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Mechatronic Systems: Solution of Exercise 4

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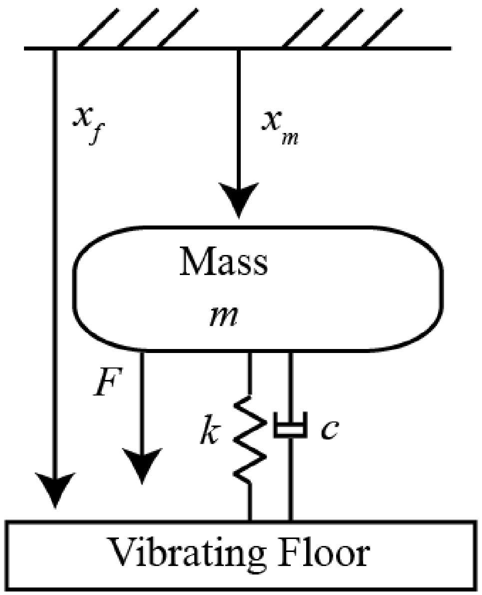
Problem (a)(b)-(i) : Transfer functions

- Differential equation

$$m\ddot{x}_m + c\left(\dot{x}_m - \dot{x}_f\right) + k\left(x_m - x_f\right) = F$$



- Laplace transformation

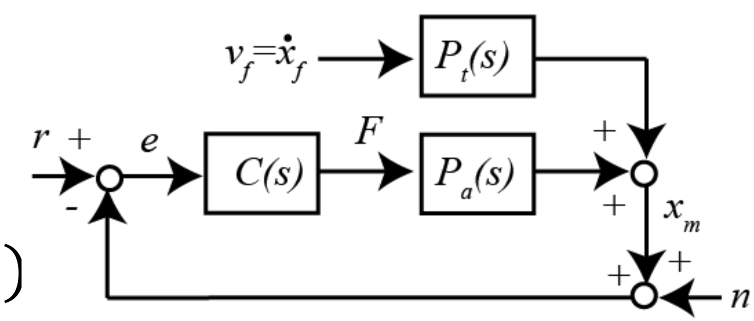
$$x_m = \frac{1}{ms^2 + cs + k} F(s) + \frac{cs + k}{ms^2 + cs + k} \frac{s}{s} x_f$$

$$x_m = \underbrace{\frac{1}{ms^2 + cs + k}}_{P_a(s)} F(s) + \underbrace{\frac{cs + k}{ms^3 + cs^2 + ks}}_{P_t(s)} v_f$$

- Transfer functions

$$P_n(s) = \frac{x_m}{n} = -\frac{C(s)P_a(s)}{1 + C(s)P_a(s)} = -T(s)$$

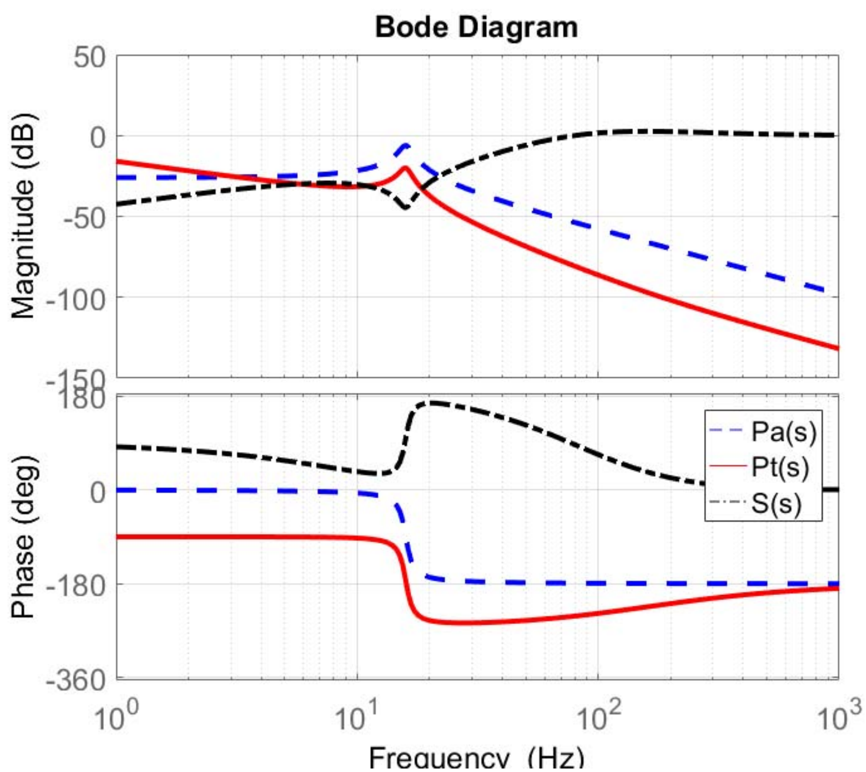
$$P_{vf}(s) = \frac{x_m}{v_f} = \frac{P_t(s)}{1 + C(s)P_a(s)} = P_t(s)S(s)$$

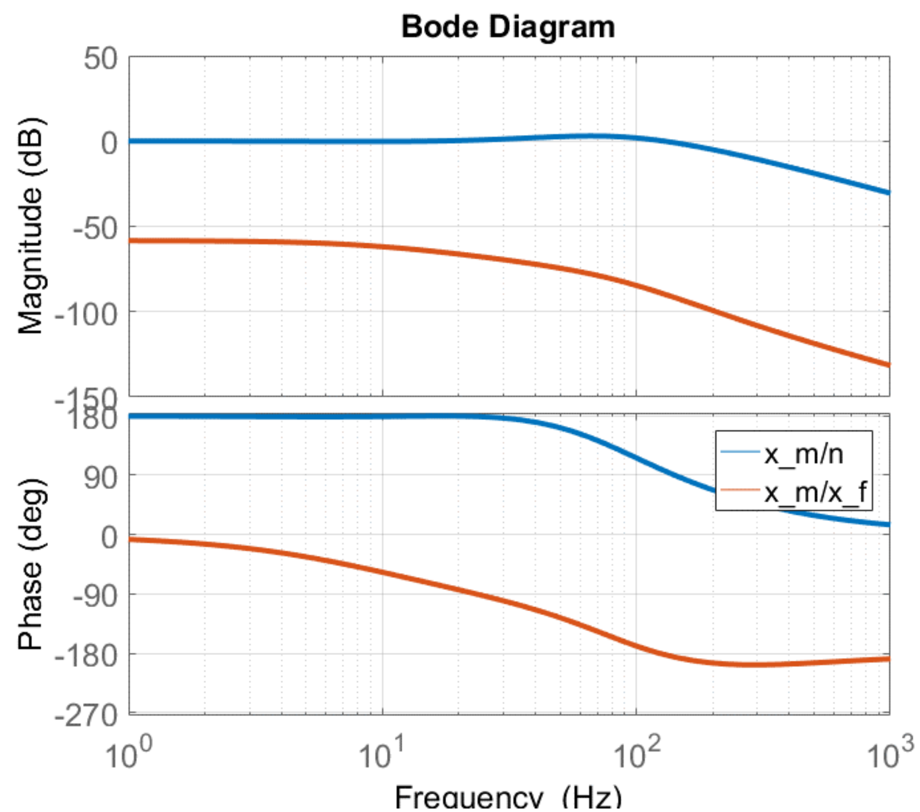


Problem (a)-(ii): Bode plot

$$P_n(s) = \frac{x_m}{n} = -T(s)$$

$$P_{vf}(s) = \frac{x_m}{v_f} = P_t(s)S(s)$$

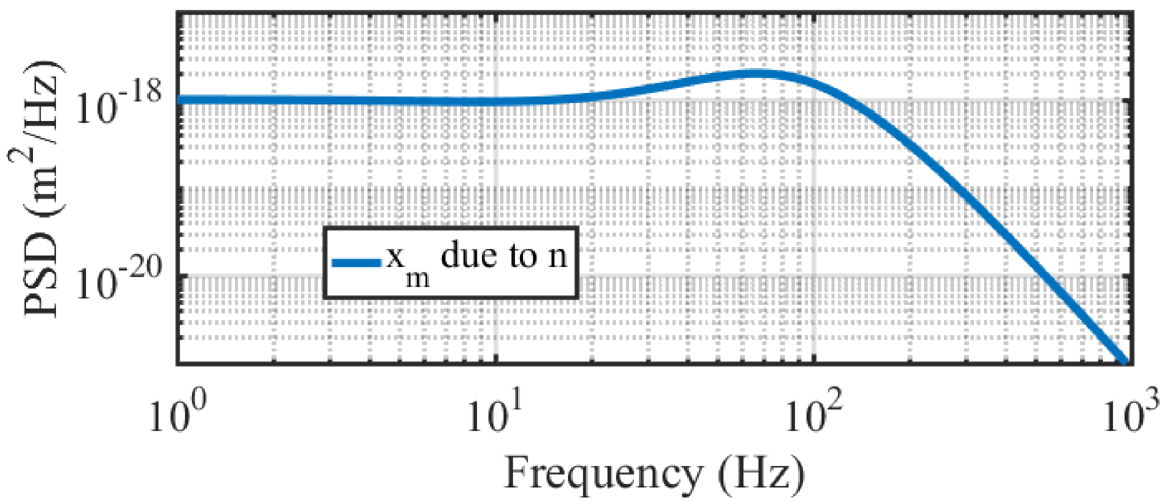




Problem (a)-(iii) (iv): PSD & Resolution

- PSD of x_m

$$PSD_{xm,n}(f) = |P_n(f)|^2 PSD_n(f) \quad (\mathrm{m}^2/\mathrm{Hz})$$
$$= |T(f)|^2 PSD_n(f)$$



$$PSD_n(f) = (1nm/\sqrt{Hz})^2$$

Param.	Description
n	1 nm/$\sqrt{Hz}$

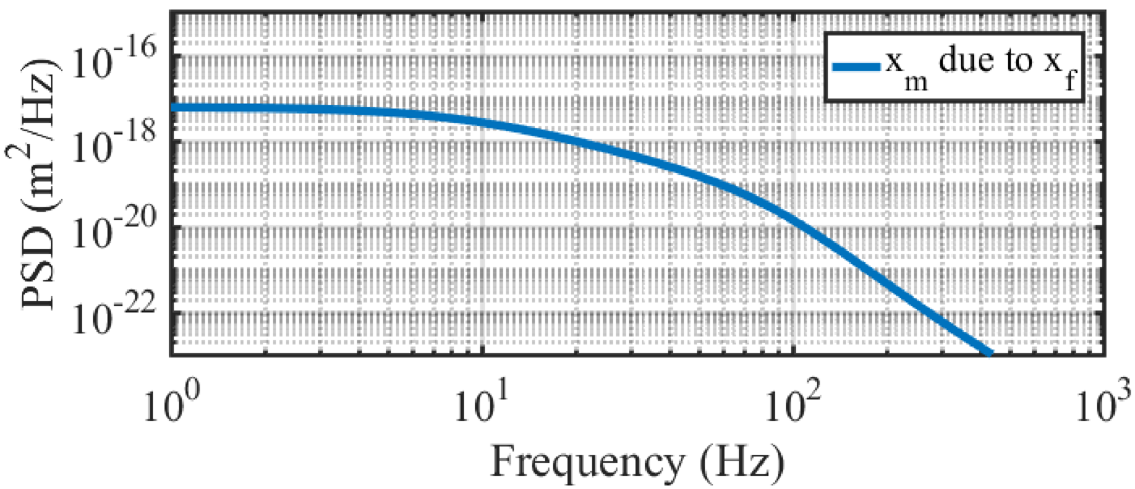
- Resolution (standard deviation)

$$x_{m,n} = \sqrt{\int_{1\mathrm{Hz}}^{1\mathrm{kHz}} PSD_{xm,n}(f)\,\mathrm{d}f} = 16.1\,nm$$

Problem (a)-(v) : Resolution due to v_f

- PSD of x_m

$$PSD_{xm,vf}(f) = \left|P_{vf}(f)\right|^2 PSD_{vf}\,(f) \quad (\mathrm{m}^2/\mathrm{Hz})$$



$$PSD_{vf}(f) = (2(\frac{um}{s})/\sqrt{Hz})^2$$

Param.	Description
v_f	2 (um/s)/$\sqrt{Hz}$

- Resolution (standard deviation)

$$x_{m,vf} = \sqrt{\int_{1\mathrm{Hz}}^{1\mathrm{kHz}} PSD_{xm,vf}(f)\,\mathrm{d}f} = 8.45\,nm$$

Problem (a)-(vi) : Overall resolution

- Total positioning resolution

$$x_m = \sqrt{{x_{m,n}}^2 + {x_{m,vf}}^2} = 18.8\,nm$$

Problem (a) -(vii) : Relation with B/W

- Simulated resolution

ω_c	Resolution		
	$x_{m,n}$	$x_{m,vf}$	x_m
100 Hz	16.1 nm	8.45 nm	18.8 nm
50 Hz	10.1 nm	42.5 nm	43.8 nm

  - By decreeing the control bandwidth, the floor vibration dominates the overall resolution.

Problem (a) -(vii) : Relation with B/W

- PSDs

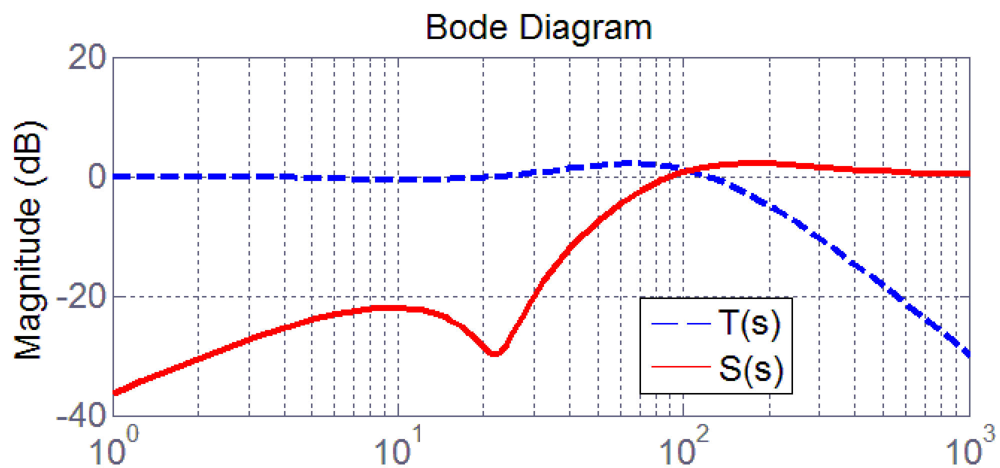
$$PSD_{xm,n}(f) = |P_n(f)|^2 PSD_n\,(f),\;\; PSD_{xm,vf}(f) = \left|P_{vf}(f)\right|^2 PSD_n\,(f)$$

- Transfer functions

$$P_n(s) = \frac{x_m}{n} = -T(s)$$

$$P_{vf}(s) = \frac{x_m}{v} = P_t(s)S(s)$$

$$S(s) + T(s) = 1$$



- Increased control bandwidth
  - Floor vibrations are better rejected, improving resolution.
  - Feedback control pick up more sensor noise degrading resolution.

→ Design trade-off to determine B/W