Optimization-based feedforward control of the strip thickness profile in hot strip rolling

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Abstract
A new feedforward control approach for the thickness profile of the strip in a tandem hot rolling mill is developed. In industry, the automatic gauge control (AGC) concept is widely used for thickness control. The AGC has the disadvantage that it does not consider known disturbances from upstream entities. This is why a number of disturbance feedforward control concepts have been proposed in the literature. These feedforward control strategies typically rely on linearized models and only provide symmetric control inputs for the mean thickness to the hydraulic adjustment system. In this work, an optimization-based feedforward controller for the lateral thickness profile is proposed that fully exploits all degrees of freedom available, i.e., the hydraulic cylinder positions and the bending forces at the drive side and at the operator side of the mill stand. Moreover, it is shown that by linearizing the mill stand model while keeping the nonlinearities from the roll gap model leads to a numerically efficient optimization problem, which is a good compromise between accuracy and computational efficiency. The feedforward controllers are further combined with the AGC in the feedback path in a two-degree-of-freedom controller structure to account for model-plant mismatch. Simulation results for a validated mathematical model and first experimental results from an industrial pilot installation show a significant benefit compared to the existing AGC without feedforward control.

Keywords: Hot strip rolling, Metals industry, Thickness control, Shape and profile control, Model-based optimization, Feedforward control

Nomenclature

Abbreviations

AGC Automatic gauge control
BR Backup roll
CVC Continuous variable crown
DS Drive side
FF Feedforward control
HGC Hydraulic gap control
MIMO Multiple input, multiple output
OS Operator side
SISO Single input, single output
TCW Thermal and wear crown
WR Work roll
WRB Work roll bending
WRS Work roll shifting

Variables

$\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ Cartesian coordinates
$\tilde{b}$ Parameter used in cost function
$b_{\text{br}}$ Distance between hydraulic cylinders
$b_c$ Face length of backup roll
$b_R$ Material width
$b_{\text{wrb}}$ Distance between hydraulic WRB cylinders
$c_m$ Mill modulus
c_i Crown of the roll
c $\varepsilon$ Thickness error defined in cost function
$F_B$ Force applied to the BR bearing
$F_f$ Frictional force
$F_R$ Roll force
$F_{\text{bal}}$ Balancing force
$F_h$ Force of the hydraulic main cylinder
$f_R$ Roll gap model
$f_s$ Static mill stand model
$F_{\text{wrb}}$ Work roll bending force
g Gravitational acceleration
$h_0$ Height of unloaded roll gap
$h_{\text{en}}$ Material entry thickness
$h_{\text{ex}}$ Material exit thickness
$k_{\text{fB}}$ Feedback gain of AGC
$k_{0}, m_1, m_2, m_3$ Coefficients for yield stress
$K_{B,i}$ Bending stiffness of the beam $i$
$k_{\text{fB}}$ Yold stress
$K_{S,i}$ Shear stiffness of the beam $i$
$L$ Length of the finished strip
$l_d$ Length of the contact arc
$m$ Mass of the moving parts (upper roll stack)

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In hot strip rolling, a central objective is the production of strips with an accurate and uniform thickness. Therefore, at each mill stand of the tandem hot rolling mill considered in this paper, the strip thickness can be adjusted by hydraulic cylinders that move the upper roll stack. Additionally, the mill stands are equipped with a work roll bending system (WRB) to compensate for the bending deflection of the work rolls and to control the strip crown.

1.1. Literature review

The widely used automatic gauge controller (AGC) causes a non-zero steady-state control error of the strip exit thickness for any entry disturbance or deviation from its operating point because of its limited feedback gain, [3; 7; 10]. The disturbance feedforward control concept proposed in [24] estimates the strip temperature and thickness at upstream mill stands and adjusts the setpoint for the hydraulic cylinder position at downstream mill stands. This position adjustment is derived from a linearized model, where only the mean thickness value but not the lateral profile is considered. An additional cylinder position that is equal for both sides of the mill stand is used as control input (SISO FF). In [14], it is suggested to additionally use the bending force in feedforward control. In other feedforward concepts, the additional hydraulic cylinder position is obtained based on an estimation of the yield stress of the strip material, [5; 11]. In [17; 18], the variations of rolling conditions, e.g., the measured roll force, are divided into variations caused by the roll eccentricities and those caused by strip temperature inhomogeneities. It is possible to identify and distinguish the root of these variations in the frequency domain because the distance between the skid marks and the revolution speed of the rolls are known.

The control concepts known from literature targeting the strip shape, including the flatness of the strip, are merely feedback control structures. In [2], the measurement from a downstream shape meter is used to control the strip flatness using an actuator influence matrix. This causes a dead time (transport delay) in the controlled plant. In fact, the distance between measurement and control input can be very large, like in tandem rolling, where the shape meter measurement is typically located several meters after the last mill stand. A model predictive control (MPC) approach is used in [2] to obtain the control inputs for each mill stand. The system synchronizes the control action to compensate for the transport delay of the strip. A MIMO control method for the strip flatness based on roll bending is discussed in [21].

Other research works, see, e.g., [8; 20], deal with finding the optimal interstand crowns. The aim is to prevent wavy edges and center buckles and to determine the control parameters for the Level 2 setup. These setup control parameters are constant within each strip and thus disregard inhomogeneities of the strip in longitudinal direction.

1.2. Motivation and objective of this paper

The main goal of this paper is to develop a thickness control strategy that realizes the desired target exit thickness profile over the complete length of the strip as accurately as possible. The deviation of the lateral exit thickness profile from a desired profile should be systematically minimized using all available control inputs.
i.e., the hydraulic cylinder position of the backup rolls at both sides of the mill stand, operator and drive side, and the bending forces at both sides of the work rolls. The proposed concept is a multi-input multi-output feedforward (MIMO FF) controller that yields the optimal transient control inputs, and will be described in Section 4. To calculate the (expected) exit thickness profile for the measured disturbances, a mathematical model of the mill stand is required. The model presented in Section 2 uses well-known sub-models (Sims roll gap model, Hensel-Spitell material model, Hertzian contact model, Timoshenko beam model).

2. Mathematical model of the mill stand

In this section, a detailed mathematical model of a single mill stand of the considered tandem hot rolling mill is presented. This model is used for simulation and is validated based on measurement results from the real plant. Parts of this model serve as a basis for the control design given in Section 4.

The upper roll stack of the considered mill stand is outlined in Fig. 1. The mill stand consists of the upper and lower work roll (WR) and the upper and lower backup roll (BR). In the roll gap between the work rolls, the steel strip is deformed. The backup rolls reduce the bending deformation of the work rolls. The position of the upper BR is adjusted by two hydraulic cylinders, one at the drive side (DS) and one at the operator side (OS), to control the height of the roll gap. An important output of this model is the lateral exit thickness profile of the strip. For the thickness profile, the impact of the WRB system is significant, and the continuous variable crown shape (CVC) of the rolls, [25], the thermal crown and wear (TCW) profile of the rolls, [29; 1], and the local flattening of the rolls, [9], have to be considered. The roll gap crown is adjusted by axial movement of the CVC work rolls (WRS, work roll shifting). This shifting position is kept constant within each strip and thus is not a degree of freedom for control or optimization.

The main parts of the proposed model are the roll gap model, the static model of the deformation of the rolls and the mill housing, the dynamic model of the hydraulic gap actuators, and the existing subordinate controllers. These parts of the model are now described in detail, see also Fig. 2.

2.1. Roll gap model

The mathematical relation between the local roll force \( q_{\text{roll}} \), the strip entry \( h_{\text{en}} \) and exit thickness \( h_{\text{ex}} \), the strip tensions at the entry \( p_{\text{en}} \) and exit side \( p_{\text{ex}} \), the rolling velocity \( u_R \), and the yield stress \( k_{\text{fm}} \) of the strip material is referred to as roll gap model, which in general is written in the implicit form

\[
F_R(q_{\text{roll}}, h_{\text{en}}, h_{\text{ex}}, p_{\text{en}}, p_{\text{ex}}, u_R, k_{\text{fm}}) = 0. \tag{1}
\]

In the static mill stand model, the roll force \( q_{\text{roll}} \) depending on the exit thickness \( h_{\text{ex}} \) (and the other parameters) is required. For this purpose, \( q_{\text{roll}} \) is numerically computed. The rolling conditions in general vary over the strip width, so all the parameters in (1) depend on the lateral coordinate \( x \), that is, the roll gap model is evaluated for \( -\frac{b_R}{2} \leq x \leq \frac{b_R}{2} \) to obtain the distribution of the local roll force \( q_{\text{roll}}(x) \).

According to [12], the mean yield stress \( k_{\text{fm}} \) of the strip material can be approximated in the form

\[
k_{\text{fm}} = k_0 e^{-m_1 T} \varphi^{m_2} \varphi^{m_3}, \tag{2}
\]

where \( T \) is the strip temperature, \( \varphi = \ln \left( \frac{h_{\text{ex}}}{h_{\text{en}}} \right) \) is the deformation degree, and \( \varphi \) is its time derivative (the deformation rate). The mean deformation rate is proportional to the rolling velocity \( u_R \); \( \varphi = \frac{u_R}{L_{\text{contact}}} \) with the length of the contact arc \( L_{\text{contact}} \). The parameters \( k_0, m_1, m_2, m_3 \) depend on the specific material and have to be identified for each strip material, e.g., by minimizing the deviation between the measured and the calculated roll force.

As roll gap models based on Sims’ model [27] are commonly used in the steel industry, Sims’ model is also applied in this paper but in an extended form that facilitates the consideration of the up- and downstream strip tension, [4]. However, the interface of (1) allows an easy substitution of this roll gap model with other models.

Figure 1: Section of upper roll stack (front view).

Figure 2: Structure of the dynamic simulation model.
2.2. Static mill stand model

To model the bending deflection of the rolls, the four rolls are considered as Timoshenko beams [15; 19]. In the following, the abbreviation \( s_i = \frac{x_i}{b_i} \) is used. The differential equations for the deflection \( v(x) \) along the direction \( y \), the angle \( \varphi(x) \) of rotation of the cross section, the bending moment \( M(x) \), the shear force \( q(x) \), and the distributed load \( q_i(x) \) read as

\[
\begin{align*}
\frac{d^2}{dx^2} v_i &= -\varphi_i + \frac{Q_i}{K_{x,i}} \quad \frac{d^2}{dx^2} \varphi_i &= \frac{M_i}{K_{b,i}} \\
M_i &= \frac{Q_i}{Q_{w,i}} \quad Q_i &= -q_i(v_{br,u} + v_{wr,u} + v_{wr,l} + v_{br,l}) .
\end{align*}
\]

(3)

The variable \( i \) specifies the four rolls, upper BR and WR and lower WR and BR,

\[
i \in \{ br, u; wr, u; wr, l; br, l \} .
\]

(4)

\( K_{x,i} \) and \( K_{b,i} \) are the bending and the shear stiffnesses of the rolls, respectively. They are calculated for the geometry of the rolls as outlined in Fig. 1, considering the different material properties of the soft core and the hard boundary layer of the rolls [28]. Together with the boundary conditions (see next paragraph), this results in a 16-dimensional nonlinear boundary value problem (4 equations for each roll). These beams are coupled by the load \( q_i \), which depends on the roll force \( q_{roll} \) from Section 2.1 between the WR and the strip and the contact forces \( q_{c,u} \) and \( q_{c,l} \) between the WR and the BR,

\[
q_{br,u} = q_{c,u} \\
q_{wr,u} = q_{roll} - q_{c,u} \\
q_{wr,l} = -q_{roll} + q_{c,l} .
\]

(5)

For modeling the contact between WR and BR, the Hertzian theory of elastic contact is used. It gives \( q_{c,u} \) and \( q_{c,l} \) depending on \( \delta_{bw,u} \) or \( \delta_{bw,l} \), respectively, [13]. The compression \( \delta_{bw} \) between two cylindrical bodies, i.e., the rolls that are in contact, is depending on \( v_i \) and is equal to

\[
\delta_{bw,u} = v_{wr,u} + c_{wr,u} - v_{br,u} + c_{br,u} \\
\delta_{bw,l} = v_{br,l} - c_{br,l} + c_{wr,l} + v_{wr,l} .
\]

(6)

where \( c_i \) is the total crown of the roll \( i \) (additional radius of the rolls) that includes the roll crown due to TCW, the manufactured CVC, and tapered roll ends. For the work rolls, the total local roll crown depends on the WRS \( s_{wrs} \).

The link between the gap model (1) and the roll stack deflection model (3) is the exit thickness \( h_{ex} \). It follows in the form

\[
h_{ex} = h_0 + (v_{wr,u} - c_{wr,u}) - (v_{wr,l} + c_{wr,l}) + 2\delta_{wr} .
\]

(7)

In (7), \( h_0 \) is the straight line connecting the roll gap height \( h_{DS}^{0} \) at the DS and \( h_{OS}^{0} \) at the OS,

\[
h_0(x) = h_{DS}^{0} + \frac{h_{OS}^{0} - h_{DS}^{0}}{h_{br}} \left( x + \frac{b_{br}}{2} \right) .
\]

(8)

As shown in Fig. 1, \( b_{br} \) is the distance between the HGC cylinders. The heights at the DS and at the OS depend on the cylinder positions \( x_{wrs}^{DS} \) and \( x_{wrs}^{OS} \). In a calibration routine, the position of the HGC cylinders is recorded, when the work rolls are in contact and a defined force is applied. In this state, the roll gap heights are \( h_{DS}^{0} = h_{OS}^{0} = 0 \) and the calibration offsets \( \xi_{cal}^{DS} \) and \( \xi_{cal}^{OS} \) for the considered set of rolls are determined. So, the relations between the heights, \( h_{DS}^{0} \) and \( h_{OS}^{0} \), and the cylinder positions, \( x_{wrs}^{DS} \) and \( x_{wrs}^{OS} \), are

\[
\begin{align*}
\xi_{cal}^{DS} &= x_{wrs}^{DS} - h_{DS}^{0} \quad \xi_{cal}^{OS} &= x_{wrs}^{OS} - h_{OS}^{0} .
\end{align*}
\]

(9)

In (7), \( h_{0} - c_{wrs,u} - c_{wrs,l} \) is the height of the unloaded roll gap with the roll crowns \( c_{wrs,u} \) and \( c_{wrs,l} \) of the upper and lower work roll. With \( \delta_{wr} \) in (7), the local flattening of the WR is considered. This local flattening is computed according to Boussinesq and Cerruti, [9; 13]. The pressure distribution along the arc of contact and the strip width is approximated by concentrated forces and the superposition of the corresponding deformation is computed.

The boundary value problem (3) requires 16 boundary conditions. The 8 boundary conditions for the WR are

\[
\begin{align*}
Q_{w,u}(\pm b_{wrs} / 2 + s_{wrs}) &= -F_{DS}^{w} \\
Q_{w,l}(\pm b_{wrs} / 2 + s_{wrs}) &= -F_{DS}^{w} \\
M_{w,u}(\pm b_{wrs} / 2 + s_{wrs}) &= 0 \\
Q_{w,r}(\pm b_{wrs} / 2 + s_{wrs}) &= -F_{DS}^{w} \\
Q_{w,l}(\pm b_{wrs} / 2 - s_{wrs}) &= -F_{DS}^{w} \\
M_{w,r}(\pm b_{wrs} / 2 - s_{wrs}) &= 0 ,
\end{align*}
\]

(10)

where \( b_{wrs} \) is the distance between the bearings of the OS and the DS, and \( F_{DS}^{w} \) and \( F_{DS}^{w} \) are the bending forces applied by the hydraulic cylinders at the DS and at the OS. The parameter \( s_{wrs} \) is the shifting position of the WR. For \( s_{wrs} > 0 \), the upper WR is shifted by the distance \( s_{wrs} \) towards the OS, and the lower WR is displaced by \( -s_{wrs} \) towards the DS. Hence, the bending forces are applied at the center of the bearings \( x = \pm \frac{b_{wrs}}{2} \pm s_{wrs} \). The bending moment \( M_{w} \) is zero at these points. Fig. 1 shows the upper roll stack with \( s_{wrs} = 0 \) and the applied forces. The remaining 8 boundary conditions for the BR are given at the position of the hydrodynamic bearings, \( x = -\frac{b_{br}}{2} \) at the DS, and \( x = -\frac{b_{br}}{2} \) at the OS,

\[
\begin{align*}
Q_{b,u}(\pm b_{wrs} / 2) &= F_{BR}^{w} \left( v_{br,u}(\pm b_{wrs} / 2) - \delta_{DS}^{b} \right) \\
Q_{b,l}(\pm b_{wrs} / 2) &= -F_{BR}^{w} \left( v_{br,l}(\pm b_{wrs} / 2) - \delta_{DS}^{b} \right) \\
M_{br,u}(\pm b_{wrs} / 2) &= M_{br,r}(\pm b_{wrs} / 2) = 0 \\
v_{br,l}(\pm b_{wrs} / 2) &= -\delta_{DS}^{b} \\
v_{br,r}(\pm b_{wrs} / 2) &= -\delta_{DS}^{b} .
\end{align*}
\]

(11)

The force \( F_{BR} \) as a function of the vertical position \( v_{br,u} \) at the bearings \( \pm b_{wrs} / 2 \) is obtained from a measured force-deflection curve of the mill stand frame. Additionally,
in \( \delta_{DS}^{B,u}, \delta_{OS}^{B,u}, \delta_{DS}^{B,l} \) and \( \delta_{OS}^{B,l} \) the displacement of the hydrodynamic journal bearings of the backup rolls are included. This displacement, depending on the applied force \( F_B \) and the revolution speed of the BR, is taken from a lookup-table. This lookup-table is based on measurements of the displacement at reference forces and speeds and was provided by the plant operator.

A numerically efficient solution method for the boundary value problem (3), (10), (11) of the four Timoshenko beams is presented in Appendix A. In the following, the two outputs roll force \( F_R = \int_{-\frac{b_R}{2}}^{\frac{b_R}{2}} q_{\text{coll}} \, dx \) and exit thickness profile \( h_{ex}(x) \) from (7) are used. As an abbreviation for this solution of the static mill stand model, we will write

\[
h_{ex}(x) = f_s(x_h^{DS}, x_h^{OS}, q_{\text{coll}}, F_{\text{urb}}, F_{\text{OS}}, u_R, x, \xi_{\text{cm}}) = 0
\]

(12)

for the rest of this paper.

The calculated profile \( h_{ex}(x) \) of the model was validated by comparing it to the exit thickness profile measured by a downstream thickness measurement device. Fig. 3 shows the results for a representative strip for a certain longitudinal coordinate \( x \) of the strip. The agreement between the measurement and the model results is very satisfactory for most parts of the strip. Only at the strip edges, some small deviations in the range of up to 50 \( \mu \text{m} \) do occur. They are most likely due to the simplifications of the rolling conditions as the three-dimensional edge effects of the deformation are not considered. Since there is no control input that only acts at these edge zones, this model-plant mismatch is acceptable for a model that will be mainly used for control purposes. A comparison of several strips and over the complete length of the strips was made for validation, and resulted in a similar model accuracy, that is, less than 50 \( \mu \text{m} \) of maximum deviations of the strip profile.

2.3. Dynamic model of the hydraulic gap actuators

For simulation of the actual rolling process, a description of the dynamic behavior of the hydraulic actuators is needed, i.e., the differential equations for the pressures in the hydraulic cylinders, and the dynamics and characteristics of the servo valves. The hydraulic forces \( F_h^{DS} \) and \( F_h^{OS} \) applied by the HGC cylinders to move the upper roll stack is given by the pressure conditions in the cylinder chambers. The movement \( x_h^{DS} \) and \( x_h^{OS} \) of the upper roll stack with the mass \( m \) is described by the momentum balance

\[
\frac{m}{2} \frac{d^2}{dt^2} x_h^{DS} = F_h^{DS} - F_B^{DS} - \frac{F_{\text{bal}}}{2} + \frac{m}{2} g + F_f^{DS}
\]

(13)

including the hydraulic forces \( F_h^{DS} \) and \( F_h^{OS} \), the bearing forces \( F_{B,h}^{DS} \) and \( F_{B,h}^{OS} \), the balancing force \( F_{\text{bal}} \), the gravitational force \( mg \), and some frictional forces \( F_f^{DS} \) and \( F_f^{OS} \). The latter may occur between the mill frame and the chocks of the roll bearings as well as in the hydraulic cylinders. These friction forces were identified in a measurement campaign. The bearing forces \( F_{B,h}^{DS} \) and \( F_{B,h}^{OS} \) are the outputs of the static mill stand model from the load of the upper BR of (11). These forces are obtained from the solution of the boundary value problem and they sum up the roll force \( F_R \) and the bending forces, see Fig. 1.

\[
F_{B,h}^{DS} + F_{B,h}^{OS} = F_R + F_{\text{urb}} + F_{\text{OS}}.
\]

(14)

2.4. Subordinate controllers

The positions \( x_h^{DS} \) and \( x_h^{OS} \) of the hydraulic cylinders are controlled by subordinate control loops according to [16], also referred to as HGC (hydraulic gap control). The reference signals of these HGC loops are the desired positions of the hydraulic cylinders \( x_{h^{DS,d}} \) and \( x_{h^{OS,d}} \), which are typically provided by the AGC [6; 23; 30]. In the AGC, the deviations \( \Delta \) of the force and position measurements from their reference values are used to estimate the deviation of the average exit thickness

\[
\Delta h_{ex} = \frac{\Delta x_{h}^{DS} + \Delta x_{h}^{OS}}{2} + \Delta F_{R}^{DS} + \Delta F_{R}^{OS} + \Delta h_{ex}^d,
\]

(15)

where \( c_m \) is the mill modulus that is used for a linear approximation of the deformation of the mill stand \( \Delta F_{R}^{DS} + \Delta F_{R}^{OS} \). The estimation of \( \Delta h_{ex} \) is used in a proportional control with the feedback gain \( k_B \)

\[
\Delta x_{h}^{\text{agc}} = k_B \left( \Delta h_{ex} - \Delta h_{ex}^d \right)
\]

(16)

for an additional cylinder position at both sides

\[
\begin{align*}
\Delta x_{h}^{DS,d} &= x_{h}^{DS} + \Delta x_{h}^{agc} \\
\Delta x_{h}^{OS,d} &= x_{h}^{OS} + \Delta x_{h}^{agc}
\end{align*}
\]

(17)

added to the reference for the cylinder positions, \( x_{h}^{DS} \) and \( x_{h}^{OS} \), to compensate for the deflection of the mill stand. The AGC is typically used in industry for thickness control and will thus be used as a benchmark for the controller developed in this paper. Because the feedback of

Figure 3: Comparison of the model and the measurement of the exit thickness profile at the last mill stand for a representative strip.
the AGC (16) is proportional to the thickness error, disturbances like a varying strip temperature yield a non-zero steady-state control error, [23]. To systematically reduce this control error and to improve the control performance, SISO and MIMO feedforward control strategies are developed in the next sections.

3. Simple SISO feedforward control strategy

In this section, a simple approach of a linearized feedforward control strategy at the first mill stand is described. Similar methods have already been proposed in, e.g., [5; 11; 14; 24]. In the simulation scenarios presented in Section 6, the SISO feedforward controller will be compared to the proposed optimization-based MIMO feedforward controller (Section 4).

In the considered rolling process, the rolling conditions, i.e., the temperature and the entry thickness of the strip, as well as the rolling velocity, can vary over the strip length. These inputs influence the roll force and the yield stress, (1), (2). The entry inhomogeneities of the strip temperature and the strip thickness are measured by pyrometers and by a thickness measurement device, respectively, before the first mill stand in the considered tandem mill. The rolling velocity is known from the main drives of the WRs at each mill stand. The feedforward controller makes use of this information known ahead.

By linearizing the roll force model (1) and the material model for the yield stress (2), the expected roll force difference \( \Delta F^R_R \) from the operating point is obtained in the form

\[
\Delta F^R_R = \Delta h_{en} \frac{\partial F_R}{\partial h_{en}} + \Delta T_{en} \frac{\partial F_R}{\partial T_{en}} + \Delta u_R \frac{\partial F_R}{\partial u_R} .
\]

(18)

The terms \( \frac{\partial F_R}{\partial h_{en}} \), \( \frac{\partial F_R}{\partial T_{en}} \), and \( \frac{\partial F_R}{\partial u_R} \) are the scalar sensitivities of the roll force with respect to variations of the inputs. They are calculated numerically by central difference quotients from (1) and (2) at the operating point. The deviations \( \Delta h_{en}, \Delta T_{en}, \) and \( \Delta u_R \) from their operating point are obtained from the measurements (in the lateral center of the strip). The expected roll force difference \( \Delta F^R_R \) from (18) causes a deflection \( \Delta x_{h}^D \) of the mill stand, which can be compensated by an additional cylinder position

\[
\Delta x_{h}^D = \frac{\Delta F^R_R}{2c_m} - \Delta h_{ex}^d ,
\]

(19)

where the same linearization of the mill stand deformation as in the thickness estimation (15) is used. The position \( \Delta x_{h}^D \) is added to the reference position at the DS and the OS

\[
x_{h}^{DS,ff} = \bar{x}_{h}^{DS} + \Delta x_{h}^{ff} \\
x_{h}^{OS,ff} = \bar{x}_{h}^{OS} + \Delta x_{h}^{ff} .
\]

(20)

Thus, the compensation \( \Delta x_{h}^{ff} \) of the (expected) mill stretch is symmetric.

The feedforward approach can be combined with the AGC to a two-degree-of-freedom controller as will be described in Section 5. The advantage of this SISO approach is that the implementation of (18) and (19) is simple and the required measurements of \( \Delta h_{en}, \Delta T_{en}, \) and \( \Delta u_R \) are already available. However, the control law is based on the linearized roll gap model (18), which becomes inaccurate for larger deviations from the operating point. Additionally, the lateral exit thickness profile is not considered and the control input is symmetric for the DS and the OS. An asymmetric control input of the HGC cylinders is important for possibly occurring asymmetric rolling conditions, e.g., for wedge-shaped thickness profiles or for a strip rolled outside of the lateral center of the roll gap. In [22], an asymmetric feedforward approach was proposed. In the considered rolling mill, the WR bending forces \( F_{wrb} \) are also available as control inputs. In the next section, a more advanced approach of the feedforward controller is developed.

4. Optimization-based feedforward control strategy

The control strategy developed in the following is a feedforward compensation of measured or estimated disturbances from upstream entities using all available control inputs in a systematic way. The control inputs are the positions \( x_{h}^{DS} \) and \( x_{h}^{OS} \) of the hydraulic cylinders and the bending forces \( F_{wrb} \), which are combined in the input vector

\[
u = [x_{h}^{DS}, x_{h}^{OS}, F_{wrb}, F_{wrb}]^T .
\]

(21)

The considered disturbances are the entry temperature profile \( T_{en}(x, z) \) and the entry thickness profile \( h_{en}(x, z) \) of the strip. For the design of the MIMO feedforward controller, these profiles are mapped to the strip cross section that is currently in the roll gap. The overall aim of the optimization-based control strategy is to achieve the desired exit thickness profile \( h_{ex}(x) \) of the strip as accurately as possible.

4.1. Optimization problem

The deviation between the (predicted) lateral thickness profile \( h_{ex},(x) \) and the desired profile \( h_{ex}^d(x) \) is considered in the cost function of the optimization problem in the following form

\[
e = \sqrt{\frac{1}{b_R - 2b} \int_{-\frac{b_R}{2} + b}^{\frac{b_R}{2} - b} (h_{ex}(x) - h_{ex}^d(x))^2 dx} .
\]

(22)

The quadratic deviation of the profile is integrated and scaled, so that \( e \) represents a thickness. The integration domain ranges from \( -\frac{b_R}{2} + b \) to \( \frac{b_R}{2} - b \) with the width \( b_R \) of the strip and a user-defined constant \( b \geq 0 \). By means of
\( \hat{b} \), the integration domain can be restricted to the relevant part of the strip and the edge zones can be excluded. To find the optimal control input, the cost function \( e \) is minimized,

\[
\min_{\hat{u}} e \\
\text{s.t. } h_{ex}(x) = f_{s}(x_{h}^{DS}, x_{h}^{OS}, \hat{q}_{roll}(\cdot), F_{wrb}^{DS}, F_{wrb}^{OS}, s_{wrs}, u_{R}) \\
f_{R}(\hat{q}_{roll}, \hat{h}_{ex}, p_{ex}, \hat{u}_{R}, k_{fu}) = 0 .
\]

\( (23) \)

Additionally, the constraints

\[
0 \leq x_{h}^{j} \leq x_{h}^{max}, \quad 0 \leq F_{wrb}^{j} \leq F_{wrb}^{max} \quad \forall j \in \{DS, OS\}
\]

\( (24) \)

have to be considered. This optimization problem can be solved using standard algorithms, for instance the MATLAB command fmincon.

In the iterative optimization routine, the static mill stand model (12) has to be repeatedly solved and therefore the computational time associated with (23) is too high for a real-time implementation on a state-of-the-art hardware. This is why a computationally efficient approach is presented in the following subsection.

4.2. Numerically efficient solution of the optimization problem

In a first step, the mill stand model \( f_{s} \) is linearized in (23). For the linearization, a reasonable mill setup and suitable strip entry conditions are defined as operating point \( \hat{A} \). Here, the average values over the strip length are chosen for the point \( \hat{A} \), defined by

\[
\begin{align*}
\bar{u} &= [\bar{x}_{h}^{DS}, \bar{x}_{h}^{OS}, \bar{F}_{wrb}^{DS}, \bar{F}_{wrb}^{OS}, \bar{s}_{wrs}, \bar{u}_{R} ] , \\
\bar{h}_{ex}(x) &= f_{s}(\bar{x}_{h}^{DS}, \bar{x}_{h}^{OS}, \bar{q}_{roll}(\cdot), \bar{F}_{wrb}^{DS}, \bar{F}_{wrb}^{OS}, \bar{s}_{wrs}, \bar{u}_{R}) \\
f_{R}(\bar{q}_{roll}, \bar{h}_{ex}, p_{ex}, \bar{u}_{R}, k_{fu}) &= 0 .
\end{align*}
\]

This yields the deviation \( \Delta h_{ex}(x) = h_{ex}(x) - \hat{h}_{ex}^{k}(x) \) at the grid points \( \bar{x}_{h}^{k}, k = 0, \ldots, N \). Summarizing these deviations in a vector, the linearization reads as

\[
\Delta h_{ex} = [\Delta h_{ex}(x_{k})]_{k=0,\ldots,N} = \bar{h}_{ex} + \left[ \frac{\partial f_{s}}{\partial x_{h}} \right]^{DS}_{\hat{A}} \Delta x_{h}^{DS} + \left[ \frac{\partial f_{s}}{\partial F_{wrb}} \right]^{OS}_{\hat{A}} \Delta F_{wrb}^{OS} + \left[ \frac{\partial f_{s}}{\partial s_{wrs}} \right]^{OS}_{\hat{A}} \Delta s_{wrs} + \left[ \frac{\partial f_{s}}{\partial u_{R}} \right]^{OS}_{\hat{A}} \Delta u_{R} .
\]

\( (25) \)

The vector \( \Delta h_{ex} \) contains the deviation between the exit thickness profile at the operating point \( \hat{A} \) and the desired exit thickness, \( \Delta h_{ex}(x) = h_{ex}(x) - \hat{h}_{ex}(x) \). Additionally, the deviations from the operating point are denoted by \( \Delta u = u - \bar{u} = [\Delta x_{h}^{DS}, \Delta x_{h}^{OS}, \Delta F_{wrb}^{DS}, \Delta F_{wrb}^{OS}, \Delta s_{wrs}, \Delta u_{R}]^{T} \) for the control input vector, \( \Delta q_{roll} = [\Delta q_{roll}(x_{k})]_{k=0,\ldots,N} \) for the distribution of the roll force, \( \Delta s_{wrs} = s_{wrs} - \bar{s}_{wrs} \) for the WR shifting position, and \( \Delta u_{R} = u_{R} - \bar{u}_{R} \) for the rolling speed. Because the shifting position of the work rolls \( s_{wrs} \) is not changed during a roll pass, \( \Delta s_{wrs} = 0 \) and the term \( \frac{\partial f_{s}}{\partial s_{wrs}} \) is omitted in the following.

This linearization of the mill stand model \( f_{s} \) is reasonable because the nonlinearity of this model is only weak. In contrast, the nonlinear roll gap model \( f_{R} \) is still used to obtain \( \Delta q_{roll} \). A comparison of the exit thickness of the linearized mill stand model with the full nonlinear model shows a good agreement for different variations of the system parameters. For this reason, the trajectories for the optimal control inputs \( u \) of the linearized mill stand model agree well with the inputs computed by the nonlinear optimization problem (23), see Fig. 9 in Section 6.

Using the abbreviations

\[
\begin{align*}
A &= \left[ \frac{\partial f_{s}}{\partial x_{h}} \right]^{DS}_{\hat{A}}, \left[ \frac{\partial f_{s}}{\partial F_{wrb}} \right]^{OS}_{\hat{A}}, \\
B &= \left[ \frac{\partial f_{s}}{\partial s_{wrs}} \right]^{OS}_{\hat{A}}, \\
\Delta u_{R} &= \left[ \frac{\partial f_{s}}{\partial u_{R}} \right]^{OS}_{\hat{A}} \Delta u_{R} ,
\end{align*}
\]

\( (26b) \)

\( (26c) \)

(26a) is written as

\[
\Delta h_{ex} = A \Delta u + b .
\]

\( (26d) \)

Equation (26) facilitates a fast computation of \( \Delta h_{ex} \) because the nonlinear optimization problem (23) can be replaced by the quadratic program

\[
\min_{\Delta u} \frac{1}{N+1} \sum_{k=0}^{N} \Delta h_{ex}^{2}(x_{k}) \\
\text{s.t. } \Delta h_{ex} = A \Delta u + b \quad \text{with } A = \left[ \Delta x_{h}^{DS}, \Delta x_{h}^{OS}, \Delta F_{wrb}^{DS}, \Delta F_{wrb}^{OS} \right]^{T}
\]

\( (27) \)

if the optimal value \( \Delta q_{roll}^{*} \) is known. The problem (27) can be easily solved using the matrix pseudo-inverse

\[
\Delta u^{*} = -A^{+} b
\]

(28)

to get the optimal control input \( u^{*} = u + \Delta u^{*} \). For restricting the part of the strip width that is considered in the optimization problem, as with \( \hat{b} \) in (22), the corresponding \( \frac{1}{M} N \) first and last indexes of \( \Delta h_{ex} \) are omitted in (27).

The value of \( \frac{1}{M} N \) is rounded to integer. To obtain the corresponding optimal value \( \Delta q_{roll}^{*} \), (1), (26d) and (28) are
iteratively solved in the form

\[ h_{x,i} = h_{x,i}^d + b_i + A \Delta u_i \tag{29a} \]

\[ q_{\text{roll},i} = f_R^{-1} \left( h_{x,i}, h_{x,i+1}, \alpha, \beta, x_h, u_R, k_{\text{fim}} \right) \tag{29b} \]

\[ \Delta h_{x,i} = \Delta h_{x,i}^d + \frac{\partial f_s}{\partial q_{\text{roll}}} (q_{\text{roll},i} - \bar{q}_{\text{roll}}) + \frac{\partial f_s}{\partial R} \Delta u_R \tag{29c} \]

\[ \Delta u_{i+1} = -A^i b_{i+1} \tag{29d} \]

\[ i \leftarrow i + 1, \tag{29e} \]

with the initial values \( b_0 = \Delta h_{x,0}, \Delta u_0 = 0 \), and \( i = 0 \). In (29b), \( f_R^{-1} \) is the (numerical) solution of (1) for \( q_{\text{roll}} \). Fig. 4 shows a typical evolution of the cost function \( e \) from (22) for five iterations of (29). The iteration converges within 2 or 3 steps. Hence, a fixed maximum number of iterations turns out to be a good termination criterion for the above iterations.

In this linearized optimization approach, the constraints (24) are not considered. This is why a check of the optimal inputs obtained from (29d) is added after each step. If constraints are violated, the corresponding inputs are projected onto their limits and the iteration (29) is started again with the other inputs remaining for optimization.

The Jacobian \( \frac{\partial f_s}{\partial q_{\text{roll}}} \) is numerically computed using the central difference quotient. Its values for a sample strip are shown in Fig. 5. In the same way, the sensitivities \( \frac{\partial f_s}{\partial R} \) and \( \frac{\partial f_s}{\partial u} \) are numerically computed. The values \( A = \frac{-1}{\frac{\partial f_s}{\partial R}} \) for a sample strip are shown in Fig. 6. Because \( A \) and \( \frac{\partial f_s}{\partial q_{\text{roll}}} \) have to be calculated only once per strip, the computational costs associated with (29) are quite low, which allows a real-time implementation of this static optimization algorithm.

The proposed algorithm computes the control input \( u \) for a particular strip position in \( z \)-direction based on measurements of the inhomogeneities \( T_{en}(x, z) \) and \( h_{en}(x, z) \), and the actual velocity \( u_R \). To obtain an optimal input \( u(z) \), the strip is discretized along the direction \( z \) and the algorithm (29) is applied to each grid point. The computation time of (29) is approximately 0.2 s for \( N = 100 \) discretization elements over the strip width \( b_z \) in the simulations with a standard PC in MATLAB. This means, the proposed optimization-based algorithm of (29) is real-time capable. The rolling time of one strip in a mill stand typically is about 90 s in the considered tandem rolling mill. This yields 450 discretization points of \( z \), which corresponds to an average discretization distance of \( \Delta z = 2 \text{ m} \), or in the worst case \( \Delta z = 4 \text{ m} \) if the strip is rolled with the maximum speed of \( u_R = 20 \text{ m/s} \), respectively. This discretization is sufficient to capture the variations of the entry properties \( T_{en}(x, z) \) and \( h_{en}(x, z) \), and the velocity \( u_R \). As can be inferred from Fig. 9 and as will be discussed in Section 6.2, the result of the proposed solution strategy (29), which uses the linearized mill stand model, is almost identical to the optimal control input \( u(z) \) calculated with the nonlinear model (23). Numerical differences due to the linearization in the feedforward controller are not crucial for the resulting exit thickness because they are compensated by the feedback controller as discussed in Section 5.

5. Closed-loop control structure

The control strategy described in Section 4 yields a feedforward control input where the measurements of \( F_R \) and \( x_h \) are not used by the feedforward controller. This
means, the resulting output thickness profile $h_{ex}(x)$ will be correct only if there is no model-plant mismatch and if the measurements of the strip entry temperature and strip entry thickness are exact. In the steel industry, an AGC loop is commonly used for thickness control, see also Section 2.4. The AGC calculates an additional (symmetric) cylinder position $\Delta x^*_{agc}$ if there is a deviation between the estimated and the desired thickness, see (15), (16). When using the feedforward control concepts described in Section 3 and Section 4, the AGC loop can still be used as a feedback controller in a two-degree-of-freedom controller structure. The feedforward control input $\Delta x^*_h$ from (29d) (or $\Delta x^*_R$ from (19)) will be added to the AGC output $\Delta x^*_h$ as outlined in Fig. 7. If the feedforward controller is properly working, the control input of the AGC $\Delta x^*_h$ will be much smaller compared to a plant operating with the AGC only.

6. Simulation and measurement results

The control concept is compared to the conventional AGC in simulation studies using the validated simulation model described in Section 2. The static mill stand model from Section 2.2 including the roll gap model also serves as the basis for the control design.

The simulations cover a sample strip and the first mill stand. The control objective of the first mill stand is to compensate for entry inhomogeneities, and rolling speed variations, etc. such that the exit thickness of the first mill stand is preferably uniform in the longitudinal and lateral directions, that is, $h_{ex}^d(x,z) = h_{ex}^d$. 

6.1. Strip entry properties

To test the control design developed in Section 4, a sample strip with the inhomogeneities shown in Fig. 8 is considered. These inhomogeneities are taken directly from measurements in the real plant, i.e., the temperature is measured by a thermo-graphic camera and pyrometers, and the thickness profile is measured with a radiometric unit (infrared laser, x-ray). It is assumed that these measurements agree with the conditions how the strip enters the roll gap, e.g., a temperature change between the measurement and the first mill stand is not considered for the simulation. Fig. 8 shows the entry profiles for the strip thickness $h_{en}$ and the temperature $T_{en}$ as well as the rolling velocity $u_R$. The temperature skid marks originate from the pusher-type slab reheating furnace. Moreover, to discuss the influence of asymmetric rolling conditions, a temperature gradient $T_{en}$ is assumed. Thus, starting at $z = 300$ m, the strip temperature at the DS is increased and at the OS decreased, respectively. Such asymmetric temperatures can stem from reheating the steel slab with one side close to the walls or door of the furnace and are added in the assumed strip temperature profile to show the behavior of the control approaches for asymmetries. These measurements suffice to assess the most significant features of the proposed controller. Other variations, e.g., of the strip tension, are not considered.

6.2. Simulation results

Fig. 9 shows the optimal control inputs $u^*$ for the considered strip. The optimal control inputs $\Delta x^*_h$ and $F^*_{wrb}$ computed with the algorithm (29) based on the linearized mill stand model (solid lines) are almost identical to the optimal control inputs obtained from the nonlinear optimization (23) based on the full model (dashed lines). The additional cylinder position $\Delta x^*_R$ of the SISO feedforward controller according to (18) and (19) (purple dash-dotted line) is symmetric for the DS and the OS and plotted for comparison reasons. The WR bending forces are constant for the SISO feedforward controller. At the colder zones of the strip, the expected roll forces are higher, and therefore the feedforward controller requests an additional position $\Delta x^*_h$ and higher bending forces $F^*_{wrb}$. The
increasing asymmetry of $T_{\text{in}}$ requires asymmetric control inputs, that is, $\Delta h_{\text{OS}}^{\text{DS}^*}$ and $\Delta h_{\text{OS}}^{\text{OS}^*}$, and $\Delta F_{\text{wrb}}^{\text{DS}^*}$ and $\Delta F_{\text{wrb}}^{\text{OS}^*}$ are diverging to the end of the strip.

Fig. 10 shows the simulation results of 3 control approaches: the results of the optimal feedforward controller (MIMO FF) from Section 4 (black line) and the simple symmetric SISO feedforward controller from Section 3 (blue line), both with the AGC in the feedback loop in a two-degree-of-freedom controller structure, are compared to the results of a standard AGC (green line). In the optimization-based MIMO and the SISO feedforward control, the control action mainly comes from the feedforward part whereas the output of the AGC is rather small because there is nearly no model-plant mismatch. The feedforward control inputs $\Delta x_{\text{w}}^*$ compared to the feedback control inputs $\Delta x_{\text{w}}^{\text{opt}}$ are dominant. The figure also shows the mean exit thicknesses $h_{\text{ex}}$ for all 3 control approaches. The aggregated error $e$ as defined in (22) and the wedge-shape of the strip (the difference of $h_{\text{ex}}(x)$ between the DS and the OS) are shown in the bottom of Fig. 10. The desired exit thickness profile $h_{\text{ex}}^d$ after the first mill stand is assumed constant over the strip width and length. This desired profile is best achieved by the optimization-based MIMO FF controller. As the standard AGC controller is a proportional feedback controller, the entry inhomogeneities cannot be completely compensated and the exit thickness still shows the skid marks.

For the SISO feedforward approach, the average exit thickness $h_{\text{ex}}$ is quite accurate and superior compared to the pure AGC. The skid marks are largely compensated but small deviations can occur due to the linearization in (18), e.g., in the first part of the strip. The asymmetric rolling conditions due to the asymmetric temperature in the second half of the strip cannot be counteracted with the symmetric control inputs $\Delta x_{\text{w}}$. Although the mean strip thickness $h_{\text{ex}}$ is still equal to the desired exit thickness $h_{\text{ex}}^d$, there is a rising error $e$ of the thickness profile due to the wedge shape $w$ of the strip. The exit thickness wedge $w$ (thickness difference between DS and OS edges of the strip) is the same for the SISO FF (blue lines) and the pure AGC (green lines) because both give symmetric control inputs. Even a small thickness wedge can lead to camber shape and to lateral strip movement, [26]. With the optimization-based MIMO feedforward control (black lines), those wedge-shapes are avoided and the error $e$ is almost constant over the strip length. The aggregated error $e$ is not exactly zero because there are remaining deviations between the thickness profile $h_{\text{ex}}(x)$ and the desired profile $h_{\text{ex}}^d(x)$. This is mainly because there are only four (scalar) control inputs $u$ used to control the thickness $h_{\text{ex}}(x)$ (or the load $q_{\text{roll}}(x)$) that is distributed over $x$.

The simulated profiles in lateral $x$-direction at $z = 400$ m are shown in Fig. 11. In this figure, the gray shaded area represents the integration domain $\left[\frac{1}{2b} + \hat{b}, \frac{1}{2} - \hat{b}\right]$ in the cost function (22). For these results, $b = 0.15b_0$ was used. The exit thickness profile of the optimization-based approach (black line) in the gray area, is almost identi-
6.3. Simulation results with model-plant mismatch

To analyze the robustness of the control concept against model-plant mismatches, the parameters of the simulation model were modified. For the results shown in Fig. 12, the exponent $m_1$ for the temperature $T$ in \((2)\) was increased by 10\%. As the optimal control inputs $u^∗$ are calculated based on nominal parameters, they are the same as in Fig. 9. Here, the exit thicknesses $h_{ex}$ of both feedforward approaches differ from the desired thickness. Hence, the feedback from the AGC slightly improves the results, but there is a remaining error that is almost constant. However, these control concepts are still superior to the standard AGC. The constant control error could be easily reduced by adding a further integral feedback action. This could be done either in the AGC control law itself \((16)\) or directly by a monitor thickness controller using the measured exit thickness.

6.4. Implementation and test in the real plant

In this section, first measurement results of the simple SISO feedforward control strategy according to Section 3 at the first mill stand of the considered finishing mill at voestalpine in Linz, Austria, are shown. Compared to \((18)\), the rolling velocity $u_R$ is not considered,

$$\Delta F_R^{\prime} = \frac{\partial F_R}{\partial h_{en}} \Delta h_{en} + \frac{\partial F_R}{\partial T_{en}} \Delta T_{en}$$ \tag{30}$$

is used. The scalar sensitivities $\frac{\partial F_R}{\partial h_{en}}$ and $\frac{\partial F_R}{\partial T_{en}}$ are used for the implementation in the industrial plant. These sensitivities are well known by the plant operator for the different materials. The scalar feedforward control action is obtained using \((19)\) with \((30)\).

In this first step, the expected variations of the roll force $\Delta F_R^{\prime}$ are calculated in the feedforward controller using the measured differences from the operating point of the entry thickness $\Delta h_{en}$ and of the entry temperature $\Delta T_{en}$ of the strip. These measurements represent the average over the strip width, that is, the lateral profile is not considered so far. The entry thickness $\Delta h_{en}$ is estimated at the last rolling pass of the roughing mill using \((15)\). The temperatures of the upper and the lower surface of the strip are measured by two pyrometers before the first mill stand and then averaged. Since the measurements are far enough from the first mill stand, the calculation of the evolution of $\Delta x^{\prime \prime}$ for one strip is available before the head end enters the first mill stand and the feedforward control input could be disabled in case of implausible shape. The feedforward control input is applied to the existing AGC loop as outlined in Section 5.

The strip velocity or other variations that influence the rolling process are not considered in this preliminary implementation. This means, this compensation of the inhomogeneities of the temperature and the entry thickness is in particular suitable for strips that are preheated in a pusher-type furnace that exhibit pronounced skid marks.
Figure 13: Measurement results from an industrial finishing mill for the entry properties of the strip used in the symmetric linearized SISO feedforward controller.

Figure 14: Measurement results from the first mill stand of an industrial finishing mill with (blue lines) and without (green lines) the symmetric linearized SISO feedforward controller with the variations of the strip entry temperature and thickness according to Fig. 13.

6.5. Measurement results
Fig. 13 shows the variations of the measured entry thicknesses and entry temperatures of two representative strips. The longitudinal coordinate \( z \) of the strips is normalized since the strips have different lengths \( L \). The temperature profiles of the two strips exhibit typical skid marks caused by non-uniform reheating conditions in the pusher-type slab reheating furnace. Consequently, thickness variations occur at the roughing mill. The associated longitudinal thickness profile of the strips entering the first mill stand are also shown in Fig. 13. The bottom of Fig. 13 shows the additional position \( \Delta x_{h}^{ff} \), that is the output of the feedforward control law (30). The feedforward control was added to the control input only for the first strip (blue lines) but the output \( \Delta x_{h}^{ff} \) was calculated for both strips. When comparing the control approaches, it must be kept in mind that strips are never completely identical. They can differ in their material properties, heating conditions, desired geometry, and rolling conditions, etc. The two strips shown in Fig. 13 are consecutively produced at the finishing mill.

In Fig. 14, the strip with pure AGC (green lines) is compared to the strip with active SISO feedforward control (blue lines). The figure shows the applied position \( \Delta x_{h}^{ff} \), the additional position of the feedback AGC \( \Delta x_{h}^{agc} \), and the estimated strip thickness \( \hat{h}_{ex} \) after the first mill stand. At the bottom of Fig. 14, a first-order polynomial is subtracted from the estimated thickness \( \hat{h}_{ex} \). The axis of the exit thickness deviation \( \Delta h_{ex} \) in the figure is the ratio to the nominal exit thickness after the first mill stand in %. In Fig. 14, the behavior of a properly working two-degree-of-freedom controller can be observed. The control input of the feedback controller (the AGC, \( \Delta x_{h}^{agc} \)) is smaller when it is combined with the feedforward controller. Since in this implementation only the entry thickness and the temperature of the strips are considered by the feedforward controller (as so decided by the plant operator), there is a remaining rise of the exit thickness that is due to the velocity speed up. Moreover, the cool-down of the strip between the temperature sensor and the rolling mill is not considered by the feedforward controller. The colder strip tail end causes higher roll forces and hence a rise of the exit thickness as well. The contin-
uous increase of the estimated exit thickness $\Delta \hat{h}_{ex}$ is observed for both strips of Fig. 14, and is thus masked out by subtracting the first-order polynomial $p_h(z)$. The variations due to the skid marks almost vanish for the first strip with feedforward controller (blue lines). For the strip without any feedforward control action (green lines), the skid marks can still be seen in the signals $\Delta x_{trim}^{p_h}$ and $\Delta \hat{h}_{ex}$. This strip, only the AGC control law (16) was active. These results from an industrial application confirm the feasibility of the proposed feedforward control concept.

The feedforward control strategy was also tested with other strips. The behavior shown in Fig. 14 is representative for these strips. The standard deviation $\sigma_{hex}$ of the remaining thickness deviations $\Delta h_{ex} - p_h(z)$ over the strip length is used as an aggregate measure of the accuracy of the exit thickness,

$$
\sigma_{hex} = \sqrt{\frac{1}{N_x - 1} \sum_{l=1}^{N_x} (\Delta h_{ex}(z_l) - p_h(z_l))^2}
$$

with the number of points $N_x$ along the length of the strip. The standard deviations for the two strips are shown as dashed lines in Fig. 14. The standard deviation $\sigma_{hex}$ was calculated for 2250 other strips rolled with the feedforward controller and for 3200 strips with pure AGC. Fig. 15 shows the frequency distribution of these standard deviations for both strategies, strips with active feedforward in blue bars and without feedforward (with AGC only) in green bars. This demonstrates that $\sigma_{hex}$ is significantly lower with the feedforward strategy (mean value 0.077%) than without feedforward compensation of the skid marks (mean value 0.118%). The number of strips with large deviations of $\hat{h}_{ex}$ is drastically reduced with the feedforward approach of (19) and (30).

In the next step of the realization of the proposed feedforward control strategies, the rolling velocity will also be considered in the feedforward controller. Since the measurement results for the first mill stand agree well with the expectations from the simulation results for the linearized feedforward control strategy, it is also reasonable that the more complex feedforward strategies developed in this paper will work well, and will further improve the thickness control, in particular also in terms of the lateral thickness profile.

The measurement results obtained so far clearly validate the proposed feedforward concept and encourage the implementation of more features of the proposed feedforward strategies. The accuracy of the exit thickness after the first mill stand is improved if the variations of the entry thickness and entry temperature are compensated. Therefore, it is planned to implement the feedforward strategy also at consecutive mill stands to improve the quality of the finished strip.

7. Conclusions

In this paper, SISO and optimal MIMO feedforward control strategies to compensate for the measured incoming inhomogeneities of a strip at the first mill stand in a tandem hot rolling mill are discussed. The thickness profile is optimized using all available control inputs, i.e., the cylinder positions and the bending forces at the drive side and at the operator side. Linearizing the mill stand model (but not the roll gap model) lead to an iterative quadratic optimization problem, which can be solved in a computationally efficient way and thus facilitate real-time control. Simulation results show a significant benefit compared to conventional AGC concept. Thickness fluctuations due to skid marks are practically avoided and, even in case of a constant model-plant mismatch, there is only a constant error over the strip length, which can be corrected by adding a simple integral feedback action.

For this paper, it was assumed that the desired exit thickness of the first mill stand is uniform over the width and the length of the strip. Therefore, at the following mill stands, only the remaining temperature inhomogeneities have to be considered for feedforward optimization. As suggested in [24], it could also be useful to overcompensate the temperature at one mill stand such that both the exit thickness and the roll force are constant at the following mill stand. In the future research, the model will be adapted, in particular the material parameters of (2) could be estimated based on measurements at the mill stands. These parameters should then be used at subsequent mill stands to further improve the accuracy of the exit thickness profile. A feedforward control concept for the whole tandem mill including the loopers between the mill stands is another topic of future research.

Appendix A. Numerically efficient solution of the boundary value problem

For a numerically efficient solution of the boundary value problem stated in Section 2.2, a tailor-made numerical solver was developed. The general idea of this solver is to make use of the fact that the ODE (3) is only weakly nonlinear. Basically, the ODE is locally linearized, and with the exact solution of the linearized system of equations, a new starting point for the linearization is found iteratively. This approach for solving the boundary value problem is described in detail in the next paragraph.

At the ends of the rolls, $|x| > \frac{b}{2}$, there is neither a contact between WR and BR nor between WR and the strip. Hence, $q = 0$ holds in this region, (3) is linear, and an analytical solution of (3) can be readily found. The boundary conditions (10) and (11) are replaced by boundary conditions at $x = \pm \frac{b}{2}$. Within these boundaries, the nonlinear boundary value problem is numerically solved. Therefore, the ODEs are evaluated at $N + 1$ discrete points $x_k$, $k = 0, \ldots, N$ in $x$-direction starting with a reasonable initial value for the 16 states $y_0 = [v_{i,0}, \varphi_{i,0}, M_{i,0}, Q_{i,0}]^T$ at
\[ x = x_0 = -\frac{b_k}{T} \] and ending at \( x = x_N = \frac{b_k}{T} \), with \( i \) according to (4). After solving this initial value problem, using, e.g., a Runge-Kutta method, the solution \( y_k \) at each position \( x_k \) with \( k = 0, \ldots, N-1 \) is linearized in the form

\[ \frac{\partial y}{\partial x} \approx A_k y + b_k, \quad x_k < x \leq x_{k+1}, \quad (A.1) \]

that is, a linear order Taylor series, where \( A_k \) represents the Jacobian of (3) at \( x_k \). Using the matrix exponential of \( A_k \), the discrete solution formula reads as

\[ y_{k+1} = \Phi_k y_k + \Gamma_k, \quad k = 0, \ldots, N - 1 \quad (A.2) \]

with

\[ \Phi_k = \exp (A_k (x_{k+1} - x_k)) \quad (A.3a) \]

\[ \Gamma_k = \int_{x_k}^{x_{k+1}} b_k \exp (A_k (x_k + \tau - x_k)) \, d\tau. \quad (A.3b) \]

Recursive insertion yields

\[ y_k = \sum_{j=0}^{k-2} \Phi_{k-1} \Phi_{k-2} \ldots \Phi_{j+1} \Gamma_j + \Gamma_{k-1} \Phi_k \]

\[ + \Phi_{k-1} \Phi_{k-2} \ldots \Phi_1 y_0, \quad k = 0, \ldots, N, \quad (A.4) \]

that is, an affine equation in the initial value \( y_0 \).

Using \( F(y_0, y_N) \) for the 16 boundary conditions (10), (11) and inserting (A.4) yields

\[ F(y_0, y_N) = F(y_0, \Phi_N y_0 + \Gamma_N) = 0, \quad (A.5) \]

a set of 16 nonlinear equations. They are solved for \( y_0 \).

With this new initial value \( y_0 \), the values \( \Phi_k \) and \( \Gamma_k \) can be iteratively computed and the whole procedure can be repeated until the desired accuracy is obtained. This approach of linearizing the ODEs is computationally efficient and is suitable for the boundary value problem (3) because this is only weakly nonlinear. With a reasonable initial value \( y_0 \), a sufficiently accurate solution can be found within two or three iterations. A very good initial value is the solution of a previous evaluation of the static mill stand model. When simulating the rolling of a strip with varying coordinate \( z \), the previous result is suitable because the rolling conditions are just slowly changing. This initial solution is also directly used for the linearization (A.1), that is, the Runge-Kutta method is used for the very first solution of the ODE (3) only. Compared to a standard MATLAB solver for boundary value problems as bvp4c, the computation times could be improved by a factor of 10 below 1 s for \( N = 200 \) discretization points over the width \( b_c \).

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References


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Research highlights.

- A mathematical model for a hot strip tandem mill is developed and validated.
- A new optimization-based feedforward thickness controller is developed.
- A tailored numerically efficient solution of the optimization problem is presented.
- Simulation scenarios demonstrate the performance improvement of the proposed control concept.
- Promising results are obtained from a first installation of the control concept in the industrial plant.

Graphical abstract.