AUTOMATION & CONTROL INSTITUTE
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Mechatronic Systems: Solution of Exercise 3

Course VU 376.050 (4 SWS, 6 ECTS)
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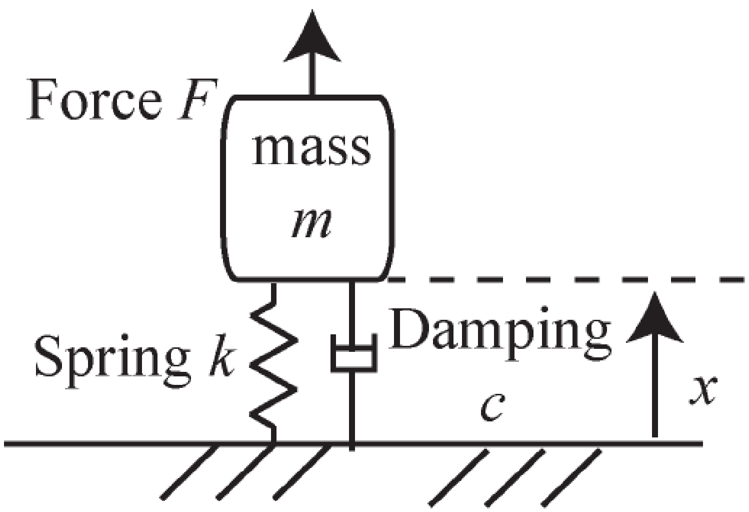
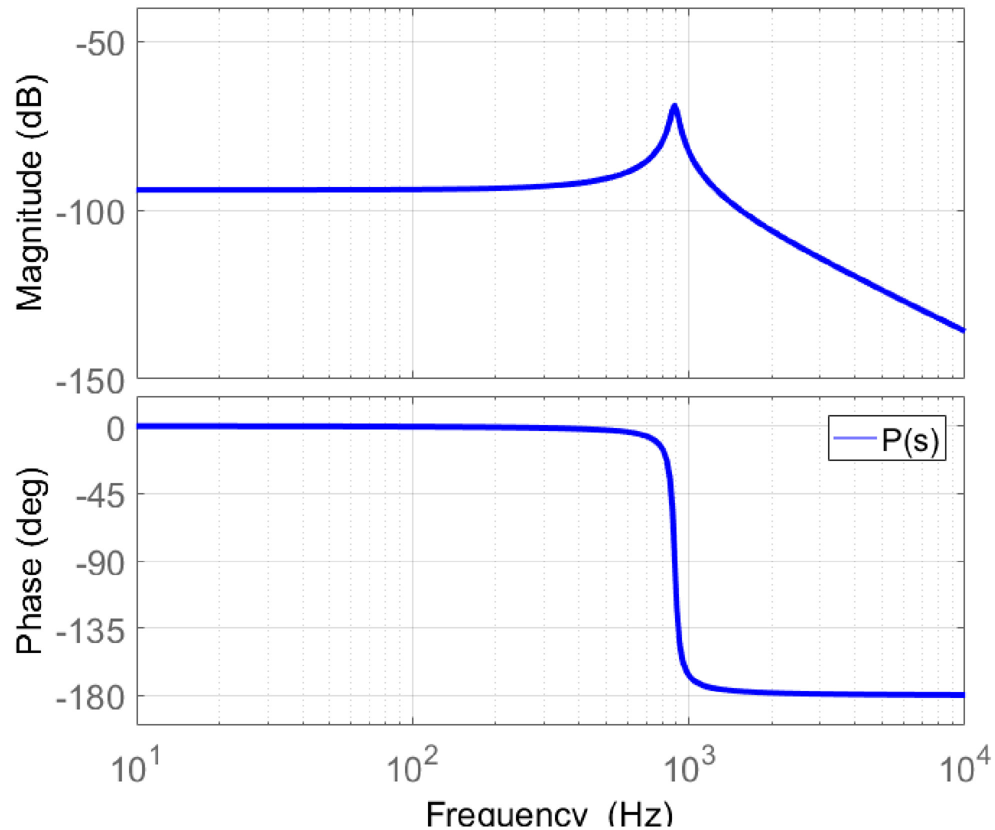
Problem (b)i: Plant model *P(s)*

$$P(s) = \frac{1}{ms^2 + cs + k}$$

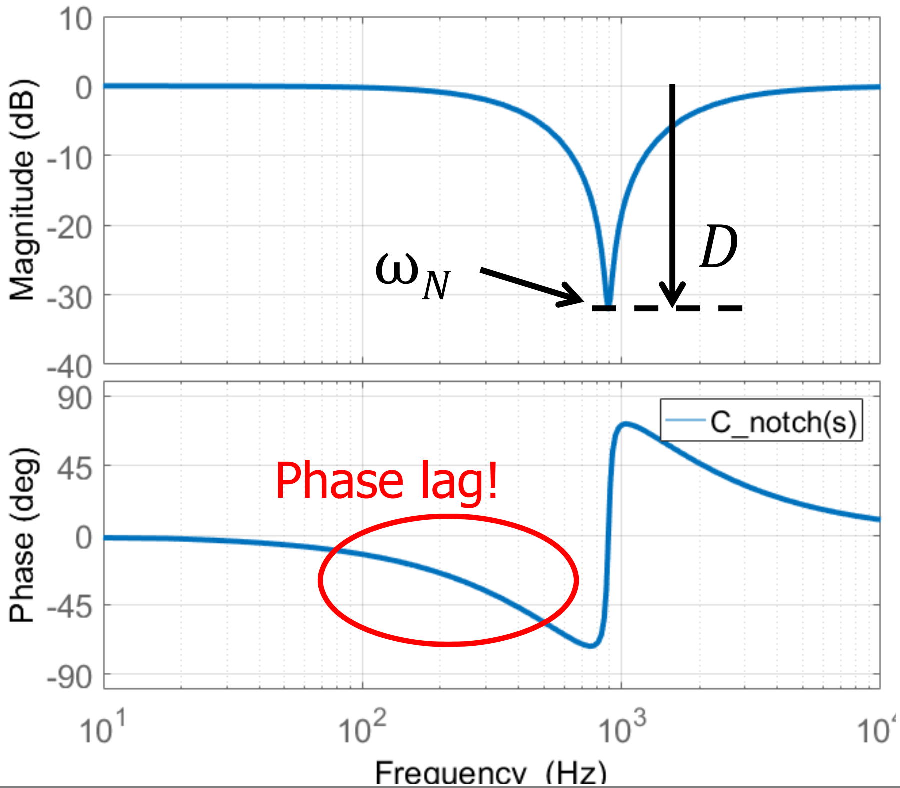


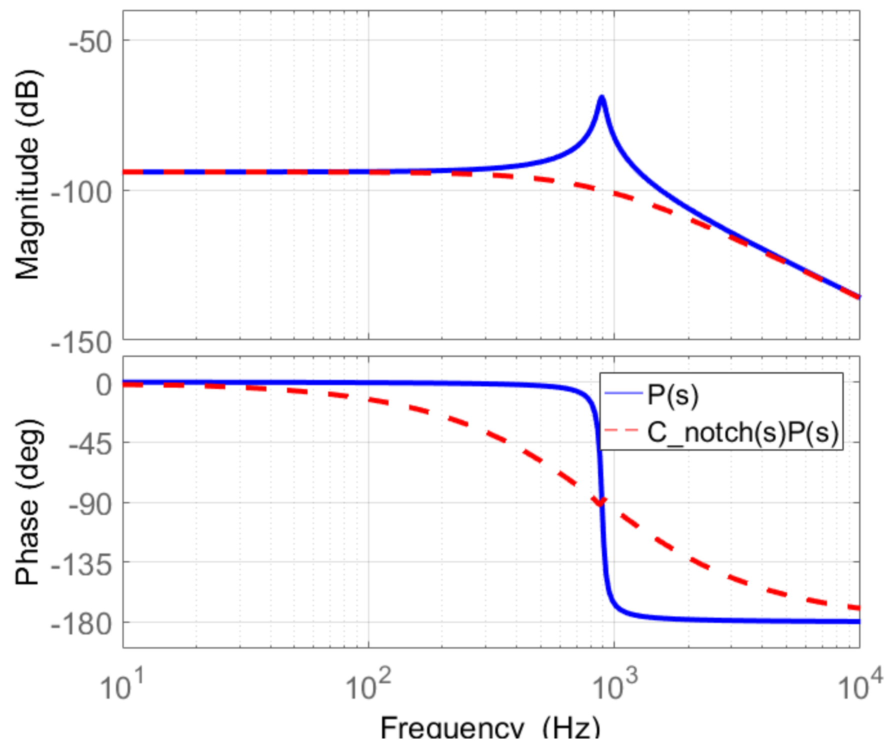


Problem (b)ii: Notch filter

$$C_{notch}(s) = \frac{s^2 + 2D\zeta\omega_N s + {\omega_N}^2}{s^2 + 2\zeta\omega_N s + {\omega_N}^2}$$

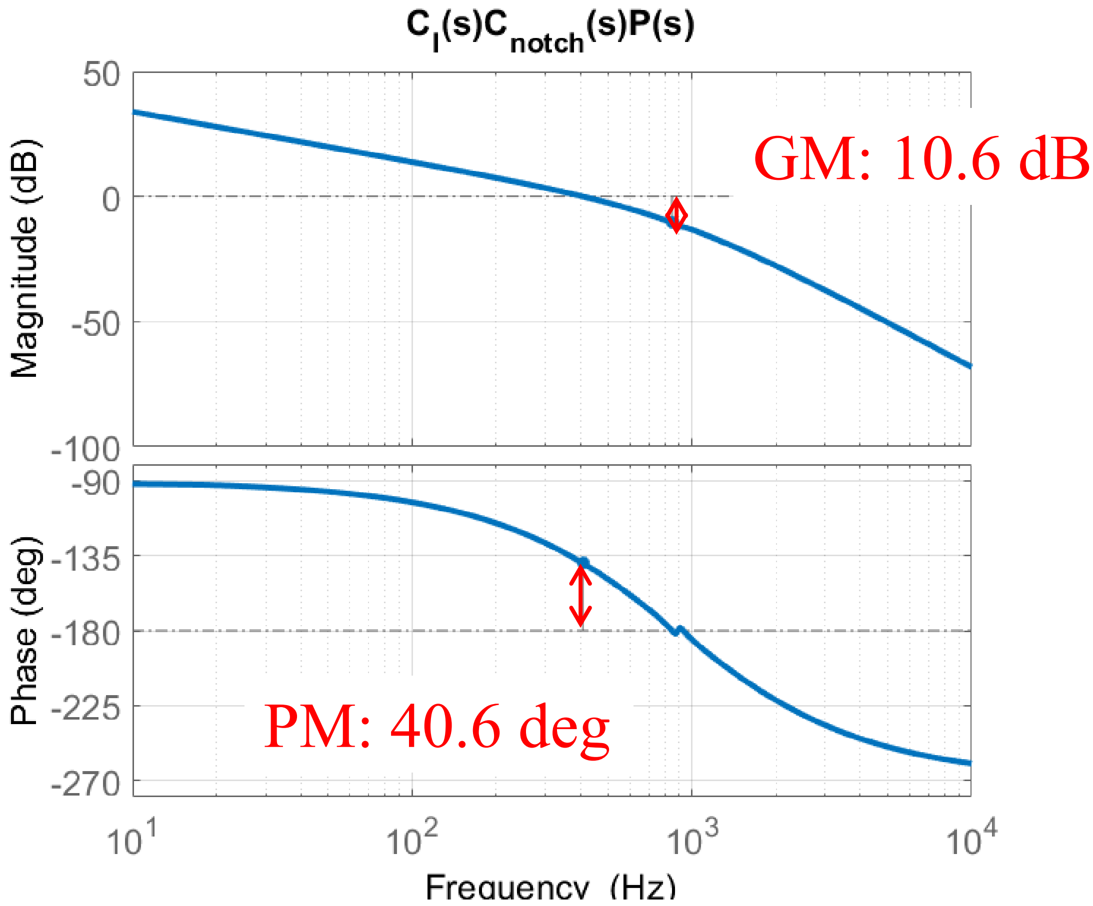
- Notch frequency is set at the resonant frequency.
- Other parameters are tuned: ζ=1, D=0.025=-32 dB



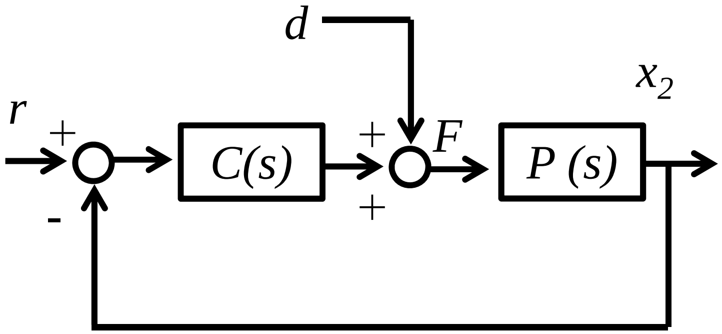


Problem (b)iii: I Controller

- Integral gain is tuned to satisfy the requirements on the gain and phase margin.
- $k_i = 1.55 \times 10^8$
- $\omega_c = 408$ Hz



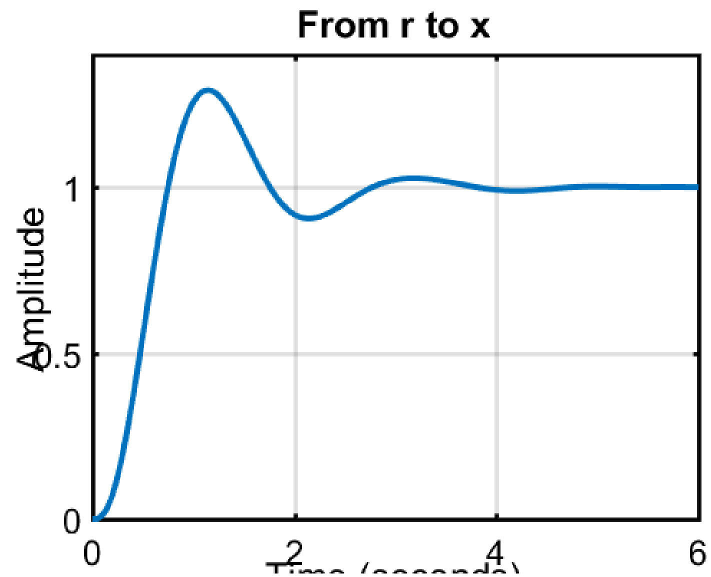
Problem (b)iv: Step responses

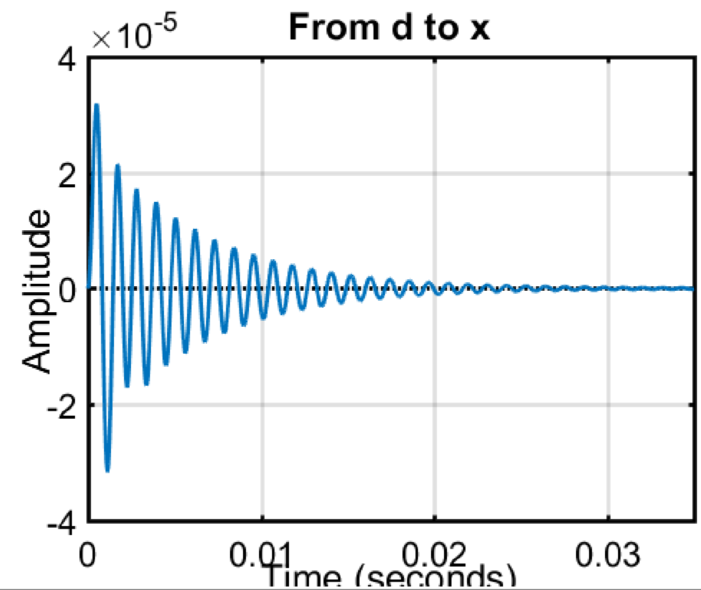


- Transfer function

$$\frac{x(s)}{r(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)}$$
$$\frac{x(s)}{d(s)} = \frac{P(s)}{1 + P(s)C(s)}$$

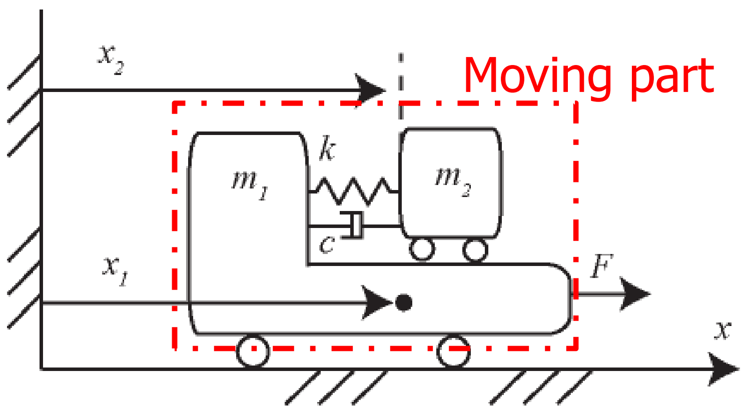
- The notch filter cannot prevents the excitation of the mechanical resonance by the disturbance *d*.

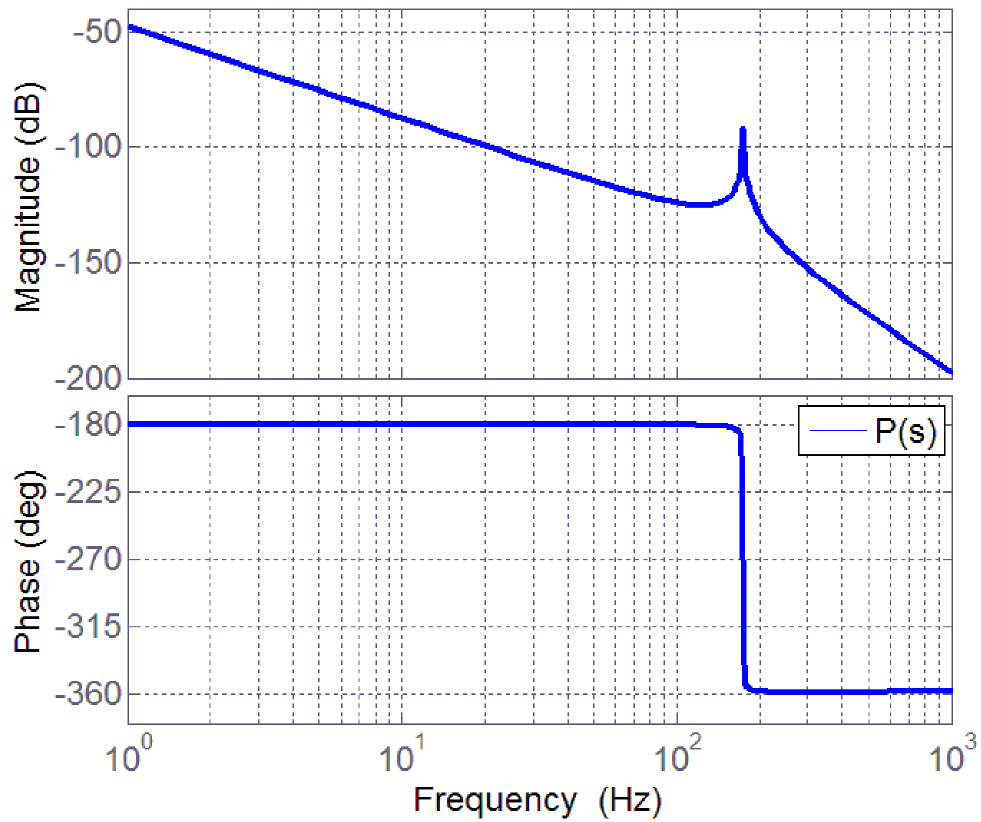




Problem (a)i: Plant model *P(s)*

$$P(s) = \frac{X_2(s)}{F} = \frac{cs + k}{s^2\{m_1 m_2 s^2 + (m_1 + m_2)(cs + k)\}}$$

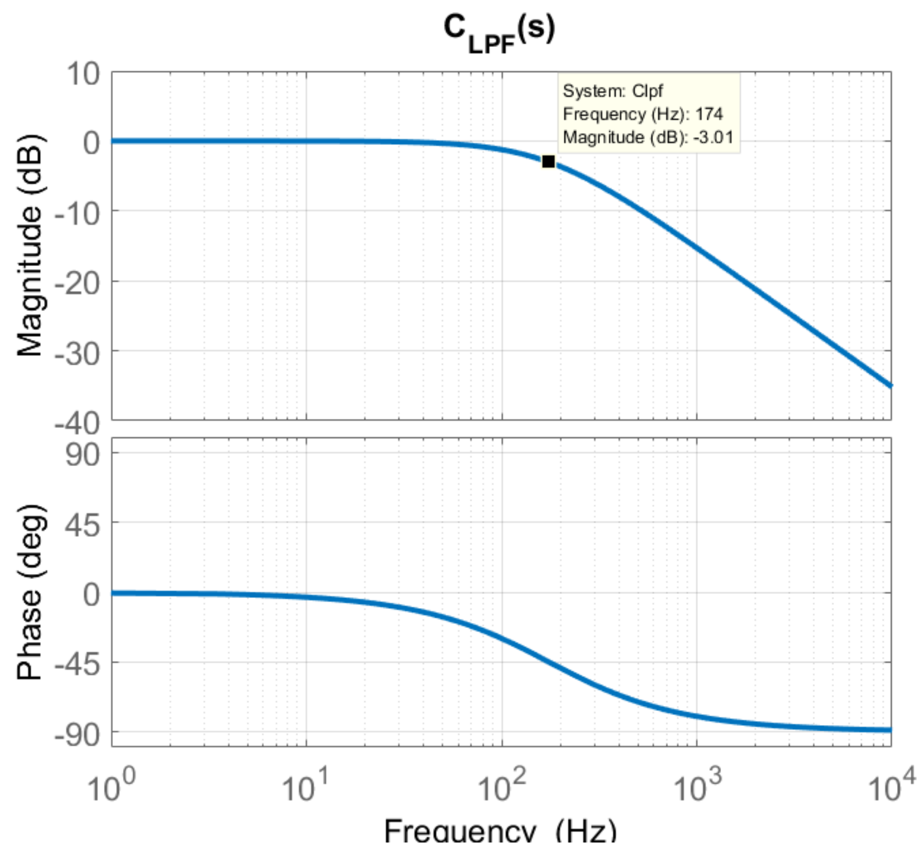




Problem (a)ii: Low-pass filter

$$C_{LPF}(s) = \frac{1}{s/\omega_f + 1}$$

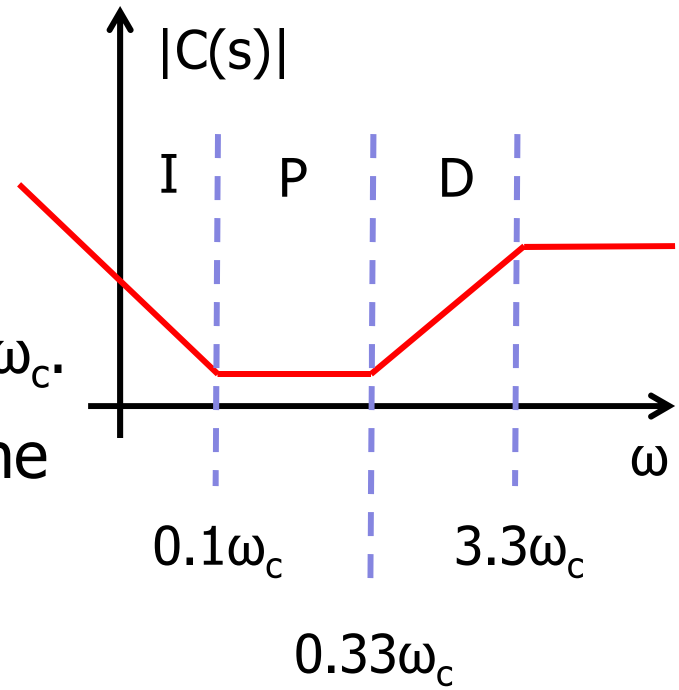
- The phase of the LPF at ω_f is -45 deg.
- The cut-off frequency is set to the resonant frequency.



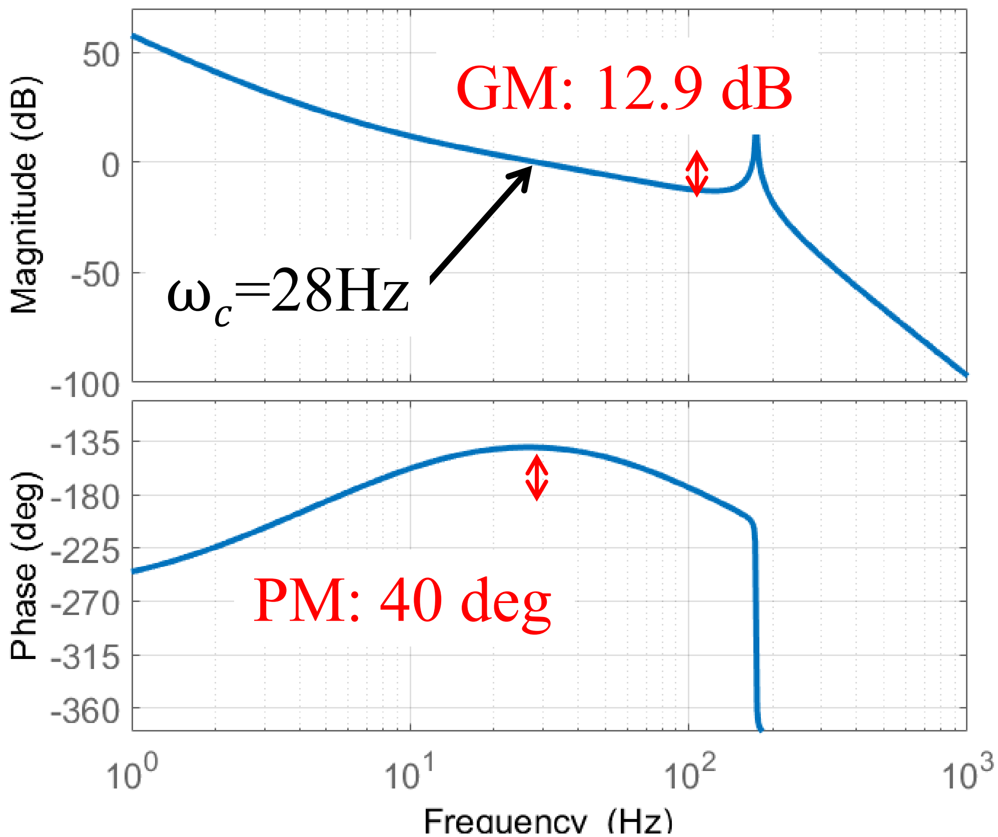
Problem (a)iii: PID Controller (1)

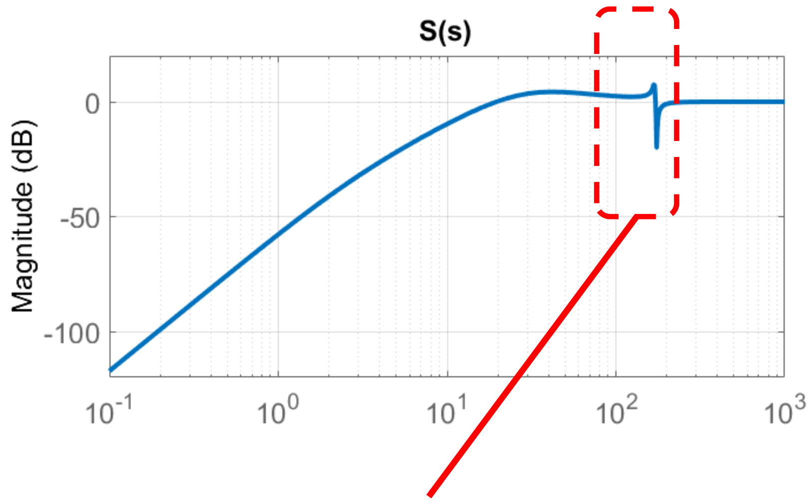
$$C_{PID}(s) = \frac{(s + 0.1\omega_c)\left(\frac{s}{0.33\omega_c} + 1\right)}{s\left(\frac{s}{3.3\omega_c} + 1\right)} 0.33(m_1 + m_2)\omega_c{}^2$$

- Straight-line approximation of a Bode plot
- Controller based on "rule of thumb"
  - The I action terminates at $0.1\omega_c$.
  - The D action starts at $0.33\omega_c$.
  - The cutoff frequency of the LPF is $3.3\omega_c$.
- ω_c is maximized to 28 Hz, satisfying the requirements on the phase and gain margin.



Problem (a)iii: PID Controller (2)





The high gain of the mechanical resonance can be utilized for disturbance rejection or motion tracking.

Problem (a)iv: Step responses

- When the locus of the Nyquist plot is close to (-1, 0), stability and robustness are impaired.
- The low-pass filter shifts the locus, such that it is away from (-1, 0) for stability.
- In the case of a notch filter, the gain is decreased, instead of shifting the phase.