AUTOMATION & CONTROL INSTITUTE
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# Mechatronic Systems: Solution of Exercise 2

## Course VU 376.050 (4 SWS, 6 ECTS)
## Winter semester 2016

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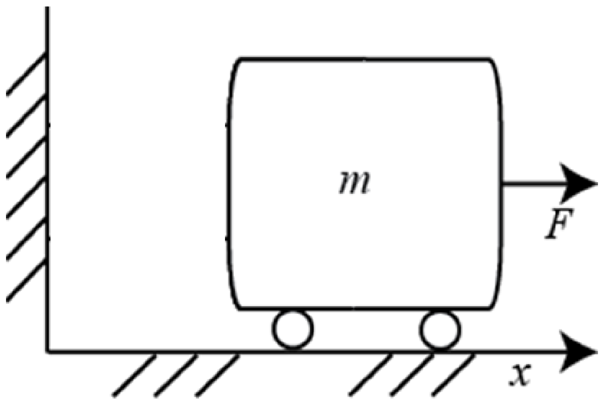
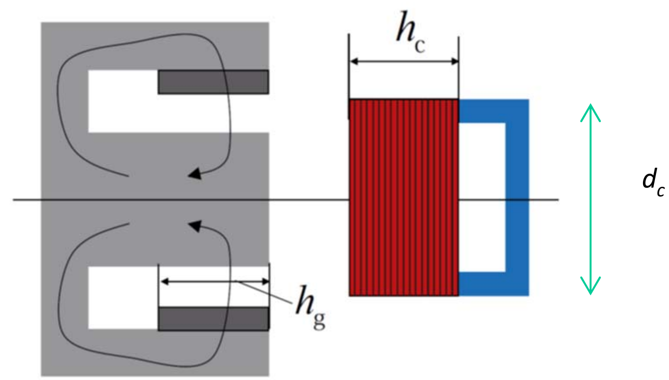
# Computation Exercise 2(a): Lorentz

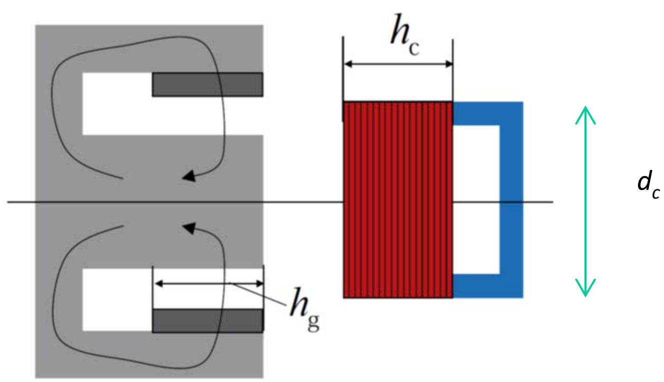


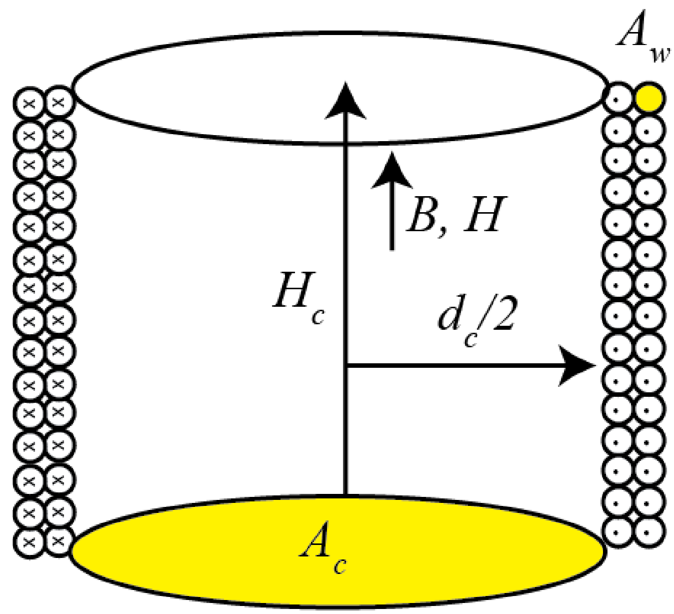




| Parameter | Value | Unit | Description |
|---|---|---|---|
| m | 0.5 | kg | Mover mass |
| n | 100 | ~ | Number of windings |
| $d_c$ | 10 | mm | Diameter coil |
| $d_w$ | 0.5 | mm | Diameter wire |
| $h_c$ | 5 | mm | Height coil |
| B | 1.2 | T | Magnetic field strength |
| ρ | $1.7 \cdot 10^{-8}$ | Ω·m | Specific resistance |
| $\mu_0$ | $4\pi \cdot 10^{-7}$ | $NA^2$ | Permeability in vacuum |
| $\mu_r$ | 100 | ~ | Relative permeability |
| Vmax | 15 | V | Max. output voltage |

# Computation Exercise 2(a)-(i): Geometry

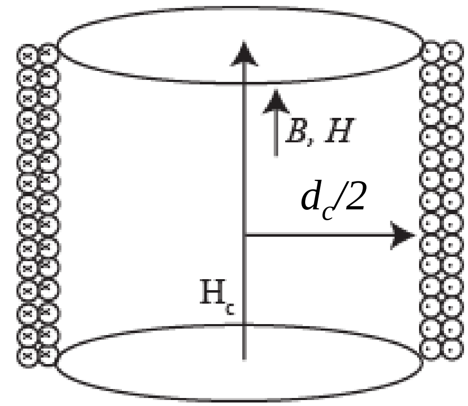
- Cross sectional area of the coil: $A_c = \pi \left(\frac{d_c}{2}\right)^2$
- Cross sectional area of the wire: $A_w = \pi \left(\frac{d_w}{2}\right)^2$
- Length wire: $l_w = n\pi d_c$
- Length wire inside the magnetic field: $l_m = l_w$



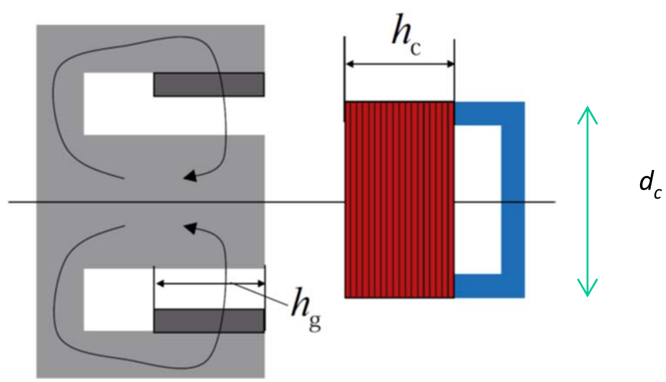



# Computation Exercise 2(a)-(i): R, L, Km

- Coil resistance: $R = \frac{\rho l_w}{A_w} = 0.272\Omega$
- Lorentz coil is a solenoid coil
  - Magnetic field strength in the coil: $H = \frac{n}{h_c} I$
  - Magnetic flux density: $B = \mu_0 \mu_r H$, Flux: $\varphi = A_c B$
  - Summed-up Flux of all windings: $\Phi = n\varphi$
  - Self inductance: $L = \frac{\Phi}{I} = \frac{n^2 \mu_0 \mu_r A_c}{h_c} = 19.7\ mH$

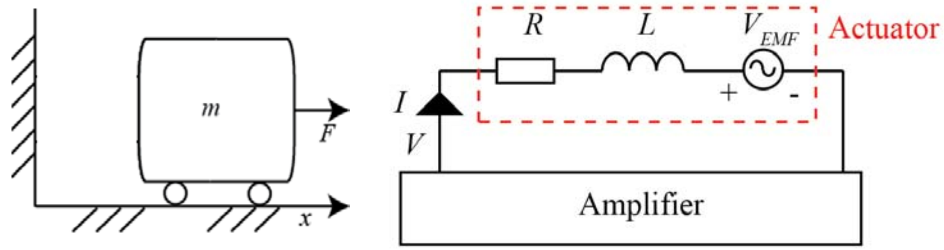



- Motor constant & Back EMF constant
  - $k_m = B l_m$
  - Motor constant : 3.77 N/A
  - Back EMF constant: 3.77 V/(m/s)



# Computation Exercise 2(a)-(ii)

- Mechanical system: floating mass



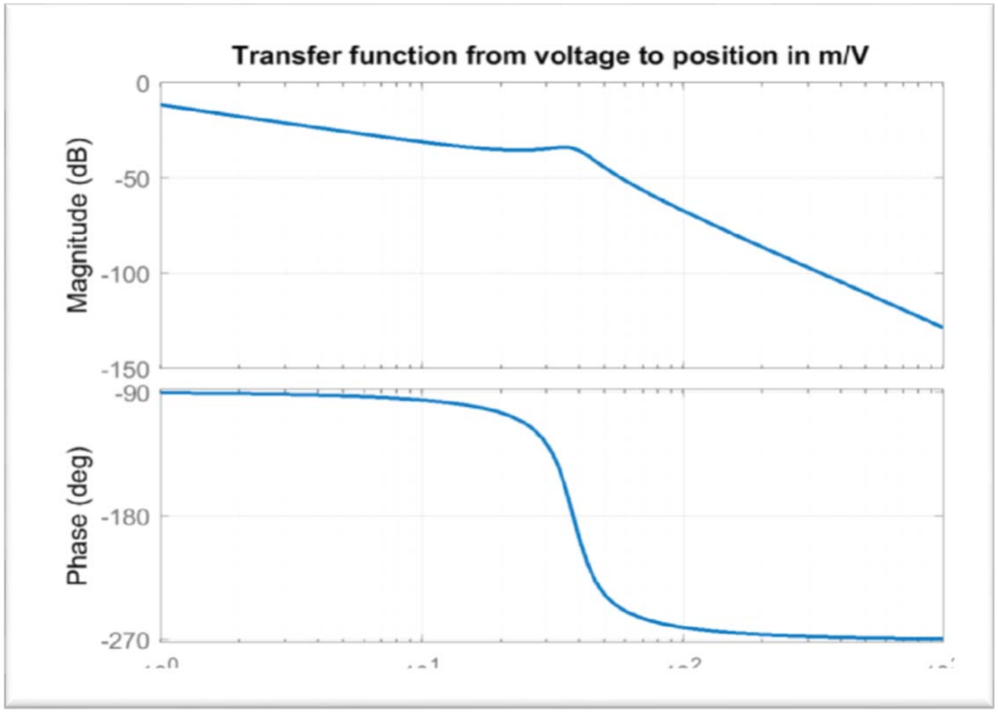

$$F = ms^2x$$

- Electrical system:

$$F = k_m I \qquad V_{EMF} = k_m s x \qquad V - V_{EMF} = (Ls + R)I$$

$$\frac{x(s)}{V(s)} = \frac{k_m}{(Ls + R)ms^2 + k_m{}^2 s}$$

- This is a 3rd-order system.



# Computation Exercise 2(a)-(iii)

$$F = k_m I \qquad V_{EMF} = k_m s x \qquad V - V_{EMF} = (Ls + R)I$$

- Maximum force
  - At a steady state, only R dominates the impedance.
  - At a static position, there is no back EMF voltage.

$$F_{max} = k_m I_{max} = k_m \frac{V_{max}}{R}$$

- Maximum velocity
  - The floating mass can accelerate until $V_{max} = V_{EMF} = \dot{x}_{max} k_m$

$$\dot{x}_{max} = \frac{V_{max}}{k_m}$$

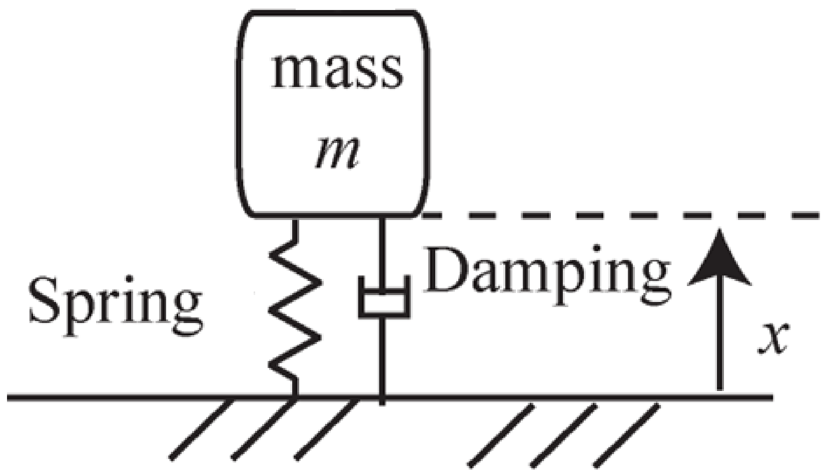
# Computation Exercise 2(a)-(iv)

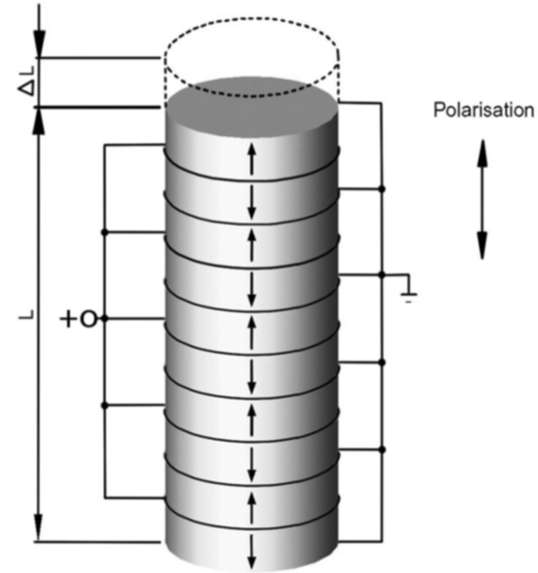
| Parameter | n=100 | n=50 |
|---|---|---|
| $F_{max}$ | 208 N | 208 N |
| $\dot{x}_{max}$ | 3.98 m/s | 7.96m/s |

$$F_{max} = k_m \frac{V_{max}}{R} \qquad \dot{x}_{max} = \frac{V_{max}}{k_m} \qquad I_{max} = \frac{V_{max}}{R}$$

- The number of windings
  - has no influence on the maximum force, as both $R$ and $k_m$ are decreased.
  - increases the maximum velocity by increasing $k_m$.
- In practice, a Lorentz actuator can exert its maximum force with a small current by increasing the windings.

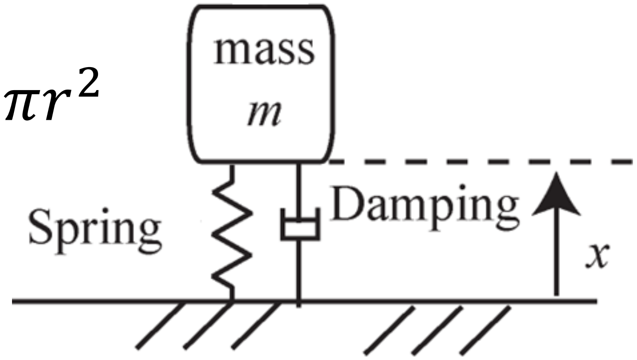
# Computation Exercise 2(b): Piezo




| Parameter | Value | Unit | Description |
|---|---|---|---|
| Y | $53 \cdot 10^{9}$ | $N/m^2$ | Young's modulus of piezo material |
| m | 100 | g | Weight mass (Load) |
| ρ | $7.85 \cdot 10^{3}$ | $Kg/m^3$ | Density of piezo material |
| c | 50 | N/(m/s) | Damping of piezo actuator |
| r | 5 | mm | Radius of piezo actuator |
| D | $195 \cdot 10^{-12}$ | m/V | Piezoelectric coefficient |
| ε | $1.68 \cdot 10^{-8}$ | F/m | Dielectric coefficient, Permittivity |
| n | 50 | - | Number of stacks |
| l | 1 | mm | Length of piezo per stack |
| R | 75 | Ω | Amplifier's output impedance |
| $V_{max}$ | 150 | V | Amplifier's maximum output voltage |

# Computation Exercise 2(b)-(i)




Geometric properties:

- Cross sectional area of the piezo: $A = \pi r^2$
- Total length of piezo: $L = Nl$

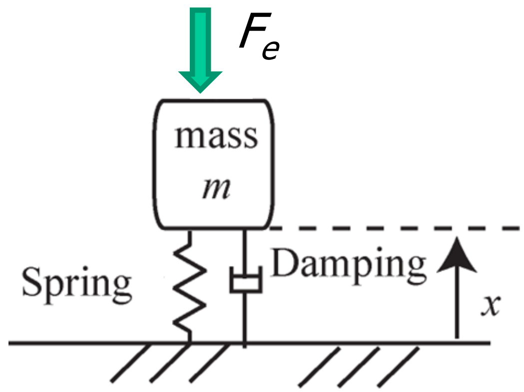
Mechanical properties:

- Mass of piezo: $m_p = AL\rho = 30.8g$
- Stiffness of piezo: $k_p = \frac{AY}{L} = 83.2N/\mu m$
- Capacitance: $C = \frac{n\varepsilon A}{l} = 66nF$

In the following calculation, it is assumed that *m* is sufficiently heavier than the piezo itself.

# Computation Exercise 2(b)-(ii)

- Behavior of Piezo: $\begin{Bmatrix} x \\ q \end{Bmatrix} = \begin{bmatrix} k_p^{-1} & D \\ D & C \end{bmatrix} \begin{Bmatrix} F_e \\ V \end{Bmatrix}$
- For multiple layers: $x = k_p^{-1} F_e + nDV$
- The external force: $F_e = -ms^2 x - csx$
- Transfer function from the applied voltage to the displacement

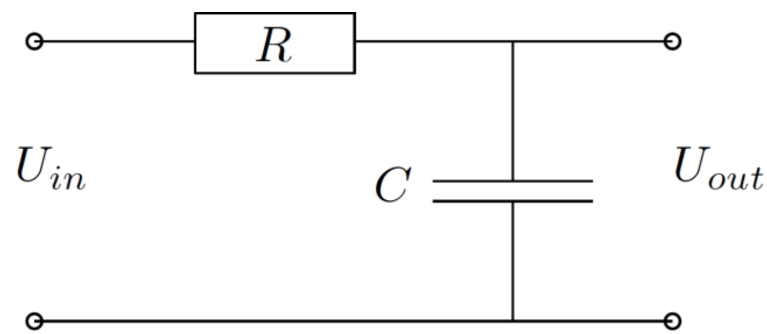



$$\frac{x}{V} = \frac{nDk_p}{ms^2 + cs + k_p}$$

# Computation Exercise 2(b)-(ii)

- Transfer function of an RC circuit

$$\frac{V_{out}}{V_{in}} = \frac{1}{RCs+1}$$

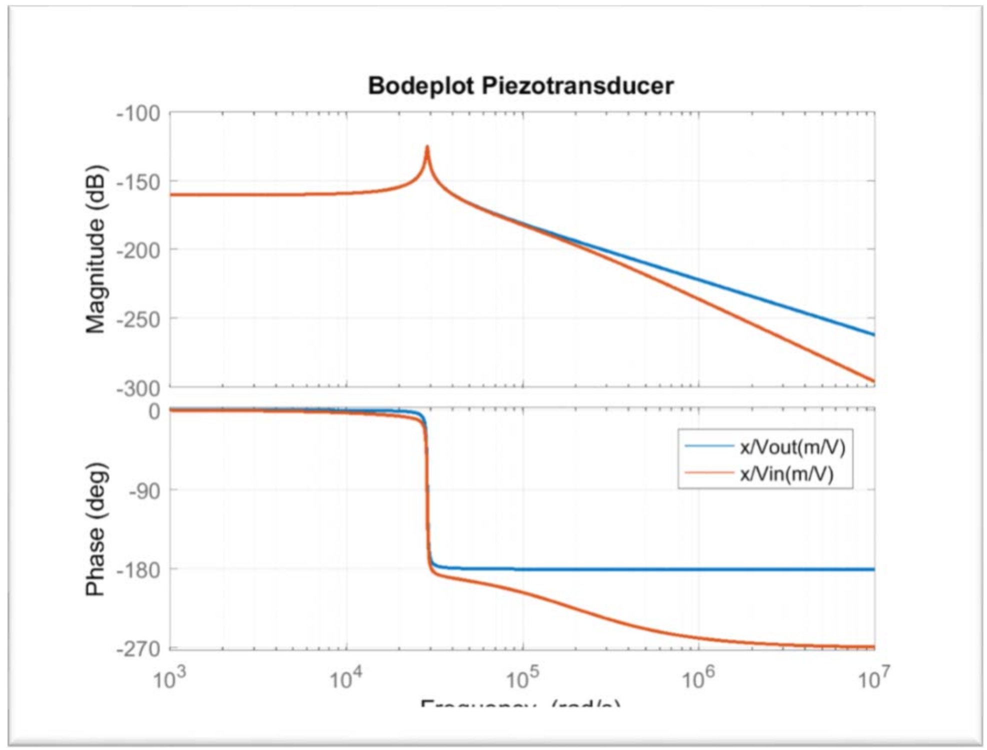



- Total transfer function

$$\frac{x}{V_{in}} = \frac{1}{RCs+1}\frac{nDk_p}{ms^2+cs+k_p}$$

- Impedanc of an RC circuit

$$Z(s) = \frac{I}{V} = \frac{Cs}{RCs+1}$$



# Computation Exercise 2(b)-(iii)

$$P(s) = \frac{x}{V_{in}} = \frac{1}{RCs + 1}\frac{nDk_p}{ms^2 + cs + k_p}$$

- Maximum displacement
  - At a steady state, the displacement is maximum.

$$x_{max} = nDV_{max}$$

- Natural frequency

$$\omega_n = \sqrt{\frac{k_p}{m}}$$

# Computation Exercise 2(b)-(iv)

| Parameter | n=50 | n=100 |
|---|---|---|
| $x_{max}$ | 1.46 µm | 2.93 µm |
| $\omega_n$ | 4.59 kHz | 3.25 kHz |

$$x_{max} = nDV_{max}$$

$$\omega_n = \sqrt{\frac{k_p}{m}} = \sqrt{\frac{AY}{mnl}}$$

- By increasing the stack number,
  - the displacement can be larger
  - However, the natural frequency decreases limiting an achievable control bandwidth
- It is typically difficult to achieve both large actuation displacement and high control bandwidth by a piezo.