CIQS Problem Sheet: Chapter 6 (Optimal Parameter & State Estimation and State Feedback)

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Instructions

- Try to solve the following exercises. If appropriate, use software tools such as Maple or Matlab / Python.
- Focus on understanding and applying the theory established in the lecture notes.
- Your solutions will serve as a basis for discussion in the next Q&A session.

1 Minimum-variance parameter estimation

In this problem, we will work with the system from Example 6.1 in the lecture notes.

- Download the Matlab data file

  `workspace_minimum_variance.mat`

  from the course homepage and load the file into your Matlab workspace. It contains a data matrix $\mathbf{S}$, covariance matrices $\mathbf{Q}$, $\mathbf{N}$, and $\mathbf{R}$ as well as a (noisy) measured output vector $\mathbf{y}_{meas}$ and a noise-free output $\mathbf{y}$ for comparison.
- Assuming that $\mathbf{y}$ is not known (only $\mathbf{y}_{meas}$), compute the minimum-variance estimate of the parameter vector $\mathbf{p} = [a_1, a_2, a_3, b_0, b_1]^{\mathrm{T}}$ with the given matrices.
- Now perform the equivalent estimate using the least squares estimator ((6.10) in the lecture notes) and compare the two estimates for $\mathbf{p}$.
- Plot the two estimated outputs $\mathbf{S}\hat{\mathbf{p}}$ from the two estimators as well as the noise-free output $\mathbf{y}$. Which estimator performs better? Why?
- Vary the matrices $\mathbf{Q}$ and $\mathbf{R}$ of the minimum-variance estimator. How does this affect the estimator's performance?

2 LQR design

- Use the estimated parameters from Problem 1, but change a_1 to 1.25. Complete Exercise 6.1 in the lecture notes and insert these parameters. You should obtain an unstable system.
- Now assume for the final time index $N = 100$ that $\mathbf{P}_N = \mathbf{S} = \mathbf{E}$, with $\mathbf{E}$ being the unit matrix. Furthermore, assume $\mathbf{N} = \mathbf{0}$, $\mathbf{Q} = \mathbf{E}$, and $\mathbf{R} = 2\mathbf{E}$. Solve the discrete Riccati equation (Theorem 6.6 in the lecture notes) to obtain the LQR control gains $\mathbf{K}_k$ for all k. Simulate the closed-loop system, with $\mathbf{x}_0$ being an arbitrary nonzero initial state. Does the LQR stabilize the system?
- Now compute the stationary Riccati controller using the Matlab command

  ```
  K = -1* dlqr(Phi,Gamma,Q,R,N);
  ```

  and simulate the corresponding closed-loop system. Check the closed-loop eigenvalues using

  ```
  eig(Phi + Gamma*K)
  ```

  to verify the asymptotic stability of the closed loop.
- Study the influence the choice of $\mathbf{S}$, $\mathbf{Q}$, and $\mathbf{R}$ has on the closed-loop performance. You may include random perturbations acting on your system to see this more clearly.

3 Kalman filtering

For this problem, we just do two Exercises from the lecture notes on each the Kalman Filter and the Extended Kalman Filter, respectively.

- Complete Exercise 6.6 from the lecture notes. compare the resulting Kalman Filter for some q with the Full-Order Luenberger Observer (Theorem 5.9) for desired poles of your choice. Include normally distributed stochastic disturbances $\mathbf{w}_k$ and sensor noise v_k in your simulations. What difference between the two observer concepts do you notice?
- Complete Exercise 6.8 from the lecture notes.