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A Mathematical Model of a Combined Direct- and Indirect-Fired Strip Annealing Furnace

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Abstract

A tractable mathematical model of a combined direct- and indirect-fired strip annealing furnace for real-time control and optimization applications is presented. The considered annealing furnace is part of a hot-dip galvanizing line of voestalpine Stahl GmbH, Linz, Austria. The furnace model is based on first principles and includes sub-models covering the combustion, the flue gas, the radiant tubes, the wall, the strip, and the rolls. These sub-models are interconnected by the heat transfer mechanisms conduction, convection, and radiation. A comparison of measurement data from the real plant and simulation data demonstrates the accuracy of the model. The model captures the essential non-linearities of the system, is computational inexpensive, and is suitable for advanced model-based control of the strip temperature.

Keywords: Steel industry, annealing furnace, hot-dip galvanizing, mathematical model, control and optimization applications

Introduction

In a hot-dip galvanizing line for the production of zinc-coated steel strip, heat treatment of the strip is an important process step. In most plants, the heat treatment is carried out by an annealing furnace. To ensure a continuous operation of the processing line, the steel strips are welded together to form an endless strip. In order to achieve the desired material properties of the final product, the strip has to be heated according to a predefined temperature trajectory. From a control point of view, this is a challenging task, in particular in transient operational situations, e. g., when a welded joint traverses the furnace or when the strip velocity changes. Since the product diversity as well as the demands on the product quality and the energy consumption are steadily increasing, there is a need for an advanced model-based temperature control concept that takes into account all these difficulties [1]. Therefore, a mathematical model with moderate complexity that captures the essential dynamical behavior of the annealing furnace is required. This paper presents a tailored real-time capable first-principles model of a strip annealing furnace that is part of a hot-dip galvanizing line of voestalpine Stahl GmbH at Linz, Austria.

The annealing furnace considered in this analysis, which is schematically shown in Fig. 1, consists of a direct-fired and an indirect-fired section that are separated by a transfer section with an air lock. However, these two sections are physically coupled by the moving steel strip that is guided through the furnace by means of deflection rolls. In the direct-fired furnace section, there are four heating zones, each equipped with a set of burners. Inside these heating zones, natural gas is burnt in a fuel-rich combustion process to avoid oxidation of the strip. Thus, the hot flue gas contains unburnt combustion products which are burnt in a so-called post combustion chamber (PCC) by adding fresh air. The heat released in this post combustion is used for preheating the strip inside the preheater (PH) and for preheating the combustion air by means of a recuperator. Inside the direct-fired section, the flue gas streams always in the opposite direction of the motion of the strip. For this reason, the furnace can be considered as a counterflow heat exchanger. The indirect-fired furnace section is divided into a radiant tube heating section (RTH) and a radiant tube soaking section (RTS). Both sections are equipped with W-shaped gas-fired radiant tubes that are grouped to several heating zones. Inside each radiant tube, natural gas is burnt in a fuel-lean combustion process. The combustion heat that is not transferred through the radiant tube wall into the furnace chamber is

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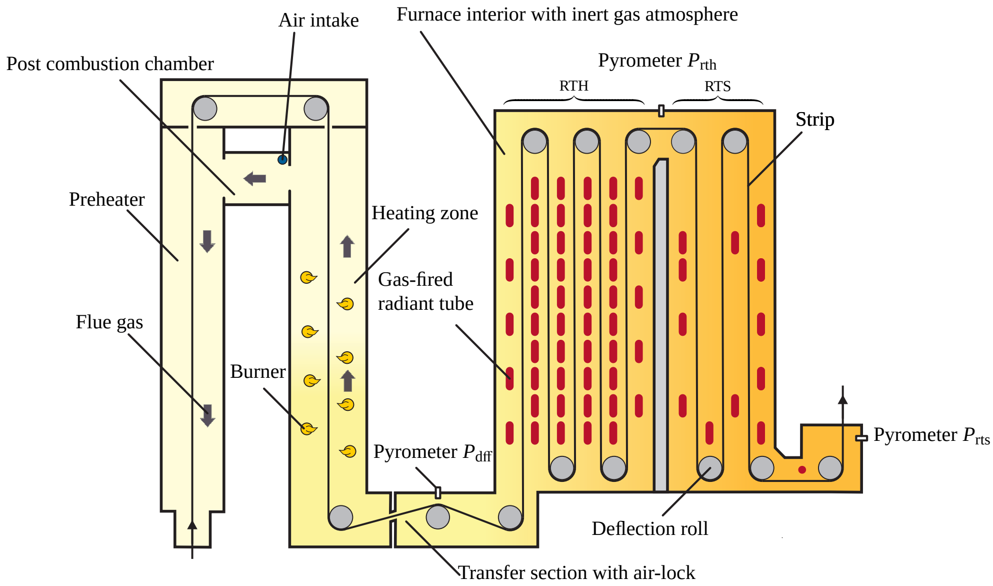

Fig. 1. Combined direct- and indirect-fired strip annealing furnace.

used for heating the combustion air in a local recuperator. Inside the furnace chamber, an inert gas atmosphere is established to prevent oxidation of the strip. This inert gas streams through the indirect-fired furnace section and enters the direct-fired furnace section through the air lock.

A comprehensive description of the model of the direct- and the indirect-fired furnace can be found in [2, 3]. Since a modern control concept for the strip temperature requires a mathematical model of the combined direct- and indirect-fired furnace, the focus of this paper is the interconnection of these two furnace models and to provide the most relevant equations that are necessary for implementing the mathematical model of the considered furnace on a computer system. The paper is structured as follows: In Sec. MATHEMATICAL MODEL, the model of the considered annealing furnace is presented. It comprises the combustion, the flue gas, the radiant tubes, the wall, the strip, and the rolls. Moreover, the heat transfer mechanisms conduction, convection, and radiation interconnect these sub-models. In Sec. EXAMPLE PROBLEM, the accuracy of the furnace model is demonstrated by a comparison of simulation results with measurement data from the real plant. An outlook of the use of the presented furnace model in control and optimization is given in Sec. OUTLOOK. Finally, Sec. CONCLUSION contains some conclusions.

MATHEMATICAL MODEL

First, the combustion process of the supplied fuel is described. Then, the individual sub-models, i. e., the flue gas, the radiant tubes, the wall, the strip, and the rolls of the considered combined direct- and indirect-fired furnace are developed. Finally, these sub-models are interconnected by the heat transfer mechanisms conduction, convection, and radiation to obtain a comprehensive mathematical model of the furnace.

Combustion

In the considered annealing furnace, the burners are operated with natural gas which is more or less pure methane (CH_4). The supply of combustion air depends on the mass flow of fuel $\dot{m}_{CH_4}$ and the excess air coefficient λ [4]. In each heating zone of the direct-fired furnace, the combustion is controlled to be fuel-rich, i. e., $\lambda < 1$. The excess air coefficient is chosen according to a constant set-point value. In contrast, the combustion inside the radiant tubes of the indirect-fired furnace is controlled to be fuel-lean, i. e., $\lambda \geq 1$, where λ is selected according to a user-defined set-point curve that is parametrized with the mass flow of fuel, i. e., $\lambda(\dot{m}_{CH_4})$.

Depending on the value of λ, the flue gas thus contains carbon monoxide, carbon dioxide, water, hydrogen, oxygen, and nitrogen. Abbreviations of these components are summarized in the set $\mathcal{S}_r = \{CO, CO_2, H_2O, H_2, N_2\}$ for the fuel-rich combustion and in the set $\mathcal{S}_l = \{CO_2, H_2O, O_2, N_2\}$ for the fuel-lean combustion. The stationary reaction equation for both a fuel-rich and a fuel-lean combustion reads as

$$CH_4 + 2\lambda\left(O_2 + 3.76 N_2\right) \longrightarrow \begin{cases} \chi^{cb}_{CO} CO + \chi^{cb}_{H_2} H_2 + \chi^{cb}_{CO_2} CO_2 + \chi^{cb}_{H_2O} H_2O + \chi^{cb}_{N_2} N_2, & \text{if } \lambda < 1 \\ \chi^{cb}_{O_2} O_2 + \chi^{cb}_{CO_2} CO_2 + \chi^{cb}_{H_2O} H_2O + \chi^{cb}_{N_2} N_2, & \text{else} \end{cases} \tag{1}$$

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where χ_ν^{cb}, $\nu \in \mathcal{S}_r \cup \mathcal{S}_l$, refers to the corresponding number of moles. For a fuel-lean combustion, the unknowns χ_ν^{cb}, $\nu \in \mathcal{S}_l$, can be easily determined by applying simple mole balances. In case of the fuel-rich combustion, the so-called water-gas-shift reaction [4, 5]

$$CO + H_2O \rightleftharpoons H_2 + CO_2$$

has to be used additionally for the determination of $\chi_\nu^{cb} \in \mathcal{S}_r$. This reaction reaches an equilibrium if

$$\frac{\chi_{CO}^{cb}\chi_{H_2O}^{cb}}{\chi_{CO_2}^{cb}\chi_{H_2}^{cb}} = \exp\left(4.33 - \frac{4577.8\,\mathrm{K}}{T_g}\right)$$

is satisfied, where T_g denotes the local gas temperature.

The incoming mass flows, i. e., the fuel and the combustion air, are coupled in the form

$$\dot{m}_\kappa = \dot{m}_{CH_4}\frac{\chi_\kappa M_\kappa}{M_{CH_4}},$$

where M_κ denotes the molar mass of the component $\kappa \in S_\kappa = \{CH_4, O_2, N_2\}$ with $(\kappa, \chi_\kappa) = \{(CH_4, 1), (O_2, 2\lambda), (N_2, 7.52\lambda)\}$. Note that the mass flow of combustion air $\dot{m}_{O_2} + \dot{m}_{N_2}$ is parametrized by the mass flow of fuel $\dot{m}_{CH_4}$. Based on Eq. (1), the mass flows $\dot{m}_\nu^{cb}$, $\nu \in \mathcal{S}_r \cup \mathcal{S}_l$, of the combustion products after reaction read as

$$\dot{m}_\nu^{cb} = \dot{m}_{CH_4}\frac{\chi_\nu^{cb} M_\nu}{M_{CH_4}}.$$

Flue Gas

Inside the direct-fired furnace section, the strip is heated by the hot flue gas from the fuel-rich combustion. The flue gas streams in the opposite direction of the strip motion. The gas flow is induced by a pressure gradient that is realized by a suction fan installed in the funnel of the furnace. In essence, the flue gas is characterized by its mass and its temperature. Inside the indirect-fired furnace section, the strip is surrounded by an inert gas atmosphere which is transparent to thermal radiation. The evolution of the strip temperature within this furnace section is mainly controlled by the surface temperatures of the radiant tubes. For this reason, only the flue gas inside the direct-fired furnace section is modeled by simple balance equations.

For modeling the flue gas, the direct-fired furnace section is discretized to N_v volume zones where each zone is assumed to be a well-stirred reactor, i. e., the gas temperature $T_{g,i}$ within each zone i is uniform. Furthermore, it is assumed that the combustion reaction is completed right after the nozzle of the burner, i. e., no additional reactions occur inside the volume zone. In view of a computationally inexpensive model of the strip temperature, it is assumed that the flue gas dynamics can be neglected. This assumption can be verified by means of singular perturbation theory [6]. Thus, the stationary mass balance for each component $\nu \in \mathcal{S}_r$ and the enthalpy balance of an individual volume zone i, see Fig. 2a, reads as

$$0 = \dot{m}_{\nu,i}^{in} + \dot{m}_{\nu,i}^{cb} - \dot{m}_{\nu,i}^{out}, \qquad \nu \in \mathcal{S}_r \tag{2}$$

$$0 = \dot{H}_i^{in} + \dot{H}_i^{f+a} - \dot{H}_i^{out} + \dot{Q}_{g,i}. \tag{3}$$

Here, $\dot{m}_{\nu,i}^{in}$ and $\dot{m}_{\nu,i}^{out}$ denote the mass flows of a component ν that enters and leaves the volume zone $i \in \{1, \ldots, N_v\}$. The extra mass flow $\dot{m}_{\nu,i}^{cb}$ enters the volume zone through the burners. Note that $\dot{m}_{\nu,i}^{in} = \dot{m}_{\nu,i-1}^{out} = \sum_{j=1}^{i-1} \dot{m}_{\nu,j}^{cb}$. Furthermore, $\dot{H}_i^{in}$ and $\dot{H}_i^{out}$ denote the enthalpy flows associated with the incoming and outgoing bulk flow, respectively. The enthalpy flow of fuel and combustion air is denoted by $\dot{H}_i^{f+a}$ and the net heat flow $\dot{Q}_{g,i}$ includes all thermal interactions (convection and radiation) of the flue gas with its environment, i. e., the remaining volume zones and the surrounding surfaces.

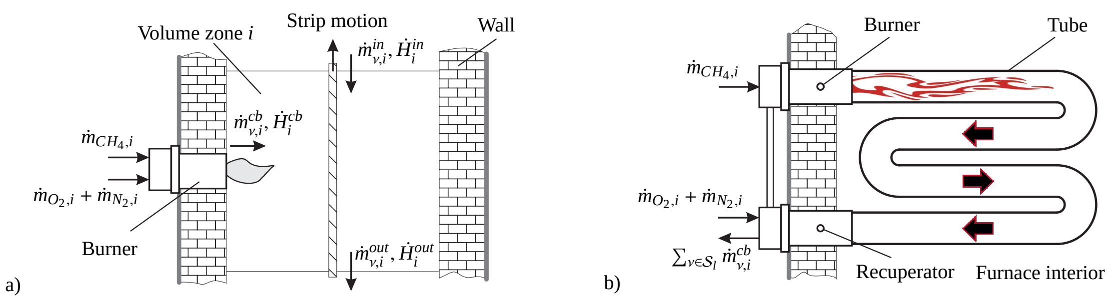

Fig. 2. a) Volume zone with moving strip. b) W-shaped radiant tube.

For determining the heat transfer by radiation and convection, the gas temperature in each volume zone i is required.

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Let $h_\nu(T) = h_\nu^0 + \Delta h_\nu(T)$ be the specific enthalpy of a component $\nu \in \mathcal{S}_r \cup \mathrm{S}_l$. Here, h_ν^0 and $\Delta h_\nu(T)$ denote the latent heat and the sensible heat, respectively. Thus, the enthalpy flow $\dot{H}$ of the flue gas reads as $\dot{H}(T) = \sum_{\nu \in \mathcal{S}_r} \dot{m}_\nu h_\nu(T)$. If the stationary mass balance (2) is used, the flue gas temperatures $T_{g,i}$, $i \in \{1, \ldots, N_v\}$ can be determined by solving the nonlinear set of algebraic equations, cf. Eq. (3),

$$0 = \sum_{\nu \in \mathcal{S}_r} \dot{m}_{\nu,i-1}^{out} \Delta h_\nu(T_{g,i-1}) - \sum_{\nu \in \mathcal{S}_r} \left(\dot{m}_{\nu,i-1}^{out} + \dot{m}_{\nu,i}^{cb} \right) \Delta h_\nu(T_{g,i}) + \sum_{\kappa \in \mathrm{S}_\kappa} \dot{m}_{\kappa,i} \Delta h_\kappa(T_{\kappa,i}) \quad \ldots$$
$$+ \sum_{\kappa \in \mathrm{S}_\kappa} \dot{m}_{\kappa,i} h_\kappa^0 - \sum_{\nu \in \mathcal{S}_r} \dot{m}_{\nu,i}^{cb} h_\nu^0 + \dot{Q}_{g,i} \tag{4}$$

W-shaped radiant tube

In the indirect-fired furnace, the fuel-lean combustion process takes place inside W-shaped radiant tubes. The radiant tubes are clustered into several heating zones. All radiant tubes of a single heating zone are supplied with the same amount of fuel and combustion air. As shown in Fig. 2b, a single radiant tube i consists of the burner, the tube, and the local recuperator for preheating the combustion air. The combustion heat is either transferred into the furnace chamber by thermal conduction through the tube wall or it is lost in form of sensible heat of the flue gas after passing the recuperator.

For the determination of the heat input $\dot{Q}_{c,i}$ into the furnace chamber, the radiant tube is considered as a quasi-static input/output system. The input quantities are the mass flow of fuel $\dot{m}_{CH_4,i}$ and the mass flow of combustion air $\dot{m}_{O_2,i}+\dot{m}_{N_2,i}$. The output quantities are the mass flow of the flue gas $\sum_{\nu \in \mathcal{S}_l} \dot{m}_{\nu,i}^{cb} = \sum_{\kappa \in \mathcal{S}_\kappa} \dot{m}_{\kappa,i} = \dot{m}_{CH_4,i} + \dot{m}_{O_2,i} + \dot{m}_{N_2,i}$ and the heat flow $\dot{Q}_{c,i}$ that follows from an energy balance in the form,

$$\dot{Q}_{c,i} = \sum_{\kappa \in S_\kappa} \dot{m}_{\kappa,i} h_\kappa^0 - \sum_{\nu \in S_l} \dot{m}_{\nu,i}^{cb} h_\nu^0 + \sum_{\kappa \in S_\kappa} \dot{m}_{\kappa,i} \Delta h_\kappa(T_{\kappa,i}) - \sum_{\nu \in S_l} \dot{m}_{\nu,i}^{cb} \Delta h_\nu(T_{c,i}). \tag{5}$$

Here, $T_{c,i}$ is the flue gas temperature after the recuperator. It can be measured only at a few points in each heating zone. Based on a linear interpolation of these values, the heat input $\dot{Q}_{c,i}(\dot{m}_{CH_4,i}, T_{c,i})$ of each radiant tube can be determined by Eq. (5).

For the following, it is assumed that the radiant tube consists of four straight pipes with a surface area S_r and a thickness d_r, cf. Fig. 2b). That is, the bendings of the radiant tube are neglected. Since the flue gas temperature decreases along the pipes $j = 1, 2, 3, 4$ of the radiant tube i, it is assumed that the heat input $\dot{Q}_{c,i}$ is (non-uniformly) distributed over the inner surface of each pipe according to weighting factors $g_{i,j} > 0$ with $\sum_{j=1}^4 g_{i,j} = 1$. Considering the Biot number [7], which is much smaller than unity, it is acceptable to model the pipe temperatures by simple heat balances, i. e., the temperature within the pipe wall is considered to be uniform. Let $T_{r,i,j}$ be the temperature of a single pipe j of the radiant tube i. The tube surface temperatures are required for the calculation of the radiative heat transfer inside the furnace chamber, see Sec. RADIATIVE AND CONVECTIVE HEAT TRANSFER. Thus, the complete model of a single radiant tube i consists of the ordinary differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t} T_{r,i,j} = \frac{1}{\rho_r c_r(T_{r,i,j}) d_r} \left(\dot{q}_{r,i,j} + \dot{q}_{c,i,j} \right) \tag{6}$$

of the four pipes $j = 1, 2, 3, 4$, where $\dot{q}_{c,i,j} = g_{i,j} \dot{Q}_{c,i} / S_r$ is the heat flux according to the combustion and $\dot{q}_{r,i,j}$ is the radiative heat flux from the furnace interior to the pipe. Moreover, ρ_r denotes the mass density and $c_r(T_{r,j,i})$ the temperature-dependent specific heat capacity of the pipe wall material.

Wall

The furnace wall consists of a steel casing and several well-insulated layers which are mainly made of fire clay, see Fig. 3a. In the indirect-fired furnace, a cladding which is an additional thin layer is attached to the inner surface to protect the insulation. Let J_i be the number of layers of a wall section i. A single layer j with $j = 1, \ldots, J_i$ is characterized by its thickness $d_{w,i,j}$ and its material parameters, i. e., the specific heat capacity $c_{w,i,j}$, the mass density $\rho_{w,i,j}$, and the heat conductivity $k_{w,i,j}$. At the outer wall surface, the temperature T_o is assumed to be equal to the constant ambient temperature, i. e., a Dirichlet boundary condition is used. At the inner wall surface, a Neumann boundary condition with the heat flux $\dot{q}_{w,i}$ due to thermal radiation and convection is assumed.

To obtain a low dimensional and computationally undemanding model of the furnace wall, the Galerkin weighted residuals method [8, 9] is applied. If the stationary solution of the heat conduction problem of a multi-layered furnace wall is used as a trial function, the lumped parameter model of the inner surface temperature $T_{w,i}$ of the wall segment i reads as

$$\frac{\mathrm{d}}{\mathrm{d}t} T_{w,i} = \frac{\dot{q}_{w,i}}{K_{1,i}} - \frac{K_{2,i}}{K_{1,i}} \left(T_{w,i} - T_o \right), \tag{7a}$$

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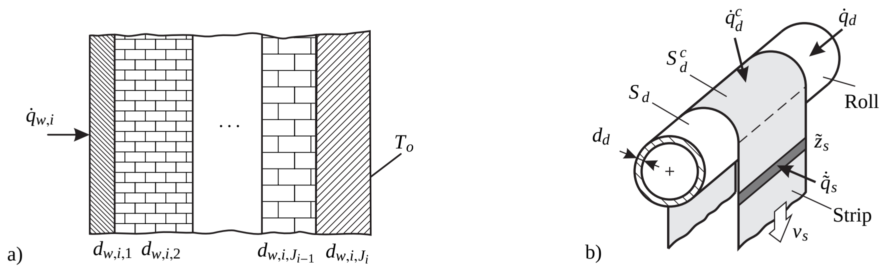

Fig. 3. a) Wall segment. b) Strip with deflection roll.

with the abbreviations

$$K_{1,i} = \left(\sum_{j=1}^{J_i} \frac{d_{w,i,j}}{k_{w,i,j}}\right)^{-2} \sum_{j=1}^{J_i} \frac{\rho_{w,i,j} c_{w,i,j} k_{w,i,j}}{3} \left(\left(\sum_{l=j}^{J_i} \frac{d_{w,i,l}}{k_{w,i,l}}\right)^3 - \left(\sum_{l=j}^{J_i} \frac{d_{w,i,l+1}}{k_{w,i,l+1}}\right)^3 \right) \tag{7b}$$

$$K_{2,i} = \left(\sum_{j=1}^{J_i} \frac{d_{w,i,j}}{k_{w,i,j}}\right)^{-1}, \tag{7c}$$

where $d_{w,i,J_i+1} = 0$ and $k_{w,i,J_i+1} = 0$. For more details, see [2]. Note that the stationary solution of Eq. (7a) equals the stationary solution of the underlying heat conduction problem.

Strip

The processed strip moves with the velocity v_s through the furnace and is mainly characterized by its geometry and material parameters. Let b_s be the width and d_s the thickness of the strip. The material parameters, i. e., the specific heat capacity c_s and the mass density ρ_s, are usually temperature dependent and may vary from strip to strip. In this analysis, the Lagrangian framework is used for describing the strip motion. In contrast to the Eulerian coordinate z which is spatially fixed, the Lagrangian coordinate $\tilde{z}$ is fixed to a material point of the strip. The mapping between these two coordinate systems can be described by $z = \tilde{z} + \int_{t_0}^{t} v_s(\tau)\mathrm{d}\tau$. Considering the Biot number [7] that is much smaller than unity, the strip temperature $T_{\tilde{s}}$ can be modeled by the simple heat balance

$$\frac{\mathrm{d}}{\mathrm{d}t} T_{\tilde{s}}(\tilde{z}, t) = \frac{2\dot{q}_{\tilde{s}}(\tilde{z}, t)}{\rho_s c_s d_s}, \tag{8}$$

where $\dot{q}_{\tilde{s}}(\tilde{z}, t)$ is the heat flux into the strip at the Lagrangian position $\tilde{z}$. For the calculation of radiative heat transfer, see Sec. RADIATIVE AND CONVECTIVE HEAT TRANSFER, the strip temperature is required in the Eulerian framework. Therefore, a simple mapping scheme, which transforms the strip temperatures to the Eulerian framework and the computed radiative heat fluxes back to the Lagrangian framework, is given in [2, 3].

The strip is guided through the furnace by means of deflection rolls, see Fig. 3. To model their temperature evolution, again heat balances and the assumption of uniform roll temperatures are used. A single roll i is constructed as hollow cylinder and is characterized by the mass density ρ_d, the thickness d_d, and the specific heat capacity c_d. Thus, the lumped parameter model of the deflection roll temperature $T_{d,i}$ reads as

$$\frac{\mathrm{d}}{\mathrm{d}t} T_{d,i} = \frac{1}{\rho_d c_d d_d} \left(\left(1 - \frac{S_d^c}{S_d}\right) \dot{q}_{d,i} + \frac{S_d^c}{S_d} \dot{q}_{d,i}^c \right), \tag{9}$$

where S_d is the total surface area of the roll and S_d^c is the contact area between the roll and the strip. The heat flux $\dot{q}_{d,i}$ for free roll surfaces includes the heat transfer mechanisms convection and radiation. The heat flux $\dot{q}_{d,i}^c$ for regions where the roll is in contact with the strip is determined by thermal conduction with the strip and can be expressed by

$$\dot{q}_{d,i}^c = \frac{T_s - T_{d,i}}{R_c},$$

where R_c is the thermal resistance and T_s corresponds to the strip temperature of the strip section that is in contact with the roll.

Radiative and convective heat transfer

Due to the high surface temperatures inside the furnace, thermal radiation is the dominant mode of heat transfer. It is a global phenomenon and interconnects most sub-models, i. e., the flue gas, the radiant tubes, the wall, the strip, and the rolls.

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Since the flue gas contains H_2O and CO_2 which are combustion products with non-symmetric patterns [4], it participates in the radiative heat exchange process. Thus, the direct-fired furnace can be considered as a multi-surface enclosure filled with a participating gaseous medium. A convenient means of analyzing this radiation problem is the zone method [10]. In contrast, the inert gas atmosphere in the indirect-fired furnace can be considered as a non-participating medium. In this case, the net-radiation method [11] is applied for modeling the radiative heat transfer.

Both the zone and the net-radiation method require spatial discretization of the furnace and assume that the surface and the volume zones have uniform temperature and act like diffuse gray radiators. Furthermore, the so-called direct-exchange areas have to be determined, which is the most intricate part of the calculation of radiative heat transfer. Generally, the determination of direct exchange areas requires the solution of multiple integrals. In this contribution, they are computed numerically. For the following, let $\mathbf{T} = [\mathbf{T}_j]_{j\in\{w,r,s,d\}}$ and $\mathbf{T}_g$ be the temperature vectors that summarize all surface and volume zone temperatures, respectively. Thus, the associated net heat flows $\dot{\mathbf{Q}} = [\dot{\mathbf{Q}}_l]_{l\in\{w,r,s,d\}}$ and $\dot{\mathbf{Q}}_g$ into the surface and volume zones can be expressed by

$$\dot{\mathbf{Q}} = \mathbf{P}_{ss}\mathbf{T}^4 + \mathbf{P}_{sv}\mathbf{T}_g^4 \tag{10a}$$

$$\dot{\mathbf{Q}}_g = \mathbf{P}_{vs}\mathbf{T}^4 + \mathbf{P}_{vv}\mathbf{T}_g^4, \tag{10b}$$

where the matrices $\mathbf{P}_{ss}$, $\mathbf{P}_{sv}$, $\mathbf{P}_{vs}$, and $\mathbf{P}_{vv}$ depend on the Stefan-Boltzmann constant, the emissivities, the attenuation coefficient, the direct-exchange areas, volumes, and the surface areas [10, 11]. The 4^{th} power in Eq. (10) is applied to each component of the respective vector. With the vector $\mathbf{S}$ that summarizes all surface areas, the required radiative heat fluxes $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_i]_{i=\{r,w,s,d\}}$ can be calculated in the form $\dot{\mathbf{q}} = \operatorname{diag}\{\mathbf{S}\}^{-1}\dot{\mathbf{Q}}$. Since the considered combined direct- and indirect-fired furnace is spatially decoupled by an air-lock in the transfer section, cf. Fig.1, and since different methods for the calculation of the radiative heat transfer inside the furnace are used, the matrices $\mathbf{P}_\sigma$ with $\sigma \in \mathcal{S}_\sigma = \{ss, sv, vs, vv\}$ have a distinct block structure. Note that the matrices depend on the strip width. For a computationally undemanding evaluation of Eq. (10), the matrices $\mathbf{P}_\sigma$, $\sigma \in \mathcal{S}_\sigma$, can be calculated in advance for various strip widths and stored. If other strip widths are required, linear interpolation might be used.

In contrast to radiation, heat transfer by convection is a local phenomenon and describes the heat exchange between the flue gas and its surrounding surfaces, especially the strip and the wall surfaces. The convective heat flux $\dot{q}_c$ can be calculated by means of the temperature difference and the (local) heat transfer coefficient α [7],

$$\dot{q}_c = \alpha\,(T_g - T)\,. \tag{11}$$

The heat transfer coefficient depends on the fluid-dynamical conditions near the considered surface. It can be determined by assuming a flat plate and a parallel flow and by using dimensionless numbers from fluid dynamics.

Discrete-time representation

The individual sub-models are defined in the continuous time domain. However, for the implementation on a computer system, the time domain has to be discretized. In this analysis, Euler's explicit method [12] with the sampling time Δt_k is used. With the heat transfer relations (10) and (11), the discrete-time representation of the mathematical model reads as

$$\mathbf{0} = \mathbf{g}_k(\mathbf{T}_{g,k}, \mathbf{T}_k, \mathbf{u}_k) \tag{12a}$$

$$\mathbf{T}_{k+1} = \mathbf{T}_k + \Delta t_k \mathbf{f}_k(\mathbf{T}_{g,k}, \mathbf{T}_k, \mathbf{u}_k). \tag{12b}$$

Here, Eq (12a) represents the nonlinear system of algebraic equations for determining the flue gas temperatures $\mathbf{T}_{g,k}$ depending on the surface temperatures $\mathbf{T}_k$ at the time instant $t = t_0 + \sum_{n=1}^{k} \Delta t_k$, cf. Eq. (4), and Eq. (12b) includes the sub-models of the radiant tubes (6), the wall (7a), the strip (8), and the deflection rolls (9). Furthermore, $\mathbf{u}_k$ summarizes the control and exogenous inputs, i. e., the mass flows of fuel to the heating zones of the direct- and indirect-fired furnace and the measured flue gas temperatures after the recuperaturs of each radiant tube.

EXAMPLE PROBLEM

For validation of the proposed furnace model, a measurement campaign was conducted at the real plant. In normal furnace operation, the strip temperature is measured by means of three radiation pyrometers (cf. Fig. 1 for their locations). Furthermore, the flue gas temperature inside the direct-fired furnace and several temperatures of radiant tubes and wall temperatures are measured by thermocouples. The system inputs, i. e., the mass flows of fuel to the heating zones and the flue gas temperatures of radiant tubes, are also available from measurements.

A radiation pyrometer measures the intensity $I_p = \sigma\varepsilon T_{\tilde{s}}^4$ incident on its detector. For a correct determination of the strip temperature $T_{\tilde{s}}$, the emissivity ε of the processed strip has to be known [13, 14]. Since the real emissivity is generally unknown and difficult to determine, it is a good choice for parameter identification. In this contribution, an estimate $\hat{\varepsilon}$ is identified to achieve an optimum match between the temperature $\hat{T}_{\tilde{s}} = \sqrt[4]{I_p/\sigma\hat{\varepsilon}}$ and its simulated counterpart $T_{\tilde{s}}$.

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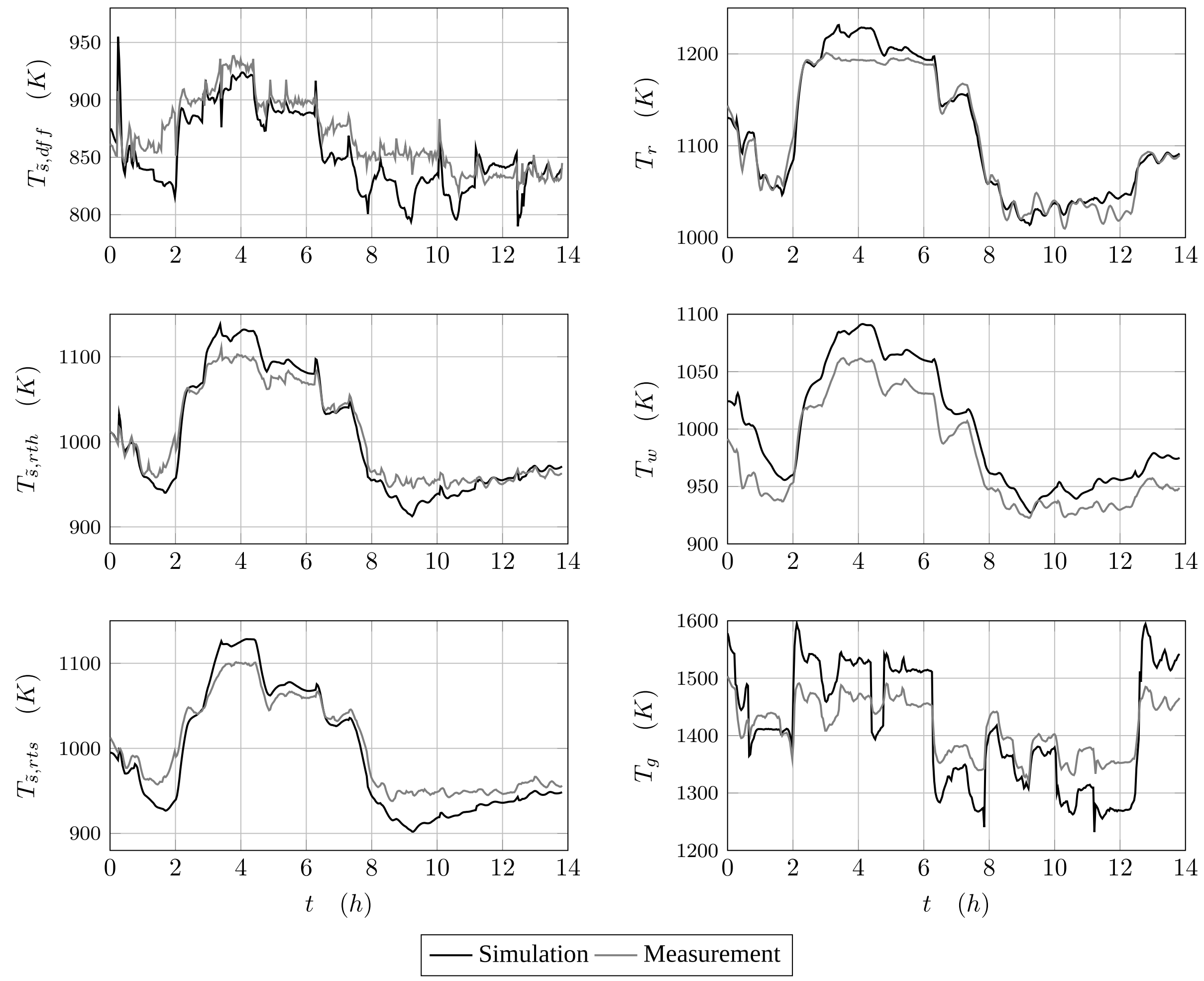

Fig. 4. Left-hand side: Strip temperatures at the pyrometer positions P_{dff}, P_{rths}, and P_{rts} given in Fig. 1. Right-hand side: Temperatures of a radiant tube, a wall segment, and the flue gas inside a heating zone of the direct-fired furnace.

During the measurement campaign 30 different strips were processed. The strip thickness varied in the range $d_s \in [0.4, 0.8]\,\mathrm{mm}$ and the strip width in the range $b_s \in [1.2, 1.5]\,\mathrm{m}$. Moreover, the strip velocity is in the range $v_s \in [1.9, 3]\,\mathrm{m/s}$. On the left-hand side of Fig. 4, the strip temperatures $T_{\tilde{s},dff}$, $T_{\tilde{s},rth}$, and $T_{\tilde{s},rts}$ at the three pyrometer positions, cf. Fig. 1, are shown. As can be inferred from Fig. 4, the simulated strip temperatures match the measured values with high accuracy. This is especially true for the strip temperatures in the indirect-fired furnace, which are most relevant for the product quality. The model accuracy can be further improved by a more detailed specification of the material parameters. In particular, the emissivity ε and the specific heat capacity c_s have a great influence on the evolution of the strip temperature. On the right-hand side of Fig. 4, the temperatures of a radiant tube, a wall segment, and the flue gas inside a heating zone of the direct-fired furnace are shown. Again, the simulated temperatures have a good agreement with the measurements.

The furnace model has been implemented and is now used for real-time simulations on a standard desktop PC (4 GHz, 16 GB RAM) with an equidistant sampling rate of $\Delta t_k = 250\,\mathrm{ms}$. On this computer, the simulation of $1\,\mathrm{h}$ furnace operation requires approximately $80\,\mathrm{s}$ CPU-time. Hence, the presented furnace model is suitable for real-time applications in control and optimization. The furnace model is currently used as an add-on device for permanent online monitoring of process variable that cannot be measured.This application of the proposed model provides the furnace operators with new insight into the annealing process and thus helps to further increase the product quality and the throughput.

Outlook

In the following, some of the intended applications of the presented furnace model are listed:

- The model is based on first principles and captures the heat flows inside the furnace accurately. For this reason, it can be used for a detailed analysis of the furnace efficiency, cf. [15, 16]. Furthermore, all model parameters have a physical

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meaning which makes the model useful for supporting the design process of new furnaces, i. e., the effect of individual parameters on the strip temperature can be verified very quickly.

- For the considered annealing furnace, a state observer should be developed. It estimates process quantities that cannot be measured, see, e. g., [17]. Such an estimation is based on available process data and measurements from the annealing furnace. The output of the estimator can be used for model-based diagnosis, e. g., the validation of measurements (soft sensing) and fault detection (condition monitoring).
- For modern model-based strip temperature control, a reduced mathematical model that includes the most important nonlinearities of the system may be required. The envisaged control concept should primarily ensure that the temperature of each strip section follows a desired trajectory, especially during transient furnace operation. For such a control system, measurements from the furnace and the estimator output are required in real time.
- The furnace model, the estimator, and the controller can be used in a simulation environment. This allows the furnace operator to simulate and optimize future operating scenarios in advance.

Conclusion

A tractable mathematical model of a strip annealing furnace is presented. The considered annealing furnace is part of a hot-dip galvanizing line of voestalpine Stahl Gmbh at Linz, Austria, and is used for heat treatment of steel strip. The focus of this paper was to interconnect the two furnace models presented in [2, 3] and to provide the most important equations necessary for the implementation on a computer system.

The considered strip annealing furnace consists of a direct- and an indirect-fired furnace section, which are physically coupled by the moving steel strip. Inside the direct-fired section, natural gas is burnt in a fuel-rich combustion process. The temperature of the flue gas is calculated by means of quasi-static mass and enthalpy balances. In a similar way, the heat which is released by a fuel-lean combustion process inside the radiant tubes of the indirect-fired furnace section is determined. The heat conduction equations of the radiant tube walls and the furnace walls are discretized by means of the Galerkin method. Furthermore, heat balances are employed to model the temperature evolution of the guiding rolls and the strip. The individual thermal sub-models are interconnected by the heat transfer mechanisms conduction, convection, and thermal radiation.

The main advantage of the presented mathematical model is that it is a good compromise between accuracy and computational effort and it is thus suitable for advanced model-based strip temperature control. Since all model parameters, variables, and sub-models have a clear physical meaning, the chosen first-principles approach facilitates an easy transfer of the model to similar furnaces.

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