased by employing an exhaust-driven turbocharger. These one-stage turbocharged engines therefore usually feature higher power and lower pollutant emissions than comparable non-charged engines. In order to further increase the engine's efficiency and to overcome the major drawback of turbocharged engines, namely the lack of boost pressure at low engine speed and load, the air system considered here comprises two turbochargers in a series connection, see Fig. 2. Additionally, an exhaust-gas recirculation (EGR) is employed to lower the engine's nitrogen oxide (NOx) emissions [1]. Thereby, the larger heat capacity of the recirculated exhaust-gas results in a reduced peak combustion temperature, which in turn reduces the NOx emissions. In this work, a two-stage turbocharged Diesel air system with high-pressure EGR and two wastegate turbochargers is considered, which is presented in Sec. II.

For one-stage turbocharged air systems, many different control concepts are presented in the literature, including Lyapunov control [2], linear parameter-varying control [3], and model predictive design methods [4]. Nonlinear internal model controllers are successfully developed for one- and two-stage turbocharged air systems with and without EGR [5], [6]. A switched single-input single-output (SISO) boost-pressure control strategy is employed in [7] for a two-stage turbocharged system without EGR. A model-based feedforward controller solving the nonlinear multivariable boost pressure and EGR control problem for a two-stage turbocharged system has not yet been presented to the best of the author's knowledge.

In order to achieve both, good trajectory tracking and robustness with respect to disturbances and model uncertainties, a two-degrees-of-freedom (2DOF) control structure is strived for the air system control, see Fig. 1. While the feedforward (FF) controller is utilized for trajectory tracking such that the output y of the plant $\mathcal{P}$ follows the desired reference trajectory y^{d} provided by the signal generator (SG), the feedback (FB) controller ensures stability and robustness against disturbances and model uncertainties.

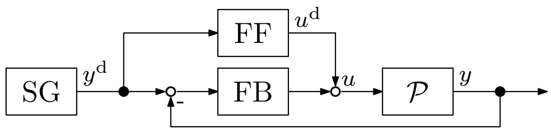

Fig. 1. Two-degrees-of-freedom control structure.

A flatness-based multivariable FF controller for a two-stage turbocharged Diesel air system with EGR is presented in this work as an extension of the respective one-stage results given in [8]. The differentially flat reduced-order model (ROM) derived in Sec. III serves as a basis for the controller design. The flatness property is shown in Sec. IV by giving the respective state and input parametrizations. The suitability of the ROM and the performance of the FF control concept are evaluated by means of simulation studies in Sec. V.

II. Two-Stage Turbocharged Air System

The two-stage turbocharged Diesel engine air system with EGR considered in this work is introduced next. In particular, the working principle of the air system is discussed in Sec. II-A, followed by the formulation of the related control problems in Sec. II-B and the introduction of the high-order reference model in Sec. II-C.

A. System Description

In addition to turbocharging and EGR, the efficiency and pollutant emissions of the overall engine system are improved by employing a charge-air cooler, an EGR cooler, and an exhaust aftertreatment system. The working principle of the system depicted in Fig. 2 is as follows. Fresh air is aspirated from the ambiance V_{a} (with constant pressure

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p_a and temperature T_a) and consecutively compressed by the low- and high-pressure compressor (LC and HC, respectively), where a check valve (CV) bypasses the HC in regions of high engine speed and load. The compressed air is then cooled by the charge-air cooler (CC), passes the throttle valve (TV), is mixed with exhaust gas in the intake manifold (IM) V_2, and is finally induced into the cylinders of the internal combustion engine (ICE), whose behavior is mainly determined by the engine speed n_E, the injected fuel mass q_I, and the boost pressure p_2 in the IM. Hot exhaust gas is exhaled into the exhaust manifold (EM) V_3 and partly fed back into the IM via the EGR cooler (EC) and valve (EV). The remaining exhaust gas drives the high- and low-pressure turbine (HT and LT, respectively), which in turn power the corresponding compressors via the connecting shafts (HS and LS). The turbines can be bypassed using the bypass (BP) and the wastegate (WG), respectively. Finally, the burned gas leaves the air system through the exhaust aftertreatment system (EA).

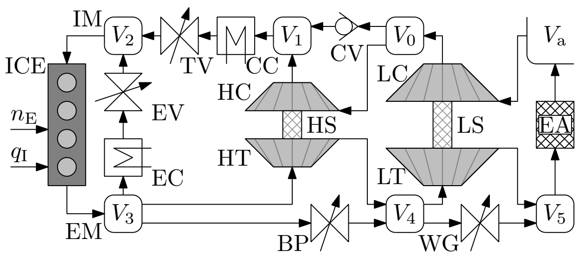

Fig. 2. Sketch of the two-stage turbocharged Diesel air system with charge-air cooling, cooled EGR, and exhaust aftertreatment system.

B. Problem Formulation

The controlled air system variables that have to be adjusted to achieve a desired combustion behavior are the boost pressure p_2 and the EGR rate r_E in the IM, where the EGR rate r_E is the mass fraction of exhaust gas in the IM. Thereby, two different operating modes have to be distinguished for the present two-stage turbocharged system. That is at first the two-stage case at low engine speed and load, where both turbochargers are used to achieve the desired boost pressure. At high engine speed and load, however, the BP and the CV have to be entirely opened to avoid damage to the high-pressure turbocharger, such that the air system behaves like a one-stage turbocharged system [7], [9]. Since the one-stage boost-pressure and EGR rate FF control problem is already solved in [8], only the two-stage case (CV closed) is considered in this work. The available control inputs are therefore the cross-sectional area u_E of the EV to manipulate the EGR mass flow $\dot{m}_\mathrm{EV}$ as well as the cross-sectional areas u_B of the BP and u_W of the WG to adjust the mass flows through the turbines and in turn the power supplied to the compressors [1]. Note that all inputs are constrained by respective minimum and maximum values, i.e. $0 \leq u_i \leq u_i^\mathrm{max}$, $i \in \{E, B, W\}$. Furthermore, in order to have the same number of inputs and outputs, the pressure p_0 between the compressors is defined as additional controlled output. It determines the amount of pressure boost contributed by the LC and the HC, respectively [9]. Finally, it shall be mentioned that the TV is not used as a control input in this work and is hence supposed to be entirely opened.

The FF control solutions often implemented in practice usually treat the boost pressure and the EGR rate control problem as independent SISO control problems [9]. However, these quantities are actually coupled, which significantly influences the dynamics of the air system [10]. It is hence desirable to design a FF controller that fully accounts for the multi-input multi-output (MIMO) characteristics of the combined boost pressure and EGR rate control problem. Furthermore, the problem at hand is highly nonlinear, which further complicates the control design.

C. Reference Air System Model

Following the modular modeling approach presented in [11], the spatially distributed piping sections V_i, $i = 0, \ldots, 5$, see Fig. 2, are approximated by lumped parameter models describing ideally mixed plenum chambers, where the gas mixture is supposed to behave like an ideal gas. The plenum chambers are interconnected by suitable nonlinear algebraic coupling models, see, e.g., [11]. The state vector x_F of the resulting dynamical air system model is composed of the partial pressures p_i^k, $i = 0, \ldots, 5$, $k \in \{\mathrm{N_2}, \mathrm{O_2}, \mathrm{CO_2}, \mathrm{H_2O}\}$ of the gas components in the chambers, the respective temperatures T_i, $0 = 1, \ldots, 5$, as well as the rotational speeds ω_H and ω_L of the shafts HS and LS, respectively. From a system-theoretical point of view, this full-order model (FOM) is described by a nonlinear ordinary differential equation (ODE) of the form

$$\Sigma_\mathrm{F} : \begin{cases} \dfrac{\mathrm{d}x_\mathrm{F}}{\mathrm{d}t} = f_\mathrm{F}(x_\mathrm{F}, u, \theta), \quad x_\mathrm{F}(0) = x_{\mathrm{F},0}, \\ y_\mathrm{F} = \begin{bmatrix} p_0 & p_2 & r_\mathrm{E} \end{bmatrix}^\mathrm{T}, \\ u = \begin{bmatrix} u_\mathrm{B} & u_\mathrm{W} & u_\mathrm{E} \end{bmatrix}^\mathrm{T}, \end{cases} \tag{1}$$

with the state, input, and output vectors $x_\mathrm{F} \in \mathbb{R}^{32}$, $u \in \mathbb{R}^3$, and $y_\mathrm{F} \in \mathbb{R}^3$, respectively, the vector $\theta = [n_\mathrm{E}\ q_\mathrm{I}]^\mathrm{T}$ of time-dependent engine parameters, as well as the vector-valued function f_F. The FOM Σ_F (1) is further used as a reference model for the verification of the ROM and the FF control.

III. Model Reduction for Control Design

Due to the complexity of the FOM Σ_F (1), the derivation of a ROM suitable for control design is indispensable. Apart from the reduction of the model complexity an excellent approximation performance of the ROM is desired to guarantee the feasibility of the developed controllers. To this end, a reduced-order air system model is derived using the singular perturbation theory, which is briefly reviewed in Sec. III-A. Thereafter, the model-order reduction is performed in Sec. III-B. The summary of the ROM equations in Sec. III-C concludes this section.

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A. Singular Perturbation Theory

The key observation exploited for model order reduction is that the pressure and temperature dynamics of the plenum chambers are significantly faster than the rotational dynamics of the turbochargers. Thus, the singular perturbation theory [12] is applied to reduce the order of the FOM Σ_{F} (1). For this, the state vector x_{F} is separated into μ states $\tilde{x}$ with slow and ν states $\tilde{z}$ with fast dynamics, leading to a system

$$\tilde{\Sigma} : \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\tilde{x}\\ \epsilon\tilde{z}\end{bmatrix} = \begin{bmatrix}\tilde{f}(\tilde{x},\tilde{z},u,\theta)\\ \tilde{g}(\tilde{x},\tilde{z},u,\theta)\end{bmatrix}, \ \tilde{x}\in\mathbb{R}^{\mu}, \tilde{z}\in\mathbb{R}^{\nu}, \\ \qquad y_{\mathrm{F}} = \tilde{h}(\tilde{x},\tilde{z},\theta), \end{cases} \tag{2}$$

with $[\tilde{x}(0)\,\tilde{z}(0)]^{\mathrm{T}} = x_{\mathrm{F}}(0)$ and the singular perturbation parameter ϵ. Letting $\epsilon\to 0$ yields

$$\frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} = \tilde{f}(\tilde{x},\tilde{z},u,\theta) \quad \text{subject to} \quad 0 = \tilde{g}(\tilde{x},\tilde{z},u,\theta).$$

An explicit ROM with $x=\tilde{x}$ is then given by

$$\Sigma_{\mathrm{R}} : \begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = f_{\mathrm{R}}(x,u,\theta) = \tilde{f}(\tilde{x},\tilde{k}(\tilde{x},u,\theta),u,\theta), \\ y_{\mathrm{F}} = h_{\mathrm{R}}(x,u,\theta) = \tilde{h}(\tilde{x},\tilde{k}(\tilde{x},u,\theta),\theta), \end{cases} \tag{3}$$

where $\tilde{z}=\tilde{k}(\tilde{x},u,\theta)$ is, for the air system under consideration, the only isolated real root satisfying

$$\tilde{g}(\tilde{x},\tilde{k}(\tilde{x},u,\theta),u,\theta) = 0.$$

The ROM (3) is also known as *quasi-steady-state* or *slow model* [12]. Note that the singular perturbation theory has successfully been applied to derive a design model for SISO boost pressure control of a one-stage turbocharged gasoline engine without EGR in [13].

B. Derivation of the Reduced-Order Air System Model

In this section, the actual reduction of the air system model is discussed in more detail. In particular, several simplifying assumptions are made to enable the application of the singular perturbation theory as described in Sec. III-A. Furthermore, for the resulting quasi-steady-state model (referred to as the ROM) to be differentially flat, simplified models for the temperature drops and increases at the turbines and the compressors are defined in a last step.

Simplifying assumptions. First of all, gas properties such as the isobaric heat capacity c_{p} are assumed to be constant and, with regard to the control task, only fresh air (index f) and exhaust gas (index e) are distinguished in the IM V_2. Its thermodynamic state is hence uniquely defined by the respective partial pressures p_2^{f} and p_2^{e} (with $p_2^{\mathrm{f}}+p_2^{\mathrm{e}}=p_2$), while the temperature T_2 is assumed to be constant. In the remaining plenum chambers only pure gases are considered, namely fresh air in V_0 and V_1 and exhaust gas in V_3, V_4, and V_5. These chambers are thus described by the respective pressures p_i and temperatures $T_i, i\in\{0,1,3,4,5\}$. Furthermore, with these simplifications the boost pressure p_2 and the EGR rate $r_{\mathrm{E}}=p_2^{\mathrm{e}}/p_2$ are uniquely determined by the partial pressures p_2^{f} and p_2^{e} of fresh air and exhaust gas in the IM. Consequently, the output

$$y = \begin{bmatrix} p_0 & p_2^{\mathrm{f}} & p_2^{\mathrm{e}} \end{bmatrix}^{\mathrm{T}}$$

can be used instead of y_{F} in (1) for the controller design.

In order to compute a ROM of the form (3), it is furthermore necessary to define simplified models for the mass flows through the air system components. For example, the models employed for the HT and the EA read as

$$\dot{m}_{\mathrm{HT}} = (a_{\mathrm{HT}} - b_{\mathrm{HT}}\omega_{\mathrm{H}})(p_3 - p_4), \quad \dot{m}_{\mathrm{EA}} = a_{\mathrm{EA}}(p_5 - p_{\mathrm{a}}),$$

where the parameters a_{HT}, b_{HT} of the HT model and a_{EA} of the EA model are identified using measurement data. The results can e.g. be found in [8]. Note that simplified models for the remaining coupling elements are defined accordingly.

Model order reduction. With these assumptions the model order is reduced to 14 and it is possible to define an air system model $\tilde{\Sigma}$ according to (2). The state vectors of this partitioned system are given by

$$\tilde{x} = \begin{bmatrix} p_2^{\mathrm{f}} & p_2^{\mathrm{e}} & \omega_{\mathrm{H}} & \omega_{\mathrm{L}} \end{bmatrix}^{\mathrm{T}} \text{ and } \tilde{z} = \begin{bmatrix} p_i & T_i \end{bmatrix}^{\mathrm{T}},\ i\in\{0,1,3,4,5\},$$

and the perturbation parameter is the maximum volume $V_i, i\in\{0,1,3,4,5\}$ of the respective plenum chambers. This model serves as a basis for the model order reduction by means of the singular perturbation theory, leading to a ROM Σ_{R} according to (3) with the dynamical state vector $x = \tilde{x}$.

Differential flatness. The ROM Σ_{R} (3) is not yet differentially flat. To this end, simplified models for the temperature drops and increases at the turbines and compressors have to be defined. The respective models used in this work read as

$$\Delta T_{\mathrm{LC}} = \bar{\eta}_{\mathrm{LC}}T_{\mathrm{a}}(p_0/p_{\mathrm{a}} - 1), \quad \Delta T_{\mathrm{HC}} = \bar{\eta}_{\mathrm{HC}}T_0(p_1/p_0 - 1),$$
$$\Delta T_{\mathrm{HT}} = \bar{\eta}_{\mathrm{HT}}T_3(p_3/p_4 - 1), \quad \Delta T_{\mathrm{LT}} = \bar{\eta}_{\mathrm{LT}}T_4(p_4/p_5 - 1),$$

where the model parameters $\bar{\eta}_i$, $i \in \{\mathrm{LC},\mathrm{HC},\mathrm{HT},\mathrm{LT}\}$, are identified using measurement data. For the respective identification results, the reader is referred to [8].

C. The Reduced-Order Design Model in Detail

Based on the results of the previous subsection, the structure of the ROM takes the form

$$\frac{\mathrm{d}p_2^{\mathrm{f}}}{\mathrm{d}t} = k_2\big(\dot{m}_{\mathrm{TV}}(x) - K_{\mathrm{E}}(\theta)p_2^{\mathrm{f}}\big) = f_2^{\mathrm{f}} \tag{4a}$$
$$\frac{\mathrm{d}p_2^{\mathrm{e}}}{\mathrm{d}t} = k_2\big(\dot{m}_{\mathrm{EV}}(x,u,\theta) - K_{\mathrm{E}}(\theta)p_2^{\mathrm{e}}\big) = f_2^{\mathrm{e}} \tag{4b}$$
$$\frac{\mathrm{d}\omega_{\mathrm{H}}}{\mathrm{d}t} = \frac{k_{\mathrm{H}}}{\omega_{\mathrm{H}}}(P_{\mathrm{HT}}(x,u,\theta) - P_{\mathrm{HC}}(x) - P_{\mathrm{HF}}(x)) = f_{\mathrm{H}} \tag{4c}$$
$$\frac{\mathrm{d}\omega_{\mathrm{L}}}{\mathrm{d}t} = \frac{k_{\mathrm{L}}}{\omega_{\mathrm{L}}}(P_{\mathrm{LT}}(x,u,\theta) - P_{\mathrm{LC}}(x) - P_{\mathrm{LF}}(x)) = f_{\mathrm{L}} \tag{4d}$$
$$y = \begin{bmatrix} p_0 & p_2^{\mathrm{f}} & p_2^{\mathrm{e}} \end{bmatrix}^{\mathrm{T}}, \quad x = \begin{bmatrix} p_2^{\mathrm{f}} & p_2^{\mathrm{e}} & \omega_{\mathrm{H}} & \omega_{\mathrm{L}} \end{bmatrix}^{\mathrm{T}}, \tag{4e}$$

with the constants $k_2=(RT_2)/V_2$, $k_{\mathrm{H}}=J_{\mathrm{H}}^{-1}$, and $k_{\mathrm{L}}=J_{\mathrm{L}}^{-1}$, the gas constant R, the constant volume V_2 of the IM, and the moments of inertia J_{H} and J_{L} of the HS and the LS, respectively. The engine parameter $K_{\mathrm{E}}(\theta)$ is given by [11]

$$K_{\mathrm{E}}(\theta) = \frac{(a_{\mathrm{E}} + b_{\mathrm{E}}n_{\mathrm{E}})V_{\mathrm{E}}n_{\mathrm{E}}}{2\,RT_2},$$

where V_{E} is the engine's displacement and the model parameters a_{E} and b_{E} are identified using measurement data [11].

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The mass flows $\dot{m}_{\mathrm{TV}}$ through the TV and $\dot{m}_{\mathrm{EV}}$ through the EV are given by

$$\dot{m}_{\mathrm{TV}} = \frac{a_{\mathrm{TV}}(\chi_{\mathrm{H}}\chi_{\mathrm{L}}p_{\mathrm{a}} - \beta_{\mathrm{H}}\beta_{\mathrm{L}}p_2)}{a_{\mathrm{TV}}(\chi_{\mathrm{H}} + \beta_{\mathrm{L}}) + \beta_{\mathrm{H}}\beta_{\mathrm{L}}} \tag{5a}$$

$$\dot{m}_{\mathrm{EV}} = \frac{\gamma_{\mathrm{E}}(\dot{m}_{\mathrm{E}}\gamma_{\mathrm{B}}\gamma_{\mathrm{W}} + a_{\mathrm{EA}}(\dot{m}_{\mathrm{E}}\gamma_{\mathrm{TC}} - \gamma_{\mathrm{B}}\gamma_{\mathrm{W}}\Delta p_{\mathrm{a}}))}{\gamma_{\mathrm{E}}\gamma_{\mathrm{B}}\gamma_{\mathrm{W}} + a_{\mathrm{EA}}(\gamma_{\mathrm{B}}\gamma_{\mathrm{W}} + \gamma_{\mathrm{E}}\gamma_{\mathrm{TC}})}, \tag{5b}$$

where $\Delta p_{\mathrm{a}} = p_2 - p_{\mathrm{a}}$, $\dot{m}_{\mathrm{E}} = K_{\mathrm{E}}(\theta)p_2 + \dot{m}_{\mathrm{F}}(\theta)$,

$$\alpha_i = a_{i\mathrm{C}}\omega_i,\ \ \beta_i = b_{i\mathrm{C}}(\omega_i' - \omega_i),\ \ \chi_i = \alpha_i + \beta_i,\ \ i \in \{\mathrm{H},\mathrm{L}\}$$
$$\gamma_{\mathrm{B}} = a_{\mathrm{HT}} - b_{\mathrm{HT}}\omega_{\mathrm{H}} + c_{\mathrm{HT}}u_{\mathrm{B}},\ \ \gamma_{\mathrm{W}} = a_{\mathrm{LT}} - b_{\mathrm{LT}}\omega_{\mathrm{L}} + c_{\mathrm{LT}}u_{\mathrm{W}},$$
$$\gamma_{\mathrm{E}} = a_{\mathrm{EV}}u_{\mathrm{E}},\ \text{ and }\ \gamma_{\mathrm{TC}} = \gamma_{\mathrm{B}} + \gamma_{\mathrm{W}}.$$

Furthermore, $\dot{m}_{\mathrm{F}}(\theta)$ is the fuel mass flow into the engine and the constant parameters a_{kl}, b_{kl}, and c_{kl} stem from the simplified air system component models. The differential equations for the shaft speeds ω_{H} and ω_{L}, cf. (4c) and (4d), are influenced by the turbine power $P_{\mathrm{HT}}(x,u,\theta)$ and $P_{\mathrm{LT}}(x,u,\theta)$, the compressor power $P_{\mathrm{HC}}(x)$ and $P_{\mathrm{LC}}(x)$, and the friction power $P_{\mathrm{HF}}(x)$ and $P_{\mathrm{LF}}(x)$, respectively, which read as

$$P_{\mathrm{HT}} = \frac{a_{\mathrm{EA}}\bar{\eta}_{\mathrm{HT}}\gamma_{\mathrm{W}}(\dot{m}_{\mathrm{E}} + \gamma_{\mathrm{E}}\Delta p_{\mathrm{a}})\dot{H}_{\mathrm{TC}}\Theta_{\mathrm{H}}}{\Gamma\gamma_{\mathrm{B}}} \tag{5c}$$

$$P_{\mathrm{HC}} = \frac{\bar{\eta}_{\mathrm{HC}}\Lambda_{\mathrm{H}}(\dot{H}_{\mathrm{a}} + P_{\mathrm{LC}})}{a_{\mathrm{TV}}\beta_{\mathrm{H}}p_2 + \chi_{\mathrm{L}}(a_{\mathrm{TV}} + \beta_{\mathrm{H}})p_{\mathrm{a}}} \tag{5d}$$

$$P_{\mathrm{LT}} = \frac{a_{\mathrm{EA}}\bar{\eta}_{\mathrm{LT}}\gamma_{\mathrm{B}}(\dot{m}_{\mathrm{E}} + \gamma_{\mathrm{E}}\Delta p_{\mathrm{a}})(\dot{H}_{\mathrm{TC}} - P_{\mathrm{HT}})\Theta_{\mathrm{L}}}{(\Gamma - a_{\mathrm{EA}}\gamma_{\mathrm{B}}(\dot{m}_{\mathrm{E}} + \gamma_{\mathrm{E}}\Delta p_{\mathrm{a}}))\gamma_{\mathrm{W}}} \tag{5e}$$

$$P_{\mathrm{LC}} = \frac{\bar{\eta}_{\mathrm{LC}}\Lambda_{\mathrm{L}}\dot{H}_{\mathrm{a}}}{(a_{\mathrm{TV}}(\chi_{\mathrm{H}} + \beta_{\mathrm{L}}) + \beta_{\mathrm{H}}\beta_{\mathrm{L}})p_{\mathrm{a}}} \tag{5f}$$

$$P_{\mathrm{HF}} = d_{\mathrm{H}}\omega_{\mathrm{H}}^2,\quad \text{and} \quad P_{\mathrm{LF}} = d_{\mathrm{L}}\omega_{\mathrm{L}}^2. \tag{5g}$$

Here, $\dot{m}_{\mathrm{TC}} = \dot{m}_{\mathrm{E}} - \dot{m}_{\mathrm{EV}}$ and $\dot{H}_{\mathrm{TC}} = \dot{m}_{\mathrm{TC}}c_{\mathrm{p}}T_3$ are the mass and enthalpy flow used for turbocharging, $\dot{H}_{\mathrm{a}} = \dot{m}_{\mathrm{TV}}c_{\mathrm{p}}T_{\mathrm{a}}$ is the fresh-air enthalpy flow, and furthermore

$$\Theta_{\mathrm{H}} = a_{\mathrm{HT}} - b_{\mathrm{HT}}\omega_{\mathrm{H}},\ \ \Theta_{\mathrm{L}} = a_{\mathrm{LT}} - b_{\mathrm{LT}}\omega_{\mathrm{L}},$$
$$\Gamma = \gamma_{\mathrm{B}}(a_{\mathrm{EA}} + \gamma_{\mathrm{W}})(\dot{m}_{\mathrm{E}} + \gamma_{\mathrm{E}}p_2) + a_{\mathrm{EA}}(\gamma_{\mathrm{E}} + \gamma_{\mathrm{B}})\gamma_{\mathrm{W}}p_{\mathrm{a}}$$
$$\Lambda_{\mathrm{H}} = a_{\mathrm{TV}}(\alpha_{\mathrm{H}} + \beta_{\mathrm{L}})p_2 + \chi_{\mathrm{L}}(\alpha_{\mathrm{H}} - a_{\mathrm{TV}})p_{\mathrm{a}},$$
$$\Lambda_{\mathrm{L}} = a_{\mathrm{TV}}(\alpha_{\mathrm{L}} - \alpha_{\mathrm{H}})p_{\mathrm{a}} + \beta_{\mathrm{H}}(a_{\mathrm{TV}}\Delta p_{\mathrm{a}} + \alpha_{\mathrm{L}}p_{\mathrm{a}}).$$

Note that here and in the sequel, the dependencies on the state x, the input u, and the parameter θ are omitted for brevity. Finally, the first component $y_1 = p_0$ of the output y (4e) takes the form

$$p_0 = \frac{a_{\mathrm{TV}}\beta_{\mathrm{H}}p_2 + (a_{\mathrm{TV}} + \beta_{\mathrm{H}})\chi_{\mathrm{L}}p_{\mathrm{a}}}{a_{\mathrm{TV}}(\chi_{\mathrm{H}} + \beta_{\mathrm{L}}) + \beta_{\mathrm{H}}\beta_{\mathrm{L}}}. \tag{6}$$

IV. Flatness-Based Feedforward Control

In this section, a FF controller for the Diesel air system is designed by means of the ROM (4) and by exploiting its flatness property [14], [15]. In fact, it will be shown that the output y (4e) constitutes a flat output of the system (4).

A. State Parametrization

At first, the parametrization of the state x in terms of the output y and its time derivatives will be presented. For the first two components of the state vector x (4e) this is trivially given by

$$p_2^{\mathrm{f}} = y_2,\ \ p_2^{\mathrm{e}} = y_3, \tag{7a}$$

where y_2 and y_3 are the second and third component of the flat output y, respectively. The parametrization of the shaft speeds ω_{H} and ω_{L} is obtained from

$$y_1 = \frac{a_{\mathrm{TV}}\beta_{\mathrm{H}}p_2 + (a_{\mathrm{TV}} + \beta_{\mathrm{H}})\chi_{\mathrm{L}}p_{\mathrm{a}}}{a_{\mathrm{TV}}(\chi_{\mathrm{H}} + \beta_{\mathrm{L}}) + \beta_{\mathrm{H}}\beta_{\mathrm{L}}}$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = k_2\left(\frac{a_{\mathrm{TV}}\chi_{\mathrm{H}}\chi_{\mathrm{L}}p_{\mathrm{a}} - a_{\mathrm{TV}}\beta_{\mathrm{H}}\beta_{\mathrm{L}}p_2}{a_{\mathrm{TV}}(\chi_{\mathrm{H}} + \beta_{\mathrm{L}}) + \beta_{\mathrm{H}}\beta_{\mathrm{L}}} - K_{\mathrm{E}}p_2^{\mathrm{f}}\right),$$

leading to

$$\omega_{\mathrm{H}} = \frac{(a_{\mathrm{TV}} + b_{\mathrm{HC}}\omega_{\mathrm{H}}')\Xi + a_{\mathrm{TV}}b_{\mathrm{HC}}\omega_{\mathrm{H}}'k_2(y_2 + y_3 - y_1)}{a_{\mathrm{TV}}k_2((a_{\mathrm{HC}} - b_{\mathrm{HC}})y_1 + p_2) + b_{\mathrm{HC}}\Xi} \tag{7b}$$

$$\omega_{\mathrm{L}} = \frac{b_{\mathrm{LC}}k_2\omega_{\mathrm{L}}'(y_1 - p_{\mathrm{a}}) + \Xi}{k_2(a_{\mathrm{LC}}p_{\mathrm{a}} + b_{\mathrm{LC}}(y_1 - p_{\mathrm{a}}))}, \tag{7c}$$

with $\Xi = k_2K_{\mathrm{E}}y_2 + \mathrm{d}y_2/\mathrm{d}t$. By means of (7), the nonlinear state transformation

$$\xi = \begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\end{bmatrix} = \begin{bmatrix}y_1\\ y_2\\ \mathrm{d}y_2/\mathrm{d}t\\ y_3\end{bmatrix} = \begin{bmatrix}p_0\\ p_2^{\mathrm{f}}\\ f_2^{\mathrm{f}}\\ p_2^{\mathrm{e}}\end{bmatrix} = \Phi(x)$$

proves to be a diffeomorphism, i.e. the inverse mapping $x = \Phi^{-1}(\xi)$ given by (7) exists and is smooth as well.

B. Input Parametrization

With the state parametrization $x = \Phi^{-1}(\xi)$ (7), the parametrization of the input is obtained from the r_i-th derivative of the flat output y

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{\partial p_0}{\partial p_2^{\mathrm{f}}}\frac{\mathrm{d}y_2}{\mathrm{d}t} + \frac{\partial p_0}{\partial p_2^{\mathrm{e}}}\frac{\mathrm{d}y_3}{\mathrm{d}t} + \frac{\partial p_0}{\partial \omega_{\mathrm{H}}}f_{\mathrm{H}}(P_{\mathrm{HT}},\ldots) + \frac{\partial p_0}{\partial \omega_{\mathrm{L}}}f_{\mathrm{L}}(P_{\mathrm{LT}},\ldots) \tag{8a}$$

$$\frac{\mathrm{d}^2y_2}{\mathrm{d}t^2} = \frac{\partial f_2^{\mathrm{f}}}{\partial p_2^{\mathrm{f}}}\frac{\mathrm{d}y_2}{\mathrm{d}t} + \frac{\partial f_2^{\mathrm{f}}}{\partial p_2^{\mathrm{e}}}\frac{\mathrm{d}y_3}{\mathrm{d}t} + \frac{\partial f_2^{\mathrm{f}}}{\partial \theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\partial f_2^{\mathrm{f}}}{\partial \omega_{\mathrm{H}}}f_{\mathrm{H}}(P_{\mathrm{HT}},\ldots) + \frac{\partial f_2^{\mathrm{f}}}{\partial \omega_{\mathrm{L}}}f_{\mathrm{L}}(P_{\mathrm{LT}},\ldots) \tag{8b}$$

$$\frac{\mathrm{d}y_3}{\mathrm{d}t} = k_2(\dot{m}_{\mathrm{EV}} - K_{\mathrm{E}}y_3), \tag{8c}$$

where $r = [r_1, r_2, r_3]^{\mathrm{T}} = [1,2,1]^{\mathrm{T}}$ is the system's vector relative degree, see [15] for a definition. In a first step, (8) is solved for $\dot{m}_{\mathrm{EV}}$, P_{HT}, and P_{LT}, leading to

$$\dot{m}_{\mathrm{EV}} = \frac{1}{k_2}\frac{\mathrm{d}y_3}{\mathrm{d}t} + K_{\mathrm{E}}y_3, \tag{9a}$$

$$\begin{bmatrix}P_{\mathrm{HT}}\\ P_{\mathrm{LT}}\end{bmatrix} = \begin{bmatrix}P_{\mathrm{HC}} + P_{\mathrm{HF}}\\ P_{\mathrm{LC}} + P_{\mathrm{LF}}\end{bmatrix} + D\begin{bmatrix}\frac{\mathrm{d}y_1}{\mathrm{d}t} - E_1\\ \frac{\mathrm{d}^2y_2}{\mathrm{d}t^2} - E_2\end{bmatrix} \tag{9b}$$

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where the matrix D and the scalars E_1 and E_2 read as

$$D = \begin{bmatrix} \frac{k_\mathrm{H}}{\omega_\mathrm{H}} \frac{\partial p_0}{\partial \omega_\mathrm{H}} & \frac{k_\mathrm{L}}{\omega_\mathrm{L}} \frac{\partial p_0}{\partial \omega_\mathrm{L}} \\ \frac{k_\mathrm{H}}{\omega_\mathrm{H}} \frac{\partial f_2^\mathrm{f}}{\partial \omega_\mathrm{H}} & \frac{k_\mathrm{L}}{\omega_\mathrm{L}} \frac{\partial f_2^\mathrm{f}}{\partial \omega_\mathrm{L}} \end{bmatrix}^{-1}, \; E_1 = \frac{\partial p_0}{\partial p_2^\mathrm{f}} \frac{\mathrm{d} y_2}{\mathrm{d}t} + \frac{\partial p_0}{\partial p_2^\mathrm{e}} \frac{\mathrm{d} y_3}{\mathrm{d}t},$$

$$E_2 = \frac{\partial f_2^\mathrm{f}}{\partial p_2^\mathrm{f}} \frac{\mathrm{d} y_2}{\mathrm{d}t} + \frac{\partial f_2^\mathrm{f}}{\partial p_2^\mathrm{e}} \frac{\mathrm{d} y_3}{\mathrm{d}t} + \frac{\partial f_2^\mathrm{f}}{\partial \theta} \frac{\mathrm{d} \theta}{\mathrm{d}t}.$$

Note that the decoupling matrix D exists and is regular for all admissible operating points of the Diesel air system. In the next step, the control inputs u_E, u_B, and u_W are computed from the EGR mass flow $\dot{m}_\mathrm{EV}$ (5b), the HT power P_HT (5c), and the LT power P_LT (5e), respectively. The EV actuation is thus obtained by

$$u_\mathrm{E} = \frac{a_\mathrm{EA} \gamma_\mathrm{B} \gamma_\mathrm{W} \dot{m}_\mathrm{EV}}{a_\mathrm{EV}((\gamma_\mathrm{B}\gamma_\mathrm{W} + a_\mathrm{EA}\gamma_\mathrm{TC})\dot{m}_\mathrm{TC} - a_\mathrm{EA}\gamma_\mathrm{B}\gamma_\mathrm{W}\Delta p_\mathrm{a})}. \quad (10)$$

This, in combination with (9a), structurally guarantees a closed EGR valve if the EGR rate and its derivatives are equal to zero, which is especially important from a practical point of view. Using (10), the HT power P_HT (5c) and the LT power P_LT (5e) can be simplified to

$$P_\mathrm{HT} = \frac{\Psi_\mathrm{H} \Theta_\mathrm{H} \dot{H}_\mathrm{TC} \gamma_\mathrm{W}}{(\Upsilon \gamma_\mathrm{W} + a_\mathrm{EA} \dot{m}_\mathrm{TC}) \gamma_\mathrm{B}^2} \quad (11a)$$

$$P_\mathrm{LT} = \frac{\Psi_\mathrm{L} \Theta_\mathrm{L} (\dot{H}_\mathrm{TC} - P_\mathrm{HT})}{\Upsilon \gamma_\mathrm{W}^2}, \quad (11b)$$

where $\Psi_i = a_\mathrm{EA} \bar{\eta}_{i\mathrm{T}} \dot{m}_\mathrm{TC}$, $i \in \{\mathrm{H,L}\}$ and $\Upsilon = \dot{m}_\mathrm{TC} + a_\mathrm{EA} p_\mathrm{a}$ are used for brevity. Equations (11a) and (11b) are easily rearranged to

$$\gamma_\mathrm{B}^2 = \frac{\Psi_\mathrm{H} \Theta_\mathrm{H} \dot{H}_\mathrm{TC} \gamma_\mathrm{W}}{(\Upsilon \gamma_\mathrm{W} + a_\mathrm{EA} \dot{m}_\mathrm{TC}) P_\mathrm{HT}} \quad (12a)$$

$$\gamma_\mathrm{W}^2 = \frac{\Psi_\mathrm{L} \Theta_\mathrm{L} (\dot{H}_\mathrm{TC} - P_\mathrm{HT})}{\Upsilon P_\mathrm{LT}}, \quad (12b)$$

which is finally solved for the control inputs

$$u_\mathrm{B} = -\frac{\Theta_\mathrm{H}}{c_\mathrm{HT}} \pm \sqrt{\frac{\Psi_\mathrm{H} \Theta_\mathrm{H} \dot{H}_\mathrm{TC} \gamma_\mathrm{W}}{c_\mathrm{HT}^2 P_\mathrm{HT} (\Upsilon \gamma_\mathrm{W} + a_\mathrm{EA} \dot{m}_\mathrm{TC})}} \quad (13a)$$

$$u_\mathrm{W} = -\frac{\Theta_\mathrm{L}}{c_\mathrm{LT}} \pm \sqrt{\frac{\Psi_\mathrm{L} \Theta_\mathrm{L} (\dot{H}_\mathrm{TC} - P_\mathrm{HT})}{c_\mathrm{LT}^2 \Upsilon P_\mathrm{LT}}}. \quad (13b)$$

It shall be mentioned that only the $+\sqrt{\ldots}$ solutions in (13) are meaningful since $-\Theta_\mathrm{H}/c_\mathrm{HT}$ and $-\Theta_\mathrm{L}/c_\mathrm{LT}$ are strictly negative and the controls u_B and u_W can only take positive values. Additionally, both u_B and u_W are well-defined because the radicands in (13) are strictly positive for all admissible air system operating points.

Thus, the parametrization of the state x (4e) and the input u according to (1) in terms of the flat output y (4e) and its time derivatives is given by (7), (9), (10), and (13). The FF controller is now easily obtained by replacing y_1, y_2, and y_3 in the flat parametrization of the input u by sufficiently smooth desired trajectories y_1^d, y_2^d, and y_3^d, respectively. Finally, it is important to keep in mind that the input constraints mentioned in Sec. II-B are not taken into account by the developed FF controller.

V. Simulation Results

In this section, the feasibility of the model reduction performed in Sec. III and the FF controller developed in Sec. IV are illustrated by means of simulation studies. In particular, the approximation performance of the ROM is evaluated in Sec. V-A and the FF controller is tested in Sec. V-B.

A. Evaluation of the Reduced-Order Model

The approximation performance of the ROM (4), which is the basis for the controller design, is evaluated by means of the FOM (1). To this end, the resulting stationary errors δp_2 and δr_E of the boost pressure and the EGR rate are depicted for all operating points of the air system in Fig. 3. The plots in Fig. 3 show that, considering the significant simplifications assumed in the course of the model reduction, the ROM (4) features an excellent approximation performance. Note further that all simulation results are presented in terms of scaled quantities.

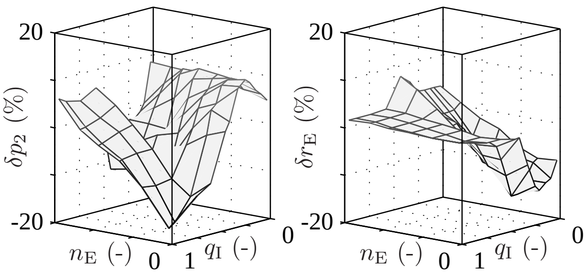

Fig. 3. Approximation performance of the ROM (4) compared to the FOM (1). Left: boost pressure error δp_2. Right: EGR rate error δr_E.

B. Simulation Study with the Full-Order Model

The performance of the FF controller is finally tested with the FOM (1). The resulting outputs p_2 and r_E of the FOM are compared to the respective desired values p_2^d and r_E^d in the first two plots of Fig. 4. The corresponding control inputs u_B, u_W, and u_E of the FF controller are given in the third plot. In the bottom plot, the time evolution of the engine speed n_E and the injected fuel mass q_I can be seen. At this point it is important to note that these simulation results are generated without employing any FB controller.

In addition to the excellent control performance that can be inferred from Fig. 4, two more interesting observations can be made. At first, the interaction of the control inputs u_B, u_W, and u_E during the performed set-point changes reveals that the MIMO behavior of the air system is indeed taken into account by the FF controller. The input constraints $0 \leq u_i \leq u_i^\mathrm{max}$, $i \in \{E, B, W\}$, however, are not taken into account by the FF controller, as can e.g. be inferred from the kink of the p_2 trajectory during its first transition and the corresponding saturation of the WG actuation at its lower value $u_\mathrm{W} = 0$.

Finally, Fig. 5 depicts the influence of the desired value p_0^d for the pressure between the two compressors on the power of the two compressors. In particular, the increase of the desired value p_0^d shown in the top plot of Fig. 5 causes the

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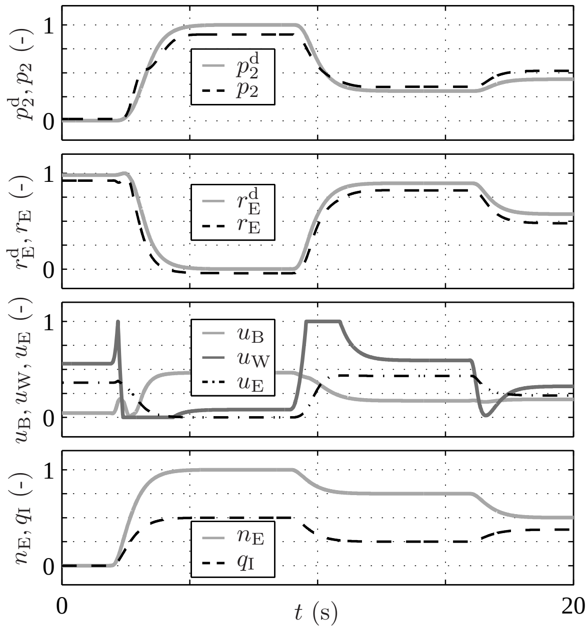

Fig. 4. Simulation study of the FF controller. From top to bottom: boost pressure p_2, EGR rate r_E, control inputs u_B, u_W and u_E, as well as engine speed n_E and injected fuel mass q_I.

increase of the LC power P_{LC} and the decrease of the HC power P_{HC} depicted in the bottom plot. From these results it can be concluded that the pressure boost contributed by each compressor can be influenced by the desired pressure p_0^d between the two compressors.

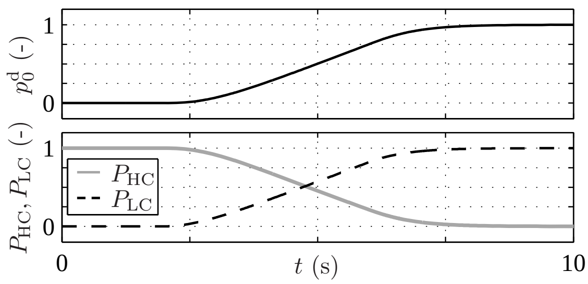

Fig. 5. Simulation study of the effect of the desired pressure p_0^d on the power of the two compressors.

VI. Conclusions & Outlook

A flatness-based feedforward (FF) controller solving the nonlinear multivariable boost pressure and EGR rate control problem for a two-stage turbocharged Diesel air system is presented. The controller design relies on a reduced-order model (ROM) derived from a respective high-order reference model by means of the singular perturbation theory. For this model to be differentially flat, different air system component models are replaced by simplified models. The FF controller of the non-input-affine ROM can directly be inferred from the flatness-based parametrization of the state and input variables. Both, the approximation performance of the ROM as well as the control performance of the FF controller are evaluated in simulation studies. In this context, the ability of the proposed controller to systematically account for the MIMO characteristics of the boost pressure and EGR rate control problem is discussed as well. It is further shown how the amount of pressure boost contributed by each of the two compressors is influenced by the chosen value for the pressure between the two compressors.

At first, in order to further improve the performance and to circumvent difficulties with saturating inputs, an appropriate trajectory planning explicitly accounting for the input constraints has to be developed. Next, in order to fully exploit the 2DOF control structure, future work will be directed towards the design of a corresponding feedback controller. Furthermore, as already mentioned earlier, the two-stage turbocharged air system has to be considered as a one-stage system at high engine speed and load. To this end, the FF controller presented here has to be combined with a FF controller solving the respective one-stage problem. The definition of a suitable criterion to distinguish between these two operational modes is a crucial step. Finally, the practical applicability of the developed FF control concept will be evaluated by means of tests on a real system.

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