Computation exercise 3(a): Control design

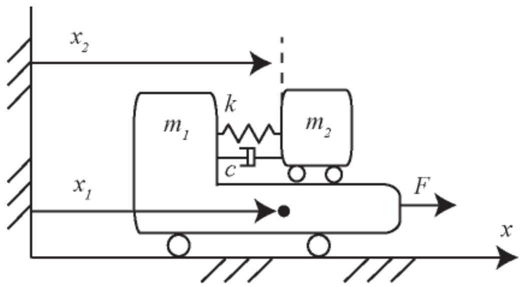
Mechatronic systems
376.050
2016W

Important: Answers must be a hard copy and submitted to the office in CA0421 by 20th of December, 2016 at 4pm. The work must be original.

Fig. 1 shows a lumped mass model of a positioning system using a Lorentz actuator. Fig. 2 shows a control block diagram to regulate the position x_2 by using the system input *F*, where *P(s)* is the plant. The transfer function *C(s)* is a controller and has the following structure:

$$C(s) = C_{PID}(s)C_{LPF}(s),$$

where $C_{PID}(s)$ is a tamed PID controller and $C_{LPF}(s)$ is a first-order low-pas filter $C_{LPF}(s) = (\frac{s}{\omega_f} + 1)^{-1}$.
Following the questions below, design a feedback controller *C(s)*.



Param.	Value	Unit
m_1	1	kg
m_2	5	kg
k	10^6	N/m
c	5	N/(m/s)

Fig. 1: A lumped mass model of a positioning system.

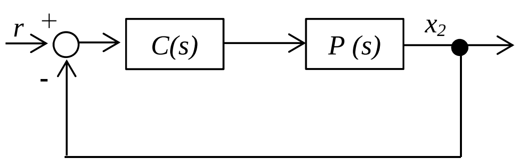


Fig. 2: Control block diagram.

i. Derive transfer function *P(s)* and simulate a Bode plot. [10%]

ii. Design the low-pass filter, such that it decreases the phase of P(s) at the resonant frequency by 45 degrees. For validation, simulate a frequency response of the filter. [20%]

iii. Design $C_{PID}(s)$ to regulate the plant cascaded with the low-pass filter (i.e. $C_{LPF}(s)P(s)$), fulfilling the following conditions. (See "rule of thumb" in the textbook for design details.) Also simulate a Bode plot of the open-loop transfer function $C_{PID}(s)C_{LPF}(s)P(s)$ for validation, as well as a Bode plot of the sensitivity function. [35%]

- Phase margin should be 40 deg or more (at the lowest open-loop cross-over frequency).
- Gain margin should be 10 dB or more.
- The (lowest) open-loop cross-over frequency should be as high as possible.

iv. Using the designed $C_{PID}(s)$, simulate a Nyquist plot of $C_{PID}(s)P(s)$ and $C_{PID}(s)C_{LPF}(s)P(s)$, and discuss the influence of the low-pass filter on closed-loop system stability. [35%]